



GRADE 5 · NUMBER & OPERATIONS – FRACTIONS · ANSWER KEY

DIVIDING WITH UNIT FRACTIONS

Today's goal: I can divide unit fractions by whole numbers and whole numbers by unit fractions.

How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

12 problems + Mr DJ's worked example

Use: In-class practice after the deck. Items 1 and 2 are printed adjacently on purpose, and the marking signal is strong: a child who gives 12, 6, 10 for BOTH has merged the two directions into a single procedure, which is a different problem from getting one division wrong. Item 3 then asks for the principle that separates them, and the direction-of-answer check it produces is what makes an unfamiliar division checkable. Item 8 is the sheet's most demanding item and the units are the whole point — 12 glasses and $\frac{3}{4}$ of a litre are not comparable quantities, and no amount of correct arithmetic decides which one a question wants. Item 6 quietly removes a whole future difficulty: 0.5 and $\frac{1}{2}$ are one number with two notations, so $6 \div 0.2 = 30$ in the “Decimal arithmetic” sheet (Grade 5) needs no separate rule. Item 7's answer should contain no rule at all — just that small pieces fit into a whole more than once. **Timing:** 20–25 min · pairs · fraction strips · items 3, 8 and 7 on the board.

1 12**2** 6**3** 10**4** $\frac{1}{12}$ **5** $\frac{1}{6}$ **6** 8 pieces**7** $\frac{1}{12}$ of the cake each

8 item 1's answers are all bigger, because they count pieces smaller than one; item 2's are all smaller, because an amount is being shared out

9 because a half is being shared between six, so each share must be smaller than a half — and 12 is much bigger than a half

10 the glasses question asks how many quarter-litres fit in 3 litres, giving 12 GLASSES; the jugs question shares 3 litres between 4, giving $\frac{3}{4}$ of a LITRE each

11 because 0.5 IS a half — the same number written two ways, so both ask how many halves fit in 6

12 because dividing asks how many of the pieces fit, and pieces smaller than one whole fit more than once per whole — so the count exceeds the number of wholes

1**MILD**

How many quarters in 3

12

1. Four quarters per whole, three wholes — 12.
2. Bigger than the 3 started with, because the pieces are smaller than one.
3. Check: 12 quarters is 3.

Watch for: Answered $\frac{3}{4}$ — divided BY the denominator.

Multiplying back catches it: $\frac{3}{4}$ of a quarter is nowhere near 3.

2**MILD**

How many thirds in 2

6

1. Three thirds per whole, two wholes — 6.
2. Bigger than 2, for the same reason.
3. Check: 6 thirds is 2.

Watch for: Answered $\frac{2}{3}$ — divided by the denominator.

3**MILD**

How many halves in 5

10

1. Two halves per whole, five wholes — 10.
2. Bigger than 5.
3. Check: 10 halves is 5.

Watch for: Answered $\frac{5}{2}$ — divided by the denominator. $\frac{5}{2}$ is smaller than 5, and dividing by a number under one cannot shrink anything.

4**MILD**

A quarter shared three ways

 $\frac{1}{12}$

1. A quarter cut into three gives twelfths — $\frac{1}{12}$.
2. Smaller than the quarter started with, as sharing requires.
3. Check: three twelfths make a quarter.

Watch for: Answered 12 — the answer to the OPPOSITE question, which is the clearest possible sign the two directions have been merged into one procedure.

5**MILD**

A third shared two ways

 $\frac{1}{6}$

1. A third cut into two gives sixths — $\frac{1}{6}$.
2. Smaller than the third started with.
3. Check: two sixths make a third.

Watch for: Answered 6 — again the opposite question's answer.

6**MEDIUM**

The 4-metre plank

8 pieces

1. How many half-metres fit in 4 metres? Two per metre, four metres — 8.
2. As a division: $4 \div \frac{1}{2} = 8$.
3. Multiplying back: $8 \times \frac{1}{2} = 4$ metres. Correct.
4. The answer is 8 PIECES, not 8 metres.

Watch for: Answered 2 metres, halving the plank. That answers “what is half of 4”, which is $4 \times \frac{1}{2}$ rather than $4 \div \frac{1}{2}$.

7**MEDIUM**

A third of a cake between 4

 $\frac{1}{12}$ of the cake each

1. A third of a cake is the amount being shared, and there are 4 children.
2. Cut that third into 4 equal pieces. Twelve such pieces make a whole cake, so each is $\frac{1}{12}$.
3. As a division: $\frac{1}{3} \div 4 = \frac{1}{12}$.
4. Check: 4 lots of $\frac{1}{12}$ is $\frac{4}{12}$, which is a third. Correct.

Watch for: Answered $\frac{4}{3}$ or 12, treating it as a how-many-fit question. A third of a cake shared between four children must give each less than a third, which rules both out immediately.

8



SPICY

Bigger or smaller, and why

item 1's answers are all bigger, because they count pieces smaller than one; item 2's are all smaller, because an amount is being shared out

1. In item 1 the fraction is the SIZE of each piece, and the pieces are smaller than one, so many of them fit — more than the number of wholes.
2. In item 2 the fraction is the whole AMOUNT being shared, so each share is a part of it and must be smaller.
3. So the direction of the answer is decided by which role the fraction plays.
4. And that is a check: an item-1 answer smaller than the starting number, or an item-2 answer bigger, is wrong before any arithmetic is examined.

Watch for: Answered bigger or smaller correctly without the reason. The reason is the reusable part — it is what lets a child check an unfamiliar division, and it is the same size-judgement habit as the “Multiplication as scaling” sheet (Grade 5).

9



SPICY

 $1/2 / 6$ cannot be 12

because a half is being shared between six, so each share must be smaller than a half — and 12 is much bigger than a half

1. Sharing always gives each person less than the whole amount.
2. The whole amount here is $\frac{1}{2}$, so each share is under $\frac{1}{2}$.
3. 12 is bigger than $\frac{1}{2}$, so it cannot be a share of it.
4. No division was needed — only recognising that the half is what is being shared out.

Watch for: Did the division to get $1/12$ and compared. The item asked for a reason without dividing, because that is the check available when a child is unsure which way round to work — exactly when they need one.

10



SPICY

Twelve glasses or three quarters of a litre

the glasses question asks how many quarter-litres fit in 3 litres, giving 12 GLASSES; the jugs question shares 3 litres between 4, giving $3/4$ of a LITRE each

1. In the first, the quarter-litre is the size of each piece and 3 litres is the total, so the answer counts pieces: 12.
2. In the second, 3 litres is the total and 4 is the number of shares, so the answer is an amount: $3/4$ of a litre.
3. The units differ because the questions differ — one answer is a count of glasses, the other a volume.
4. Both check out: $12 \times \frac{1}{4} = 3$ litres, and $4 \times \frac{3}{4} = 3$ litres. Same total, different questions.
5. So the arithmetic cannot tell you the units. Only the question can.

Watch for: Explained the two calculations without naming the units. The units are the point of the item: 12 and $3/4$ are not comparable quantities, and the “Multistep problems and remainders” sheet (Grade 4) made the same demand about remainders.

11 **SPICY** Halves and 0.5

because 0.5 IS a half — the same number written two ways, so both ask how many halves fit in 6

1. 0.5 means five tenths, which is a half.
2. So $6 \div 0.5$ and $6 \div \frac{1}{2}$ are the same division written differently.
3. Both ask how many halves fit in 6, and both give 12.
4. This is why $6 \div 0.2 = 30$ in the “Decimal arithmetic” sheet (Grade 5) is not a separate rule: it asks how many fifths fit in 6, and there are five per whole.

Watch for: Said “because you can turn a fraction into a decimal”. True and it stops short: they are not two numbers that CONVERT, they are one number with two notations, which is why no separate method is needed.

12 **EXTRA SPICY** Why dividing by less than 1 enlarges

because dividing asks how many of the pieces fit, and pieces smaller than one whole fit more than once per whole — so the count exceeds the number of wholes

1. Dividing 6 by 2 asks how many 2s fit in 6: three, because each piece is bigger than one whole so fewer fit.
2. Dividing 6 by $\frac{1}{2}$ asks how many halves fit: each whole holds two, so there are twice as many pieces as wholes.
3. The smaller the piece, the more of them fit — so as the divisor shrinks below 1, the answer grows above the total.
4. Dividing by exactly 1 gives the total back, because each piece is one whole. So 1 is the dividing line, just as it is for multiplying in the “Multiplication as scaling” sheet (Grade 5).

Watch for: Said “because that is what happens with fractions”. The reason is the counting one: a piece smaller than a whole fits into a whole more than once, so there are more pieces than wholes — and stated that way it needs no rule at all.

Exit Ticket

5 problems

Use: End of the dividing-with-unit-fractions lesson. This bank is the “Multiplication as scaling” sheet (Grade 5) from the other side and the two belong in the same week: one says multiplying does not always enlarge, the other that dividing does not always shrink, and a child taught only one still holds half a wrong rule. Item 3 mirrors that bank's Zed item deliberately — both ask why a false rule survived, because a child who can explain that is less likely to swap it for another over-general one, and both ask for an independent check. Item 4 is the diagnostic and the answer must name WHICH NUMBER IS THE TOTAL and which is the piece; “the numbers are the other way round” describes the notation and gives a child nothing to use on a real problem. Note that the standard explicitly excludes the general algorithm for dividing fractions — that is Grade 6 — so nothing here should say “invert and multiply”, and nothing does. The how-many-fit question is what will make that algorithm sensible when it arrives. **Timing:** 5–10 min · independent · fraction strips available.

1 12

2 1/12

3 12 glasses

4 3 halves is only $1\frac{1}{2}$, not 6, so 3 cannot be right; her rule held because every divisor she had used before was bigger than 1, and dividing by anything bigger than 1 really does make a number smaller

5 $6 \div 1/2$ counts how many halves fit inside 6, so the answer is a count and is bigger than 6; $1/2 \div 6$ shares a half between six, so the answer is an amount and is smaller than a half — 12 and $1/12$

1

MEDIUM

The counting direction

12

1. In words: how many quarters fit in 3?
2. Four per whole, three wholes, so 12.
3. Check: 12 quarters is 3. The answer is BIGGER than 3, as this question requires.

Watch for: Answered $1/12$ — the other division's answer. Saying the question in words first is what separates them, which is why the item asks for it.

2

MEDIUM

The sharing direction

1/12

1. In words: a quarter shared between three.
2. Cutting a quarter into three gives twelfths, so $1/12$.
3. Check: three twelfths is a quarter. The answer is SMALLER than a quarter.

Watch for: Answered 12 — the other division's answer. The two divisions use the same numbers and their answers sit on opposite sides of them.

3

MEDIUM

Quarter-litre glasses

12 glasses

1. How many quarter-litres fit in 3 litres? Four per litre, three litres, so 12.
2. As a division: $3 \div \frac{1}{4} = 12$.
3. Multiplying back: $12 \times \frac{1}{4} = 3$ litres. Correct.
4. The answer is a number of GLASSES, not a number of litres — the question decides the units.

Watch for: Answered $3/4$, which is 3 litres shared between 4 — the other question. $3/4$ of a litre is an amount and the item asked how many glasses, so even the units are wrong.

4

SPICY

Kim's 3

3 halves is only $1\frac{1}{2}$, not 6, so 3 cannot be right; her rule held because every divisor she had used before was bigger than 1, and dividing by anything bigger than 1 really does make a number smaller

1. Multiplying back: $3 \times \frac{1}{2} = 1\frac{1}{2}$. The question started with 6, so 3 is far too small.
2. The right answer is 12, and $12 \times \frac{1}{2} = 6$ checks out.
3. Kim's rule is a fair summary of her evidence: $6 \div 2$, $6 \div 3$, $6 \div 5$ all give less than 6, and until now every divisor was a whole number of 2 or more.
4. 1 is the dividing line: dividing by more than 1 shrinks a number, by exactly 1 leaves it, and by less than 1 makes it bigger.
5. She had never divided by anything below 1, so her evidence could not have shown her this case — and “divide by a half” also sounds like “halve it”, which points the same wrong way.

Watch for: Showed the answer is 12 without multiplying back, or without explaining why the rule held. Both halves are asked for: the multiply-back check is what a child can use unaided, and understanding why the rule survived is what stops it being replaced by another over-general one — the same pairing as the “Multiplication as scaling” sheet (Grade 5)’s Zed.

5

SPICY

Two questions, same numbers

6 / $1/2$ counts how many halves fit inside 6, so the answer is a count and is bigger than 6; $1/2 \div 6$ shares a half between six, so the answer is an amount and is smaller than a half — 12 and $1/12$

1. $6 \div \frac{1}{2}$: the half is the SIZE OF THE PIECES and 6 is the total, so the question is how many pieces. Twelve.
2. $\frac{1}{2} \div 6$: the half is the TOTAL and 6 is the number of shares, so the question is how much each. One twelfth.
3. So the roles of the two numbers are swapped — which is the total and which is the piece.
4. The sizes confirm it: counting pieces smaller than one gives more than 6, and splitting a half between six gives less than a half.
5. And the two answers multiply to 1, which is what makes them look related when they are answers to opposite questions.

Watch for: Said “because the numbers are the other way round”. True of the notation and it does not say what CHANGED — the useful answer names which number is the total and which is the piece, because that is what a child has to decide when reading a real problem.

Slide	Time	What to do
1	0:00	Teach this with the “Multiplication as scaling” sheet (Grade 5), not after it. That bank says multiplying does not always enlarge; this one says dividing does not always shrink. Same evidence problem, opposite direction, and a child who has met only one of the two still has half a wrong rule.
2	0:30	Ask the words BEFORE any notation, every single time. “How many halves fit in 6” gets a confident 12 from a class that would answer 3 to the symbols.
3	1:10	THE slide, and it is worth being explicit: the maths was never the difficulty. The class answered correctly a moment ago in words. What the symbols do is hide the question, so the fix is to restore the question.
4	2:00	Concede both reasons. “Divide by a half” really does sound like “halve”, and every divisor they have used has been above 1. Neither is carelessness.
5	2:50	Multiply back, as in the “Dividing by two-digit divisors” sheet (Grade 5). 12 halves is 6; 3 halves is one and a half. The check is independent of the method that produced the answer, which is what makes it a check.
6	3:40	Now the other direction, and do the cutting: a half cut into six pieces. Each is a twelfth, because twelve of them make a whole. This is the “A fraction as division” sheet (Grade 5)’s sharing again.
7	4:50	The pair on one slide, which is the point of the lesson. Same two numbers, 12 and $\frac{1}{12}$. Have the class say which question each answers, and note that the SIZE gives it away without any working.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	6:00	Both directions plus a real context. Insist on the words first — a child who writes the notation first will divide by 4 and not notice. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #6, #9 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.