



A fraction as division

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the fraction-as-division lesson. Item 3 forbids doing the correct calculation, deliberately: the under-or-over-one judgement is the check a child can apply when they cannot remember which way round to divide, which is precisely the moment they need one. “He divided the wrong way round” names the error without giving the child anything reusable. Item 4 asks for a general statement about what a denominator MEANS, for the same reason — a child who is simply told $3\frac{1}{4}$ will substitute the wrong number again next time. Note that item 1 teaches the OPPOSITE reading from the “Multistep problems and remainders” sheet (Grade 4): there the remainder had to stay whole because a bus cannot be cut, here the fraction is the answer because a cake can. The deciding question is whether the thing being shared can be cut, and the two banks are worth putting side by side.

Timing: 5–8 min · independent · paper circles or strips available.

1. 3 cakes between 5 **APPLICATION**

Answer: **less than one cake each — exactly $\frac{3}{5}$ of a cake**

1. There are fewer cakes than children, so each share is less than a whole cake.
2. 3 divided by 5 is $\frac{3}{5}$.
3. Checking: cut each cake into 5, giving 15 pieces, and each child takes 3 of them — and 3 fifths is what 3 pieces of a fifth are.
4. All the cake is used: 5 lots of $\frac{3}{5}$ is 3.

If wrong: Answered “0 remainder 3” or “you cannot”. A cake can be cut, so the fraction IS the answer — the opposite reading from the “Multistep problems and remainders” sheet (Grade 4), where the remainder had to stay whole because a bus cannot be cut.

2. Two fractions as divisions FLUENCY

Answer: **$5/8$ is 5 divided by 8, which is five eighths (or 0.625); $17/5$ is 17 divided by 5, which is $3\frac{2}{5}$**

1. $5/8$ means $5 \div 8$. Since 5 is less than 8 the answer is less than 1: five eighths.
2. $17/5$ means $17 \div 5$. Five goes into 17 three times with 2 left over.
3. The 2 left over is 2 of FIVE pieces, so $2/5$.
4. So $17/5 = 3\frac{2}{5}$.

If wrong: Wrote $8 \div 5$ and $5 \div 17$, taking the larger number first. The $5/8$ case gives 1.6 instead of 0.625, which is more than 1 — impossible for a fraction whose top is smaller than its bottom.

3. Tom's $1\frac{1}{3}$ REASONING

Answer: **because $1\frac{1}{3}$ is more than one sandwich each, and there are only 3 sandwiches for 4 children — four children with more than a sandwich each would need more than 4 sandwiches**

1. There are fewer sandwiches than children, so nobody can get a whole one.
2. Tom's answer gives each child more than a whole sandwich.
3. Four children at $1\frac{1}{3}$ each would need $5\frac{1}{3}$ sandwiches, and only 3 exist.
4. So the answer is impossible from the situation alone, before any calculation.
5. His error was dividing 4 by 3 instead of 3 by 4 — understandable, since every earlier division put the larger number first.

If wrong: Said Tom “divided the wrong way round”. That names the error and the item asked for why the ANSWER is impossible — the under-or-over-one judgement is the check that works when a child cannot remember which way round to divide, which is exactly when they need it.

4. Priya's $3\frac{1}{4}$ REASONING

Answer: **the bottom number says how many the whole is being shared between, not how many are left over — 4 children means quarters, so the answer is $3\frac{1}{4}$**

1. 13 apples between 4 children: 3 each uses 12, leaving 1 apple.
2. That 1 apple is shared between the same 4 children, so it is cut into 4 pieces and each child gets one of them — a quarter.
3. So the answer is $3\frac{1}{4}$.
4. Priya put the 1 over the 3, which is how many apples each child already had. The bottom of a fraction is always the number of equal parts the whole was cut into.
5. Check: 4 lots of $3\frac{1}{4}$ is 13. Four lots of $3\frac{1}{3}$ is $13\frac{1}{3}$, which is more apples than there were.

If wrong: Gave $3\frac{1}{4}$ without saying what the denominator means. The item asks for the general statement, because a child who fixes this one answer by being told will make the same substitution the next time a remainder appears.

Quick answers for fast checking:

#	Level	Answer
1	application	less than one; $\frac{3}{5}$ of a cake
2	fluency	$5 \div 8 = \frac{5}{8}$; $17 \div 5 = 3\frac{2}{5}$
3	reasoning	more than one each from only 3 for 4 — impossible
4	reasoning	the bottom is how many are sharing; $3\frac{1}{4}$

Practice Worksheet — answer key

Use: In-class practice after the deck. The check that runs through this whole sheet is comparing a fraction's top with its own bottom: smaller top means under one, larger top means over one. It settles items 1, 5 and 8 with no calculation at all, and item 8 is worth taking to the board because most children will find a common denominator when a glance would do. Item 6 is the sheet's most important item and it pairs directly with the “Multistep problems and remainders” sheet (Grade 4) — the same division, two interpretations, and the CONTEXT decides which. A child who can only say “you cannot cut a bus” has the example and not the principle. Item 7 asks for the cutting argument rather than the notation: “the bar means divide” is what the symbol says, and the 12 quarter-pieces is why it is true.

Timing: 15–20 min · pairs · paper strips · items 6, 7 and 8 on the board.

1. Four fractions as divisions FLUENCY

Answer: $1 \div 2 = \frac{1}{2}$; $2 \div 3 = \frac{2}{3}$; $3 \div 4 = \frac{3}{4}$; $5 \div 8 = \frac{5}{8}$

1. Each one is top divided by bottom.
2. In every case the top is smaller than the bottom, so every answer is less than 1.
3. As decimals: 0.5, about 0.67, 0.75 and 0.625.

If wrong: Reversed one or more of them, giving answers above 1. Whenever the top is smaller than the bottom the answer must be under 1, which catches every reversal at once.

2. Three improper fractions FLUENCY

Answer: $3\frac{1}{4}$, $3\frac{2}{5}$ and $4\frac{1}{2}$

1. $13 \div 4$: 3 each, 1 left, and the 1 is a quarter. So $3\frac{1}{4}$.
2. $17 \div 5$: 3 each, 2 left, and the 2 is $\frac{2}{5}$. So $3\frac{2}{5}$.
3. $9 \div 2$: 4 each, 1 left, and the 1 is a half. So $4\frac{1}{2}$.
4. In each, the leftover goes over the number that is sharing.

If wrong: Answered $3\frac{1}{5}$ for the first — the leftover put over the quotient instead of the divisor. Checking backwards catches it: 4 lots of $3\frac{1}{5}$ is $13\frac{4}{5}$, which is more than 13.

3. 4 pizzas between 6 APPLICATION

Answer: **less than one each — $4/6$, which is $2/3$ of a pizza**

1. Fewer pizzas than people, so each share is under one whole.
2. $4 \div 6 = 4/6 = 2/3$.
3. Check: 6 lots of $2/3$ is 4, so all the pizza is used.

If wrong: Answered $6/4$ or $1\frac{1}{2}$, taking the larger number first — more than a pizza each from four pizzas for six people.

4. 7 metres into 4 lengths APPLICATION

Answer: **$1\frac{3}{4}$ m each**

1. $7 \div 4 = 7/4$.
2. 4 goes into 7 once, with 3 left over.
3. The 3 metres left are shared between 4 lengths, so $3/4$ m.
4. Each length is $1\frac{3}{4}$ m. Check: $4 \times 1\frac{3}{4} = 7$.

If wrong: Answered 1 remainder 3, leaving it there. Ribbon can be cut, so the fraction is the answer — and “1 metre remainder 3” does not tell anyone how long to cut.

5. $5/8$ is not 1.6 REASONING

Answer: **because $5/8$ has a smaller top than bottom, so it is less than one whole — and 1.6 is more than one**

1. 5 shared between 8 means fewer things than sharers, so each share is under 1.
2. 1.6 is more than 1, so it cannot be the answer.
3. No division was needed to see this — only the comparison of 5 with 8.
4. The child divided 8 by 5. The right way round is $5 \div 8 = 0.625$, which is under 1 as required.

If wrong: Did the division correctly and compared. That works and the item asked for a reason without dividing, because comparing the top with the bottom is the check that is available instantly and always.

6. Litres against buses REASONING

Answer: **because litres can be cut and buses cannot — a fraction answer is only meaningful when the thing being shared can be divided into parts**

1. 9 litres between 2 buckets: half a litre is a real amount of water, so $4\frac{1}{2}$ litres each is a genuine answer.
2. 9 buses between 2 schools: half a bus is not a bus, so the answer has to be whole — 4 each with 1 left to be allocated somehow.
3. So the arithmetic is the same and the INTERPRETATION differs.
4. The question to ask is always whether the thing can be cut, which is what the “Multistep problems and remainders” sheet (Grade 4) was about from the other direction.

If wrong: Said “buses are bigger” or “you cannot cut a bus in half” without the general point. The general point is the useful one: the same division needs a different interpretation depending on whether the thing is divisible, and it is the context that decides.

7. Why $3/4$ is $3/4$ REASONING

Answer: **because cutting each of the 3 into 4 pieces gives 12 pieces, each a quarter, and each of the 4 shares is 3 of those pieces — which is 3 quarters**

1. Cut all 3 wholes into quarters: $3 \times 4 = 12$ quarter-pieces.
2. Share the 12 pieces between 4 children: $12 \div 4 = 3$ pieces each.
3. Each piece is a quarter, so 3 pieces is $3/4$.
4. So the sharing and the fraction are the same thing described twice — the fraction notation records the sharing rather than merely agreeing with it.

If wrong: Said “because the fraction bar means divide”. That is the notation and not the reason; the cutting argument is what shows the two are the same act, and it is what makes item 6 answerable.

8. $\frac{7}{9}$ or $\frac{9}{7}$ REASONING

Answer: **$\frac{9}{7}$ is bigger, because its top is larger than its bottom so it is more than one, while $\frac{7}{9}$ has a smaller top so it is less than one**

1. $\frac{7}{9}$ is 7 shared between 9 — fewer things than sharers, so under 1.
2. $\frac{9}{7}$ is 9 shared between 7 — more things than sharers, so over 1.
3. Anything under 1 is smaller than anything over 1, so $\frac{9}{7}$ wins.
4. No common denominator and no division were needed: comparing each top with its own bottom was enough.

If wrong: Converted both to a common denominator ($\frac{49}{63}$ and $\frac{81}{63}$). Correct, and far more work than needed — the against-one comparison settles any pair where one is under and the other over.

9. Why it is useful REASONING

Answer: **because it means a sharing problem needs no new method, and a fraction can be turned into a decimal whenever that is more convenient**

1. Any “share these between those” question is answered by writing one number over the other — no procedure to recall.
2. It explains why improper fractions and mixed numbers are the same thing: $\frac{13}{4}$ is just $13 \div 4$ finished off.
3. It gives a route to decimals: $\frac{5}{8}$ is $5 \div 8 = 0.625$, which is how any fraction becomes a decimal.
4. And it makes the size of a fraction judgeable at a glance — top smaller than bottom means under one, which is the check used all over this sheet.

If wrong: Said “so you can work out fractions”. The concrete answers are better: no new method for sharing, and a route from any fraction to a decimal.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$ — all under one
2	fluency	$3\frac{1}{4}$, $3\frac{2}{5}$, $4\frac{1}{2}$
3	application	less than one; $\frac{2}{3}$ of a pizza
4	application	$1\frac{3}{4}$ m
5	reasoning	top smaller than bottom, so under one
6	reasoning	the context decides whether the thing can be cut
7	reasoning	12 quarter-pieces, 3 each
8	reasoning	$\frac{9}{7}$ is over one, $\frac{7}{9}$ under

9	reasoning	no new method for sharing; a route to decimals
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Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	The word to insist on is IS, not “equals”. A fraction and a division are the same object with two notations, and children who hold that need no separate method for sharing problems.
2/8	0:40	Ask for the under-or-over-one judgement BEFORE any method. Three shared among four is less than one each, and saying so first kills the reversed division without any rule about order.
3/8	1:20	THE slide. Do the cutting physically or on the board: 3 sandwiches, 4 cuts each, 12 pieces, 3 each. Then say the sentence — the fraction was the sharing all along.
4/8	2:20	The reversed answer. Judge it against the sandwiches rather than against the correct answer: more than one each, out of three, for four children. It fails on the situation, not on a rule.
5/8	3:10	Worth a moment of sympathy: every division these children have done put the larger number first, so reaching for $4 \div 3$ is a reasonable generalisation. Then the rule: top divided by bottom, always.
6/8	4:00	Move to answers above one. 13 shared among 4. Get the 3 each and the 1 left, then ask what to do with the leftover — it gets cut into 4, because 4 is who is sharing.
7/8	5:00	Name the $3\frac{1}{3}$ error explicitly. The denominator is WHO IS SHARING, never how many are left over. Children who have just met remainders are especially prone to this.
8/8	6:15	3 cakes between 5 is the item that needs a fraction answer to be a real answer. Note for later: the “Multistep problems and remainders” sheet (Grade 4) had the opposite lesson, because you cannot have part of a bus. Exit ticket after.