



Adding and subtracting unlike denominators

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the unlike-denominators lesson. Item 3 asks for the reasonableness check BEFORE the method, and that ordering is the assessment — the standard's word-problem half names reasonableness explicitly, and it is the only check available to a child who has no correct answer to compare with. The sentence to look for is that $2/5$ is less than $1/2$, so adding a positive amount produced less than what you started with. This is the additive instinct's fifth appearance: the “Multiplicative comparison” sheet (Grade 4), the “Equivalent fractions” sheet (Grade 4), the “Relating two number patterns” sheet (Grade 5), then G6 ratios and the “Similarity and dilations” sheet (Grade 8). Item 4 is a different kind of error and should be marked differently — Nell found the common denominator, which is the hard half, and her $2/6$ equals one of the fractions she started with, so the sum added nothing. Item 2's $2\frac{1}{2}$ is `smaller\_from\_larger` in fractions: the same misconception the helper names in `bank_m3_nbt_02.py` and `bank_m4_nbt_04.py`, and it is why the helper keeps that name here.

Timing: 5–10 min · independent · fraction strips available.

1. A sum, with its range **APPLICATION**

Answer: **between $2/3$ and 1, giving $11/12$**

1. Adding a positive amount to $2/3$, and $1/4$ is small, so the answer is between $2/3$ and 1.
2. In twelfths: $8/12 + 3/12 = 11/12$.
3. $11/12$ is in range, which is the check the range was written for.

If wrong: Answered $3/7$ — tops and bottoms added. $3/7$ is less than $2/3$, which the range written down would have caught immediately.

2. A difference, with its range APPLICATION

Answer: **between $\frac{1}{3}$ and $\frac{5}{6}$, giving $\frac{1}{2}$**

1. Taking away, so the answer is less than $\frac{5}{6}$; and $\frac{1}{3}$ is less than $\frac{5}{6}$, so it stays positive.
2. In sixths: $\frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.
3. $\frac{1}{2}$ is in range.

If wrong: Answered $\frac{4}{3}$ by subtracting across — bigger than $\frac{5}{6}$, which the range rejects on sight.

3. Three and a quarter minus one and three quarters FLUENCY

Answer: **$1\frac{1}{2}$**

1. Convert: $3\frac{1}{4}$ is $\frac{13}{4}$, and $1\frac{3}{4}$ is $\frac{7}{4}$.
2. $\frac{13}{4} - \frac{7}{4} = \frac{6}{4}$.
3. $\frac{6}{4} = 1\frac{1}{2}$.
4. Check: 3 and a bit less 1 and a bit should be about $1\frac{1}{2}$, and it is.

If wrong: Answered $2\frac{1}{2}$, taking $\frac{1}{4}$ from $\frac{3}{4}$ instead of the other way round because $\frac{1}{4} - \frac{3}{4}$ looked impossible. $2\frac{1}{2}$ is more than $3 - 1$, which is impossible for this subtraction.

4. Eli's $\frac{2}{5}$ REASONING

Answer: **$\frac{2}{5}$ is less than $\frac{1}{2}$, and adding a positive amount to $\frac{1}{2}$ must give more than $\frac{1}{2}$; Eli added the tops and the bottoms, which counts pieces of different sizes as though they were the same**

1. Without calculating: $\frac{1}{3}$ is positive, so $\frac{1}{2} + \frac{1}{3}$ must be MORE than $\frac{1}{2}$. And $\frac{1}{3}$ is less than $\frac{1}{2}$, so the total is less than 1. The answer lies between $\frac{1}{2}$ and 1.
2. $\frac{2}{5}$ is 0.4 and $\frac{1}{2}$ is 0.5, so $\frac{2}{5}$ is below the range — rejected.
3. Eli added $1 + 1$ for the top and $2 + 3$ for the bottom. That treats halves and thirds as if they were the same-sized pieces, and they are not, so there is nothing to count yet.
4. The fix: make the pieces the same size. $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$, so the total is $\frac{5}{6}$, which is in the range.

If wrong: Showed $\frac{5}{6}$ is the right answer and stopped. The item asks for the check FIRST — rejecting $\frac{2}{5}$ without calculating — because that is the skill the standard names, and it works when a child has no correct answer to compare against.

5. Nell's half-finished conversion REASONING

Answer: **she needed to change the numerators by the same factor as the denominators: $1/2$ becomes $3/6$ and $1/3$ becomes $2/6$, because smaller pieces means you need more of them**

1. Going from halves to sixths cuts each piece into 3, so one half becomes THREE sixths, not one.
2. Going from thirds to sixths cuts each piece into 2, so one third becomes TWO sixths.
3. So the sum is $3/6 + 2/6 = 5/6$.
4. Nell's $2/6$ is $1/3$ — the same as one of the numbers she started with, so nothing was added at all. That is the give-away.
5. She had done the harder part right: 6 is the correct common denominator.

If wrong: Said Nell “forgot to change the tops”. True, and the reason is what stops it recurring: the pieces got SMALLER, so more of them are needed to make the same amount — the same argument as the “Equivalent fractions” sheet (Grade 4).

Quick answers for fast checking:

#	Level	Answer
1	application	$11/12$, between $2/3$ and 1
2	application	$1/2$, between $1/3$ and $5/6$
3	fluency	$1\frac{1}{2}$
4	reasoning	$2/5 < 1/2$, so out of range; different-sized pieces counted together
5	reasoning	numerators scale by the same factor; smaller pieces, more of them

Practice Worksheet — answer key

Use: In-class practice after the deck. Items 1 and 2 are worth marking as SETS: a child who adds across gets three answers each smaller than one of the fractions added, and a child who subtracts across gets three each bigger than the number they started with. In both cases one range check would have caught all three, which is the argument for making the check habitual rather than occasional. Item 5 asks for two INDEPENDENT reasons and most children give one twice — the second, bounding the answer above as well as below, is the one that works for any pair. Item 9 is the sheet's best item: “find a common denominator” never mentions the numerators, which is precisely why it can be half-followed and still feel finished, and that is the Nell error from the exit ticket. Item 4 quietly checks that equivalence from the “Equivalent fractions” sheet (Grade 4) survived — a child who thinks $10/12$ and $5/6$ are different answers has lost it.

Timing: 20–25 min · pairs · fraction strips · items 5, 8 and 9 on the board.

1. Both denominators change FLUENCY

Answer: **5/6**

1. $1/2 + 1/3$: sixths take both, so $3/6 + 2/6 = 5/6$.
2. BOTH fractions had to change, because neither 2 nor 3 is a multiple of the other.
3. $5/6$ is bigger than both, as any sum of positives must be.

If wrong: Answered $2/5$ — tops and bottoms added. $2/5$ is SMALLER than $1/2$, so the range check rejects it before any arithmetic.

2. Both denominators change again FLUENCY

Answer: **11/12**

1. $2/3 + 1/4$: twelfths take both, so $8/12 + 3/12 = 11/12$.
2. $11/12$ is bigger than $2/3$, and just under 1.

If wrong: Answered $3/7$ — tops and bottoms added. $3/7$ is less than $2/3$, which the range check catches.

3. Only one denominator changes FLUENCY

Answer: **5/8**

1. $1/4 + 3/8$: eighths already take quarters, so only $1/4$ changes — $2/8 + 3/8 = 5/8$.
2. THIS IS THE ONE WORTH NOTICING: 8 is a multiple of 4, so one fraction is already in the right unit. Item 13 asks why.

If wrong: Answered $4/12$ — tops and bottoms added, and smaller than $3/8$. Or changed both unnecessarily to 24ths, which is correct but more work.

4. Halves into quarters FLUENCY

Answer: **1/4**

1. $3/4 - 1/2 = 3/4 - 2/4 = 1/4$.
2. Only one fraction had to change, because 4 is a multiple of 2.
3. The answer is less than $3/4$, as a subtraction must be.

If wrong: Answered $2/2$ by subtracting across. That is 1, which is BIGGER than the $3/4$ started with — impossible for a subtraction.

5. Thirds into sixths FLUENCY

Answer: $\frac{1}{2}$

1. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$, which is $\frac{1}{2}$.
2. Only the thirds had to change.
3. Worth naming the simplification: $\frac{3}{6}$ and $\frac{1}{2}$ are the same number.

If wrong: Answered $\frac{4}{3}$ by subtracting across — bigger than $\frac{5}{6}$, so impossible. Or left it as $\frac{3}{6}$, which is correct but unsimplified.

6. Quarters into eighths FLUENCY

Answer: $\frac{5}{8}$

1. $\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$.
2. Only the quarters had to change, because 8 is a multiple of 4.

If wrong: Answered $\frac{6}{4}$ by subtracting across — that is $1\frac{1}{2}$, bigger than $\frac{7}{8}$.

7. A mixed-number sum APPLICATION

Answer: $3\frac{5}{6}$

1. The wholes give 3.
2. $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$.
3. So $3\frac{5}{6}$.
4. Check: about $2\frac{1}{2} + 1\frac{1}{2} = 4$, and $3\frac{5}{6}$ is close.

If wrong: Added the fractions across as $\frac{2}{5}$, giving $3\frac{2}{5}$. The estimate of 4 rejects it.

8. A mixed-number difference APPLICATION

Answer: $1\frac{1}{2}$

1. Convert: $3\frac{1}{4} = \frac{13}{4}$ and $1\frac{3}{4} = \frac{7}{4}$.
2. $\frac{13}{4} - \frac{7}{4} = \frac{6}{4} = 1\frac{1}{2}$.
3. Check: 3 and a bit less 1 and a bit is about $1\frac{1}{2}$.

If wrong: Answered $2\frac{1}{2}$, taking $\frac{1}{4}$ from $\frac{3}{4}$ instead of $\frac{3}{4}$ from $\frac{1}{4}$. It exceeds $3 - 1$, so the estimate rejects it.

9. Using twelfths REASONING

Answer: **yes — $6/12 + 4/12 = 10/12$, which is the same amount as $5/6$**

1. $1/2 = 6/12$ and $1/3 = 4/12$, so the sum is $10/12$.
2. $10/12 = 5/6$, because both parts divide by 2.
3. Twelve works because it is a multiple of both 2 and 3 — that is the only requirement.
4. So the common denominator does not have to be the SMALLEST one; it only has to be common. Using twelfths just means the answer may need simplifying.

If wrong: Said $10/12$ is a different answer from $5/6$. They are the same amount written with different-sized pieces, which is the “Equivalent fractions” sheet (Grade 4)’s whole point — and worth checking, since a child who thinks they differ has not carried equivalence forward.

10. Two reasons $2/5$ is wrong REASONING

Answer: **first, $2/5$ is less than $1/2$, and adding a positive amount cannot reduce a number; second, the answer must be less than 1 but more than $1/2$, and $2/5$ is outside that range**

1. $1/2$ is 0.5 and $2/5$ is 0.4, so the answer came out smaller than one of the things being added.
2. Adding $1/3$ to $1/2$ must give more than $1/2$.
3. Also, since $1/3 < 1/2$, the total is less than $1/2 + 1/2 = 1$. So the answer lies strictly between $1/2$ and 1.
4. $2/5$ fails both tests, and neither test needed the addition to be done.

If wrong: Gave one reason twice in different words. Two INDEPENDENT reasons is the ask, and the second — bounding the answer above as well as below — is the one that generalises to any pair of fractions.

11. Ravi's walk APPLICATION

Answer: **about $1\frac{1}{2}$ miles; exactly $1\frac{5}{12}$ miles**

1. Estimate: $3/4$ is nearly 1 and $2/3$ is a bit more than a half, so about $1\frac{1}{2}$.
2. Exactly: $3/4 = 9/12$ and $2/3 = 8/12$, so $17/12$.
3. $17/12 = 1\frac{5}{12}$ miles.
4. Compare with the estimate: $1\frac{5}{12}$ is close to $1\frac{1}{2}$, so the answer is reasonable.

If wrong: Answered $5/7$ by adding across — less than either distance, so Ravi would have finished nearer the start than after his first walk.

12. Jo's litre REASONING

Answer: **6/6 is one whole litre, which is more than she started with — the answer is 3/8**

1. Jo had $7/8$ of a litre, which is less than 1.
2. She poured some away, so the answer must be less than $7/8$.
3. $6/6 = 1$, which is more than $7/8$ — impossible after pouring away.
4. Correctly: $7/8 - 1/2 = 7/8 - 4/8 = 3/8$ of a litre.
5. She subtracted across: $7 - 1 = 6$ and $8 - 2 = 6$, giving $6/6$.

If wrong: Found $3/8$ without showing why $6/6$ is impossible. Noticing that $6/6$ is a whole litre — more than she began with — is the check, and it needs no calculation at all.

13. One fraction or both REASONING

Answer: **because 8 is already a multiple of 4, so quarters can be cut into eighths and the eighths are left alone; 2 and 3 are not multiples of each other, so neither halves nor thirds can be used and both must change**

1. $1/4 + 3/8$: eighths are made by cutting quarters in half, so $1/4 = 2/8$ and $3/8$ stays as it is.
2. $1/2 + 1/3$: thirds cannot be made from halves, and halves cannot be made from thirds.
3. So a NEW size is needed that both can be cut into — sixths — and both fractions change.
4. The test is whether one denominator divides the other. If it does, one fraction changes; if not, both do.

If wrong: Said “because 4 and 8 are close together”. The property is divisibility, not closeness: 3 and 9 are further apart than 4 and 6, and $3/9$ needs only one change while quarters and sixths need two.

14. The better reason REASONING

Answer: **because it says WHY the denominators must match, so a child can work out what to do in a case they have not been shown — while “find a common denominator” is an instruction that can be followed without understanding, and half-followed, as when the tops are left unchanged**

1. “Different-sized pieces cannot be counted together” explains the whole procedure, including why the numerators must change too.
2. “Find a common denominator” does not mention the numerators at all, which is exactly the error where a child writes $1/6 + 1/6$.
3. It also explains why the common denominator need not be the smallest: any size both can be cut into will do.
4. And it transfers — the same idea explains comparing fractions (the “Comparing fractions” sheet (Grade 4)) and, later, adding algebraic fractions.

If wrong: Said “because it is easier to understand”. The specific answer is that the piece-size reason covers the NUMERATORS, and the common-denominator instruction does not — which is why the instruction can be followed halfway and still feel complete.

Quick answers for fast checking:

#	Level	Answer
1	fluency	5/6
2	fluency	11/12
3	fluency	5/8
4	fluency	1/4
5	fluency	1/2
6	fluency	5/8
7	application	3$\frac{5}{6}$
8	application	1$\frac{1}{2}$
9	reasoning	10/12 = 5/6; any common multiple works
10	reasoning	below 1/2; and outside the 1/2-to-1 range
11	application	about 1$\frac{1}{2}$; exactly 1 $\frac{5}{12}$ miles
12	reasoning	6/6 is a whole litre, more than she started with
13	reasoning	whether one denominator divides the other
14	reasoning	the piece-size reason covers the numerators; the instruction does not

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	The reasonableness check named in the standard is the spine of this lesson, not an afterthought. Establish it before any method, because it is what rejects $2/5$ without needing the method at all.
2/8	0:40	Write $2/5$ up without comment and ask whether it can be right. Do not say it is wrong. Someone will notice it is less than a half, and that is a far better moment than a correction.
3/8	1:30	THE slide. $1/3$ is less than $1/2$, so the sum is between $1/2$ and 1. Have the class say the range aloud for every sum in the lesson before calculating — it is the habit the standard is asking for and it takes seconds.
4/8	2:20	Different-sized pieces, nothing to count. Use paper strips or a bar model; the physical version is what makes “common denominator” a description rather than an instruction.
5/8	3:20	Do the conversion and the addition. Then compare with the predicted range out loud, every time, so the check closes the loop instead of being abandoned once the method starts.
6/8	4:20	The half-done error, and it is a sign of a child who is nearly there: they found the common denominator, which is the hard part. Smaller pieces means MORE of them — the “Equivalent fractions” sheet (Grade 4) argument, reused.
7/8	5:20	Mixed numbers. Converting to improper fractions avoids the whole difficulty, and the estimate is the check: $2\frac{1}{2}$ is more than $3 - 1$, so it cannot be right.
8/8	6:45	Two sums, range predicted first each time. Exit ticket after.