

GRADE 5 · GEOMETRY · ANSWER KEY

CLASSIFYING FIGURES IN A HIERARCHY

Today's goal: I can understand that attributes of a category of figures belong to all subcategories and classify two-dimensional figures in a hierarchy based on properties.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 2 is the structural item and the one children find hardest: rectangles and rhombuses, with NEITHER containing the other. Every earlier pair on the sheet nests, so a child reasonably expects this one to as well — and the two required drawings are what break the expectation. Draw the structure as overlapping rings rather than a ladder. Item 3 has a reliable wrong answer worth watching for: “all sides equal” is the one property a square does NOT inherit, since no category above it requires it, and that is exactly what makes a square a special rectangle rather than another name for one. Items 7 and 8 are the standard's own point and are worth board time: hierarchy means a property is stored once and inherited, so there is less to remember — and the reason matters more than the fact here precisely because the fact contradicts every picture of a long thin rectangle a child has ever been shown. This continues the “Classifying figures by their properties” sheet (Grade 4), where sorting by properties begins but is not yet nested. **Timing:** 20–25 min · pairs · rulers, set squares, paper shapes to rotate · items 2, 7 and 8 on the board.

- 1 yes; no
- 2 no and no — a 2-by-5 rectangle is not a rhombus, and a leaning rhombus with equal sides is not a rectangle
- 3 four right angles from rectangle, and two pairs of parallel sides from parallelogram
- 4 the square — because it is the only one meeting both sets of requirements: four right angles from rectangle, and all sides equal from rhombus
- 5 nothing about the shape changed — it still has four equal sides and four right angles, so it is still a square; orientation is not one of the requirements
- 6 draw a rectangle 2 units by 5 units — it has four right angles and unequal sides, so it is a rectangle and not a square
- 7 because a property only needs learning once, at the highest category that has it — everything below inherits it, so there is no separate list of facts for each shape
- 8 because the fact contradicts every picture a child has been shown, so on its own it is just one more thing to remember and it loses to the pictures; the reason lets a child work it out again, and settles the next question too

1 MILD Square and rectangle

yes; no

1. Rectangle requires: 4 sides, 4 right angles, 2 pairs of parallel sides. A square has all of them — so yes.
2. Square requires all of those PLUS all sides equal. A rectangle need not have equal sides — so no.
3. Every square is a rectangle; some rectangles are squares.

Watch for: Answered no to the first, on the grounds that they are “different shapes”. Different NAMES; the requirements are what decide, and a square meets all of the rectangle's.

2 MEDIUM Rectangle and rhombus

no and no — a 2-by-5 rectangle is not a rhombus, and a leaning rhombus with equal sides is not a rectangle

1. A rhombus requires all sides equal. A 2-by-5 rectangle fails that, so not every rectangle is a rhombus.
2. A rectangle requires four right angles. A leaning rhombus fails that, so not every rhombus is a rectangle.
3. So neither category contains the other — the hierarchy is not one straight chain.
4. Drawings: a long thin rectangle, and a pushed-over square with equal sides and no right angles.

Watch for: Assumed one must contain the other, since every earlier pair did. This is the pair that shows the structure is a network rather than a ladder, and the two drawings are the proof.

3 MEDIUM Two inherited properties

four right angles from rectangle, and two pairs of parallel sides from parallelogram

1. All rectangles have four right angles, and every square is a rectangle, so every square does.
2. All parallelograms have two pairs of parallel sides, and every square is a parallelogram, so every square does.
3. Four sides comes from quadrilateral in the same way.
4. None of these is a separate fact about squares; each follows from a category the square belongs to.

Watch for: Named “all sides equal” as inherited. It is not: no category above the square requires it, and it is the square's own extra requirement — which is exactly what makes the square a special case rather than a synonym.

4 SPICY Both rectangle and rhombus

the square — because it is the only one meeting both sets of requirements: four right angles from rectangle, and all sides equal from rhombus

1. Rectangle requires four right angles; rhombus requires all sides equal. Both require four sides and two pairs of parallel sides.
2. Put the two requirement lists together: four sides, four right angles, two pairs of parallel sides, all sides equal.
3. That is exactly the definition of a square.
4. So the square sits in the overlap of the two categories, and nothing else does — which is why it is the most specific shape here and inherits the most.

Watch for: Answered “a square” without the requirements. The item asks WHY, and combining the two requirement lists to recover the square's own definition is the argument — it also shows a square adds nothing beyond what the two parents demand.

5

SPICY

The rotated square

nothing about the shape changed — it still has four equal sides and four right angles, so it is still a square; orientation is not one of the requirements

1. Turn the paper and the sides do not change length, and the angles do not change size.
2. Every requirement of a square is still met.
3. No definition on this sheet mentions which way up a shape sits, so orientation cannot decide what a shape IS.
4. “Diamond” is a description of how it looks on the page, not a category with requirements.
5. The child has learned a picture rather than a definition — which is the same problem as thinking a rectangle must be long and thin.

Watch for: Said “a diamond is a different shape”. Rotating a shape changes no side and no angle, so it cannot change what the shape is — and this is worth doing physically with a paper square.

6

SPICY

One drawing against “all”

draw a rectangle 2 units by 5 units — it has four right angles and unequal sides, so it is a rectangle and not a square

1. A square requires all sides equal.
2. A 2-by-5 rectangle has two sides of 2 and two of 5, so it fails that requirement.
3. It is still a rectangle: four sides, four right angles, two pairs of parallel sides.
4. So here is a rectangle that is not a square, and one counter-example is enough to disprove any “all” statement.

Watch for: Argued at length without drawing anything. A single drawn counter-example is both quicker and more convincing, and knowing that one case disproves an “all” is worth more than the shape fact.

7

SPICY

Why less to remember

because a property only needs learning once, at the highest category that has it — everything below inherits it, so there is no separate list of facts for each shape

1. “Two pairs of parallel sides” is learned once, for parallelograms.
2. Rectangles, rhombuses and squares all get it free, because each is a parallelogram.
3. Without the hierarchy, each of those shapes needs its own list, and the same property is written four times.
4. So the hierarchy replaces four facts with one fact plus a membership.
5. And it makes the facts reliable: an inherited property cannot be forgotten for one shape and remembered for another, because it is not stored per shape.

Watch for: Said “because it is organised”. The concrete version is better: a property is stored ONCE, at the top category that has it, and everything below gets it free — so four facts become one.

8

EXTRA SPICY

Reason against fact

because the fact contradicts every picture a child has been shown, so on its own it is just one more thing to remember and it loses to the pictures; the reason lets a child work it out again, and settles the next question too

1. Told as a fact, “a square is a rectangle” sits alongside years of drawings in which rectangles are long and thin. The drawings usually win.
2. Told as a reason — a rectangle's requirements are these three, and a square meets them — the child can rebuild the conclusion whenever they doubt it.
3. The reason also answers questions nobody covered: is a rhombus a parallelogram, is a square a quadrilateral. The fact answers only itself.
4. And it generalises: the same requirements test decides any “is every A a B” question, which is what the standard is actually asking for.

Watch for: Said “because understanding is better than memorising”. True and generic; the specific answer is that this particular fact CONTRADICTS the pictures the child already has, so it needs an argument to survive them — and the argument then answers questions the fact does not.

Use: End of the hierarchy lesson. Item 3 is the diagnostic and what it assesses is not the fact but the ARGUMENT — a child who writes “a square IS a rectangle” with no reasoning has added one more entry to the list that caused the problem, and it will not survive contact with the next picture of a long thin rectangle. The answer wanted names the shift: a category is a set of requirements, not a bin. Item 4 exists because the over-correction is so common that it usually appears within the same lesson — mark it as its own error rather than as partial credit, and note that one drawn counter-example disposes of any “all” statement. Item 2 is the half of the standard usually skipped: inheritance means there is LESS to learn, not more. Watch for a child naming “all sides equal” there — that property is precisely the one NOT inherited. This continues the “Classifying figures by their properties” sheet (Grade 4), where the sorting is by properties but not yet nested. **Timing:** 5–10 min · independent · rulers and set squares available.

1 every rhombus is a parallelogram; not every parallelogram is a rhombus

2 four right angles — because every square is a rectangle, and all rectangles have four right angles

3 he is treating categories as separate bins with one picture each, so nothing can be inside anything else; but a category is a set of requirements — a rectangle needs four sides, four right angles and two pairs of parallel sides, a square has all three, so every square is a rectangle

4 wrong — a square needs all sides equal and a rectangle does not, so a 2-by-5 rectangle is a rectangle and not a square; one counter-example settles it

1  MEDIUM Rhombus and parallelogram

every rhombus is a parallelogram; not every parallelogram is a rhombus

1. A parallelogram requires four sides and two pairs of parallel sides.
2. A rhombus has four sides, two pairs of parallel sides and all sides equal — so it meets every requirement, and every rhombus is a parallelogram.
3. A rhombus also requires all sides equal, which a parallelogram does not.
4. So a parallelogram with sides 3, 5, 3, 5 is a parallelogram and not a rhombus — not every parallelogram is a rhombus.

Watch for: Answered yes to both, or no to both. The two questions have different answers, and treating them as one question is the error the whole sheet is about.

2  MEDIUM An inherited property

four right angles — because every square is a rectangle, and all rectangles have four right angles

1. All rectangles have four right angles; that is one of the requirements.
2. Every square meets the rectangle requirements, so every square is a rectangle.
3. Anything true of ALL rectangles is therefore true of every square.
4. So “a square has four right angles” is not a separate fact about squares — it follows, and there is nothing extra to memorise.
5. Two pairs of parallel sides is inherited the same way, from parallelogram.

Watch for: Named a property that squares have and rectangles do not, such as all sides equal. That one is NOT inherited — it is the extra requirement that makes a square a special rectangle, so it has to be stated rather than passed down.

3



SPICY

Tam's reasoning

he is treating categories as separate bins with one picture each, so nothing can be inside anything else; but a category is a set of requirements — a rectangle needs four sides, four right angles and two pairs of parallel sides, a square has all three, so every square is a rectangle

1. Tam has learned shapes as a LIST, where each name has its own picture and the names are what distinguish them.
2. In a list, nothing is a member of anything else, so “a square is a rectangle” sounds like a contradiction.
3. A category is really a set of requirements a shape either meets or does not.
4. Rectangle's requirements: four sides, four right angles, two pairs of parallel sides. A square has every one of them.
5. So every square is a rectangle — not because someone decided it, but because it meets the requirements. It is a rectangle with an extra property, not a different thing.
6. The reverse fails, because being a rectangle asks nothing about side lengths.

Watch for: Corrected Tam with the fact (“a square IS a rectangle”) and no argument. That is another item for the list he already has — and asserted flatly it will not survive, because it contradicts every picture he has been shown. The requirements checklist is what makes it his own conclusion.

4



SPICY

Bo's over-correction

wrong — a square needs all sides equal and a rectangle does not, so a 2-by-5 rectangle is a rectangle and not a square; one counter-example settles it

1. A square's requirements include all sides equal.
2. Draw a rectangle 2 units by 5 units: four right angles, two pairs of parallel sides, and sides that are not all equal.
3. It meets the rectangle requirements and fails the square ones, so it is a rectangle that is not a square.
4. One counter-example is enough to disprove an “all” statement, which is why this takes a single step.
5. So the correct pair of statements is: EVERY square is a rectangle, and SOME rectangles are squares.

Watch for: Said Bo is “nearly right” or has “the right idea”. He has swapped one error for its mirror image, which is a distinct mistake and a common one straight after this lesson — and one drawn counter-example disposes of it.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	The question is the lesson. Ask it cold and take a vote — most classes say no, and the no is not ignorance, it is what a list of shapes teaches.
2	0:40	Name the real problem, which is not about squares: the child has categories as BINS with one picture each. Nothing in a list is inside anything else. That is what has to change.
3	1:20	THE slide. Write the rectangle's requirements as a checklist and tick them off against a square. The conclusion follows from the ticks rather than from authority — which matters, because “a square IS a rectangle” asserted flatly is a fact to memorise and will not survive.
4	2:15	The asymmetry, and give it real time. Children accept “every square is a rectangle” and then over-correct to “all rectangles are squares” within the same lesson. A drawn 2-by-5 rectangle settles it.
5	3:10	The part usually skipped, and it is half of the standard: inherited properties come FREE. If all rectangles have four right angles then squares do too, and there is nothing extra to remember. Say that hierarchy makes there be LESS to learn, not more.
6	4:10	Rectangles and rhombuses, neither inside the other, with the square in the overlap. Draw it as two overlapping rings rather than a ladder — children who picture a single chain get stuck here.
7	5:10	The diamond. Rotate a paper square in front of them and ask what changed. Nothing in any definition mentions orientation, and this is where a child sees that the properties are the whole story.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.

10	6:15	Both directions asked separately, on purpose. Then the free-property question, which is what shows the inheritance landed. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #4 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.
