



Decimal arithmetic

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the decimal arithmetic lesson. This bank carries two rules that CONTRADICT each other in appearance, and the failure mode is a child who has mastered the first: lining up the point is exactly right for adding and exactly wrong for multiplying, so success on items 1 and 3 predicts nothing about item 2. Item 4 is the diagnostic and Rafi's objection is about MEANING — he is generalising correctly from every multiplication he has ever seen, all of them by numbers above 1. A procedural answer leaves the belief intact, and a child who holds it will “correct” right answers by moving the point until they look bigger. the “Multiplication as scaling” sheet (Grade 5) is the full lesson on this and the two should be taught close together. Item 3 wants the coincidence named: right-alignment was never a separate rule, because a whole number's last digit is always the ones. Item 1's subtraction reuses the zero-is-a-quantity claim from the “Dividing by two-digit divisors” sheet (Grade 5).

Timing: 5–10 min · independent · no calculators.

1. An addition **FLUENCY**

Answer: **16.15**

1. Write 12.40 and 3.75 so the places line up.
2. 40 hundredths + 75 hundredths = 115 hundredths, so carry: 16.15.
3. Estimate: about $12 + 4 = 16$, so the answer is the right size.

If wrong: Answered 15.15 or 16.05 — the carry out of the hundredths dropped.

2. A subtraction from a whole number **FLUENCY**

Answer: **2.64**

1. Write 5.00 and 2.36.
2. 500 hundredths – 236 hundredths = 264 hundredths, so 2.64.
3. Estimate: about $5 - 2 = 3$.

If wrong: Answered 3.36 or 2.36, by leaving the hundredths of 5 alone because “there is nothing to take from”. 5 is 5.00 and the zeros are real quantities — the same claim as the zero in a quotient (the “Dividing by two-digit divisors” sheet (Grade 5)).

3. A multiplication FLUENCY

Answer: **0.12**

1. Multiply $3 \times 4 = 12$.
2. One decimal place in each factor, two altogether, so two in the answer: 0.12.
3. Size check: both factors are under 1, so the answer is smaller than both.

If wrong: Answered 1.2 — the point “lined up” rather than the places counted. It is also bigger than both factors, which is the tell.

4. A division by less than 1 FLUENCY

Answer: **30**

1. How many 0.2s fit in 6? Five per whole, six wholes, so 30.
2. Check: $30 \times 0.2 = 6$.
3. The answer is far bigger than 6, because the pieces are smaller than 1.

If wrong: Answered 3, as though it were $6 \div 2$. Asking “how many fit” makes 3 obviously too small — six wholes hold five fifths each.

5. Ivy's addition REASONING

Answer: **she lined up the ends instead of the places, adding tenths to hundredths; right-aligning worked for whole numbers because in a whole number the last digit is always the ones, so the ends and the places line up together**

1. $3.5 + 0.42$ should be written $3.50 + 0.42$, giving 3.92.
2. Ivy added the 5 (five tenths) to the 2 (two hundredths), which are different-sized pieces and cannot be combined as they stand.
3. Her method was not arbitrary: for $34 + 275$, or any whole numbers, the last digit is the ones column, so aligning the ends aligns the places automatically.
4. Decimals break that coincidence, because the last digit might be tenths or hundredths or thousandths. The rule was always “align the places” — the ends were just where the ones happened to be.
5. An estimate also rejects 0.77: 3.5 plus something positive must be more than 3.5.

If wrong: Said Ivy “should have lined up the decimal points”. That is the correct instruction and it does not explain why her old method worked, so it leaves her with two unrelated rules. The coincidence — last digit IS the ones in a whole number — is what makes it one rule.

6. Rafi's objection REASONING

Answer: **0.12 is right; multiplying makes a number bigger only when you multiply BY more than 1, and 0.4 is less than 1, so it makes 0.3 smaller**

1. 0.3×0.4 means four tenths OF three tenths — a part of 0.3, so it must be less than 0.3.
2. Compare: 0.3×1 is 0.3, and 0.3×0.5 would be half of it. Multiplying by anything under 1 gives less than you started with.
3. Rafi's belief is true for whole numbers greater than 1, which is every multiplication he has met so far, so it is a reasonable generalisation from his evidence.
4. And the arithmetic confirms it: $3 \times 4 = 12$, two decimal places in, two out, so 0.12 — and 0.12 really is a bit more than a tenth, which is about what four tenths of three tenths should be.
5. Lining up the point gives 1.2, which is bigger than both factors and therefore impossible for this multiplication.

If wrong: Said “because that is the rule for decimals”. Rafi has an objection about MEANING, not procedure, so a procedural answer will not move him — and a child who cannot answer him will quietly change correct answers to look bigger.

Quick answers for fast checking:

#	Level	Answer
1	fluency	16.15
2	fluency	2.64
3	fluency	0.12
4	fluency	30
5	reasoning	places not ends; a whole number's last digit IS the ones
6	reasoning	multiplying by less than 1 makes it smaller

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 3 is the one to mark first: 9.0, 6.0 and 1.2 are all exactly ten times too big, which is the signature of a point placed by alignment rather than by counting places — and a child who scored full marks on items 1 and 2 is MORE likely to make it, not less, because they have just been rewarded for lining things up. Item 9 is the sheet's best item and its answer is the reason the two rules differ: adding needs matching units, while multiplying manufactures a smaller unit — tenths of tenths are hundredths, which is exactly why the places are added. Item 5 forbids doing the subtraction, deliberately: the estimate is the check that works when there is no correct answer to compare against. Item 6 sets up the “Multiplication as scaling” sheet (Grade 5), and item 8 reuses the “Multistep problems and remainders” sheet (Grade 4) — 6.25 pieces is a correct division and a wrong answer.

Timing: 20–25 min · pairs · no calculators · items 3, 6 and 9 on the board.

1. Adding with a shorter number FLUENCY

Answer: **3.92**

1. Write $3.50 + 0.42$ so the places match — the 0 changes nothing, since 3.5 IS 3.50.
2. Tenths: $5 + 4 = 9$. Hundredths: $0 + 2 = 2$. Ones: 3.
3. So 3.92. Note it contains no 7 at all.

If wrong: Answered 0.77 or 7.7, from $35 + 42$ with the ENDS aligned. Tenths were added to hundredths, which are different-sized pieces. This is the bank's core misconception and this is the item that catches it.

2. Adding two two-place numbers FLUENCY

Answer: **16.15**

1. $12.40 + 3.75$.
2. Hundredths: $0 + 5 = 5$. Tenths: $4 + 7 = 11$, so 1 tenth and carry 1.
3. Ones: $2 + 3 + 1 = 6$. Tens: 1.
4. So 16.15.

If wrong: Answered 15.15 or 16.05 — the carry from the tenths dropped. An estimate of $12 + 4 = 16$ confirms the whole part.

3. Adding when both are under 1 FLUENCY

Answer: **0.95**

1. $0.80 + 0.15$.
2. Hundredths: $0 + 5 = 5$. Tenths: $8 + 1 = 9$.
3. So 0.95.

If wrong: Answered 0.23, from $8 + 15$ with the ends aligned. Writing 0.80 makes the tenths column line up with the tenths.

4. Subtracting with a borrow FLUENCY

Answer: **2.45**

1. $4.20 - 1.75$.
2. Hundredths: $0 - 5$ needs a borrow, giving $10 - 5 = 5$.
3. Tenths: $1 - 7$ needs a borrow, giving $11 - 7 = 4$. Ones: $3 - 1 = 2$.
4. So 2.45, and an estimate of $4 - 2 = 2$ agrees.

If wrong: Answered 3.55 by taking the smaller digit from the larger in each column, which avoids the borrow entirely and always gives too big an answer. That is ``smaller_from_larger()'``, and item 12 asks about it directly.

5. Subtracting from a whole number FLUENCY

Answer: **2.64**

1. 5 is 5.00 — the tenths and hundredths are there, and they are zero.
2. $5.00 - 2.36$: hundredths $0 - 6$ borrows to $10 - 6 = 4$.
3. Tenths $9 - 3 = 6$. Ones $4 - 2 = 2$.
4. So 2.64.

If wrong: Abandoned it, saying there is nothing to take the 6 from. Writing 5.00 first is the whole fix: the zeros are quantities, not blanks.

6. Borrowing across three places FLUENCY

Answer: **9.93**

1. 10 is 10.00.
2. $10.00 - 0.07$ borrows across the tenths and the ones.
3. So 9.93, and an estimate says the answer must be just under 10.

If wrong: Answered 9.03 or 10.07. Writing 10.00 first makes the borrowing ordinary; an estimate rules out both wrong answers immediately.

7. Multiplying: count the places FLUENCY

Answer: **0.9**

1. Do NOT line up the point — multiply the digits, then count the places.
2. $15 \times 6 = 90$.
3. One decimal place in 1.5 and one in 0.6, so two places out: $0.90 = 0.9$.
4. Size check: 0.6 is under 1, so the answer must be less than 1.5.

If wrong: Answered 9.0 — ten times too big, the signature of one place lost, from lining up the point instead of counting places.

8. Multiplying with three places in FLUENCY

Answer: **0.6**

1. $24 \times 25 = 600$.
2. One decimal place in 2.4 and two in 0.25, so three places out: $0.600 = 0.6$.
3. Size check: 0.25 is a quarter, and a quarter of 2.4 is 0.6.

If wrong: Answered 6.0 or 0.06 — the places miscounted. Counting them (one plus two) before doing any arithmetic makes the answer's size predictable.

9. Both factors under 1 FLUENCY

Answer: **0.12**

1. $3 \times 4 = 12$.
2. One place in each, so two places out: 0.12.
3. THE ONE WORTH DWELLING ON: 0.12 is smaller than BOTH 0.3 and 0.4. Every child's model of multiplication says it should be bigger.

If wrong: Answered 1.2 — ten times too big, and bigger than either number being multiplied, which is the tell. See the “Multiplication as scaling” sheet (Grade 5) for why multiplying by less than 1 shrinks.

10. Dividing by a half APPLICATION

Answer: **9 — nine 0.5s fit in 4.5**

1. How many halves in four and a half? Two per whole, four wholes, plus one more for the half.
2. So 9.
3. The answer is BIGGER than 4.5, because the pieces are smaller than 1.

If wrong: Answered 0.9, dividing as though by 5 and then placing a point. Asking “how many fit” makes 0.9 obviously impossible — you cannot fit less than one half into four and a half.

11. Dividing by eight tenths APPLICATION

Answer: **9 — nine 0.8s fit in 7.2**

1. $0.8 \times 9 = 7.2$, so nine 0.8s fit in 7.2.
2. Or scale both by 10: $72 \div 8 = 9$, which item 14 asks about.
3. Again the answer is bigger than 7.2, for the same reason.

If wrong: Answered 0.9. The “how many fit” reading is the check: 0.9 of an 0.8 is less than one piece, and 7.2 plainly holds several.

12. Jo's subtraction REASONING

Answer: **because 3.55 is bigger than the difference could be — 4.2 minus something near 1.75 must be around 2.5, and taking away nearly 2 cannot leave more than 3**

1. Estimate: $4.2 - 1.75$ is about $4 - 2 = 2$.
2. 3.55 is more than 3, so more than $4.2 - 1.2$ — far too big.
3. Jo took each column's smaller digit from its larger, ignoring which number it belonged to: in the hundredths she did $5 - 0$ instead of $0 - 5$.
4. That method always gives too big an answer when borrowing was needed, because it avoids the borrow entirely.
5. The real answer is 2.45, from $4.20 - 1.75$.

If wrong: Did the subtraction properly and compared. That shows 3.55 is wrong and the item asked for a reason not needing the subtraction — the estimate is the reusable half, and it works when a child has no correct answer to compare with.

13. Why 0.9 sits between REASONING

Answer: **because 1.5 was multiplied by 0.6, which is less than 1, so it shrank; and 0.6 was multiplied by 1.5, which is more than 1, so it grew**

1. 1.5×0.6 is six tenths OF 1.5, which is a part of it — so less than 1.5.
2. The same product is also 1.5 lots of 0.6, which is more than one lot — so more than 0.6.
3. Both descriptions are of the same multiplication, read from either factor.
4. So whether multiplying makes something bigger or smaller depends entirely on whether the OTHER factor is above or below 1.

If wrong: Said “because 0.9 is in between them”, which restates the observation. The reason is about which factor is above 1 and which below, and it is the general rule — the “Multiplication as scaling” sheet (Grade 5) is where it is developed in full.

14. Why $72 / 8$ gives the same REASONING

Answer: **because multiplying both numbers by 10 makes each piece ten times bigger and the amount ten times bigger too, so the number of pieces that fit is unchanged**

1. $7.2 \div 0.8$ asks how many 0.8s fit in 7.2.
2. $72 \div 8$ asks how many 8s fit in 72.
3. Both the pieces and the whole have been scaled by 10, so the count is the same — nine either way.
4. This is why the point can be moved in BOTH numbers together: it turns an awkward division into an easy one without changing the answer. Moving it in only one number would change the answer.

If wrong: Said “because you can move the decimal point”. That is the technique; the item asks why it is allowed, and the answer is that scaling both by the same factor leaves the count of pieces unchanged.

15. The ribbon APPLICATION

Answer: **6 pieces, with 0.1 m left over**

1. How many 0.4s fit in 2.5? $0.4 \times 6 = 2.4$, and $0.4 \times 7 = 2.8$, which is too much.
2. So 6 pieces.
3. $2.5 - 2.4 = 0.1$, so 0.1 m is left over.
4. The leftover is 0.1 m — a real length, not a fraction of a piece. Saying 6.25 pieces would answer a different question.

If wrong: Answered 6.25 pieces. The division does give 6.25, and the question asked how many pieces — a quarter of a piece is not a piece, which is the “Multistep problems and remainders” sheet (Grade 4) remainder-in-context skill again.

16. Two different rules REASONING

Answer: **because adding combines quantities, so they must be measured in the same-sized pieces; multiplying does not combine them at all — it scales one by the other, and the places are counted afterwards**

1. To add, you need the same units: tenths added to tenths, hundredths to hundredths. Lining up the places is what puts the same units together.
2. $0.3 + 0.4$ is seven tenths, and both were already tenths.
3. To multiply, nothing is being combined. 0.3×0.4 is a part OF a part, and the answer is in a smaller unit than either — tenths of tenths, which are hundredths.
4. That is why the places are ADDED: one place times one place gives two places. Lining anything up would be answering the wrong question.
5. So the two rules are not arbitrary, and they are not in competition — they come from what each operation does.

If wrong: Said “because they are different operations”, which is true and explains nothing. The distinction to reach for is that adding needs matching units while multiplying MAKES a new, smaller unit — tenths of tenths are hundredths, and that is where the place count comes from.

Quick answers for fast checking:

#	Level	Answer
1	fluency	3.92
2	fluency	16.15
3	fluency	0.95
4	fluency	2.45
5	fluency	2.64

6	fluency	9.93
7	fluency	0.9
8	fluency	0.6
9	fluency	0.12
10	application	9
11	application	9
12	reasoning	the estimate is about 2, so 3.55 is impossible
13	reasoning	0.6 is under 1 so it shrinks; 1.5 is over 1 so it grows
14	reasoning	both scaled by 10, so the count of pieces is unchanged
15	application	6 pieces, 0.1 m over
16	reasoning	adding needs matching units; multiplying makes a smaller unit

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	The single most important thing to say in this lesson is that addition and multiplication need DIFFERENT rules. Children arrive expecting one rule for decimals, and the mastery of lining up the point is precisely what wrecks their multiplying.
2/8	0:40	Do both alignments side by side. Then remind them of the “Reading and comparing decimals” sheet (Grade 5): 3.5 and 3.50 are the same number, so writing the 0 invents nothing.
3/8	1:30	Worth the minute: right-alignment was never a separate rule, it was place-alignment in disguise, because a whole number’s last digit is always the ones. Children who hear this stop feeling their old method was arbitrary.
4/8	2:20	5 – 2.36. Ask where the hundredths of 5 are and wait — “there aren’t any” is the answer to draw out, followed by “so how many?”. Zero is a quantity, which is the same claim as the quotient zero in the “Dividing by two-digit divisors” sheet (Grade 5).
5/8	3:20	THE slide, and the pivot of the lesson. Say plainly: the rule you just learned does NOT apply here. Count the places in and the places out. Then show 1.2 and let the class reject it.
6/8	4:30	Do not rush this. 0.12 being smaller than both factors contradicts everything a child believes about multiplying, and if it is not addressed they will “fix” correct answers by moving the point. the “Multiplication as scaling” sheet (Grade 5) is the whole lesson on it.
7/8	5:30	Ask division as “how many fit” every time. 30 being bigger than 6 is the same shock as slide 6 in reverse, and the counting question makes it obvious rather than surprising.
8/8	6:45	Four questions, one per operation, and the estimate first each time. The real question is the one on the slide: which rule does this need? Exit ticket after.