

GRADE 5 · MEASUREMENT & DATA · ANSWER KEY

LINE PLOTS WITH FRACTIONS

Today's goal: I can make a line plot to display a data set of measurements in fractions.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: In-class practice after the deck. The cup data (items 1–3) is chosen because the label shortcut does NOT come out right on it — $\frac{3}{4}$ against $\frac{3}{8}$, a factor of two — while the beaker data in items 3–5 forgives it exactly. Setting them on the same sheet is the argument: the same wrong method, one data set flattering it and one exposing it. Item 4 is a small but reliable catch: $\frac{3}{4}$ is what one beaker holds and 3 litres is what four of them hold, and children give the axis value. Items 7 and 8 are the axis thread from the “Line plots with fractions” sheet (Grade 4) and an earlier lesson — note that a plot WITHOUT the empty marks looks tidier, so “neatness” is exactly the wrong instinct here and worth saying aloud. Item 9's answer should reach the point that a count and a volume are different KINDS of number, which is the same units argument as the “Dividing with unit fractions” sheet (Grade 5) item 7. **Timing:** 20–25 min · pairs · squared paper, fraction strips · items 5, 7 and 8 on the board.

1 an eighths axis from 0 to $7/8$, with 3 dots above $1/8$, 2 above $4/8$ (a half), and 1 above $7/8$

2 (a) 6 cups (b) $2\frac{1}{4}$ (c) $3/8$

3 total 5, equal share $5/8$ of a litre

4 3 litres, which is more than half the total

5 $1\frac{1}{2}$ — different because it counts each amount once instead of counting every cup, so three of the six cups are left out

6 the beaker share is $5/8$, and $5/8$ is not one of the plotted values — it sits between $1/2$ and $3/4$

7 because the axis is a number line, and the spacing is what shows how far apart the values are — removing the empty marks makes distant values look adjacent

8 it makes the three values look evenly spread when they are not — $1/8$ and $1/2$ are three eighths apart while $1/2$ and $7/8$ are also three eighths apart, but the plot loses the four missing marks and so hides where nothing was measured

9 the number of measurements has been confused with the amount measured — 8 dots means 8 beakers, and the total is 5 litres

1 MEDIUM The line plot

an eighths axis from 0 to $7/8$, with 3 dots above $1/8$, 2 above $4/8$ (a half), and 1 above $7/8$

1. Smallest unit is an eighth, so the axis is eighths: 0, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$.
2. Equal spacing throughout.
3. $1/2$ is $4/8$, so its two dots go at the fourth mark.
4. $2/8$, $3/8$, $5/8$ and $6/8$ are drawn with no dots above them.

Watch for: Marked $1/2$ before $1/8$ because 2 is less than 8, or left out the empty marks. Both make the plot unreadable as a number line — the first reverses the order, the second hides the spacing.

2 MEDIUM Cups, total, equal share

(a) 6 cups (b) $2\frac{1}{4}$ (c) $3/8$

1. Dots: $3 + 2 + 1 = 6$ cups.
2. Total: $3 \times 1/8 = 3/8$; $2 \times 1/2 = 1$; one $7/8$. So $3/8 + 1 + 7/8 = 21/8$.
3. In eighths: $3/8 + 8/8 + 7/8 = 18/8$, which is $21/8$.
4. Equal share: $18/8 \div 6 = 18/48 = 3/8$.
5. Check: 6 lots of $3/8$ is $18/8 = 21/8$. Correct.

Watch for: Divided by 3 rather than 6, giving $3/4$ — the number of labels used instead of the number of cups. Unlike the beaker data, this one does not forgive it: $3/4$ is twice the right answer.

3 MEDIUM The beaker data

total 5, equal share $5/8$ of a litre

1. Dots: $2 + 1 + 4 + 1 = 8$ beakers.
2. Total: $2 \times 1/4 + 1/2 + 4 \times 3/4 + 1 = 1/2 + 1/2 + 3 + 1 = 5$ litres.
3. $5 \div 8 = 5/8$ of a litre each.
4. Check: $8 \times 5/8 = 5$. Correct.

Watch for: Answered $21/2$ and $5/8$. The share is right and the total is not, which is the coincidence this data set is chosen to expose — see the exit ticket.

4 MEDIUM The four beakers at $3/4$

3 litres, which is more than half the total

1. $4 \times 3/4 = 3$ litres.
2. Half of the total 5 litres is $2\frac{1}{2}$.
3. 3 is more than $2\frac{1}{2}$, so yes — those four beakers hold more than half the liquid.
4. They are half the beakers and more than half the liquid, because $3/4$ is above the average of $5/8$.

Watch for: Answered $3/4$, giving the value on the axis rather than the amount those four beakers hold together. The axis says how much each one holds; the dots say how many there are.

5**SPICY**

The label total

1 1/2 — different because it counts each amount once instead of counting every cup, so three of the six cups are left out

1. Labels: $1/8 + 1/2 + 7/8 = 1/8 + 4/8 + 7/8 = 12/8 = 1 1/2$.
2. The real total is $21/4$, which is $18/8$.
3. What was missed is exactly the extra dots: two more cups at $1/8$ and one more at $1/2$, which come to $2 \times 1/8 + 1/2 = 3/4$.
4. And $1 1/2 + 3/4 = 2 1/4$, the real total — so the label sum is short by $3/4$.
5. So the labels tell you which amounts occurred, and the dots tell you how often. A total needs both.

Watch for: Got $1 1/2$ and said it was “nearly right”. It is $3/4$ short — a third of the true total — and on the beaker data the same error was exactly half.

6**SPICY**

The share need not be on the plot

the beaker share is $5/8$, and $5/8$ is not one of the plotted values — it sits between $1/2$ and $3/4$

1. The plotted values are $1/4$, $1/2$, $3/4$ and 1 .
2. The equal share is $5/8$, which is none of them.
3. $5/8$ lies between $1/2$ ($= 4/8$) and $3/4$ ($= 6/8$), so it would need an eighths axis even to be drawn.
4. A shared-out amount is what everyone WOULD have if it were levelled; there is no reason for it to be an amount anybody actually had.

Watch for: Said the share “should be” $3/4$ because that is the commonest value. The commonest value and the equal share are different questions, and the second need not be a value that occurred at all.

7**SPICY**

Why empty marks are drawn

because the axis is a number line, and the spacing is what shows how far apart the values are — removing the empty marks makes distant values look adjacent

1. A number line's marks are equally spaced, and the position of a dot is what says how big the measurement was.
2. If $2/8$, $3/8$, $5/8$ and $6/8$ are missing, then $1/8$, $1/2$ and $7/8$ sit next to each other and look evenly spread.
3. In fact the data is clustered near the bottom, with one value far off — and that pattern is the whole reason for drawing a plot.
4. So the empty marks carry information: they show where nothing was measured.

Watch for: Said “so it looks neater”. The opposite is often true — the plot looks tidier without them, and that tidiness is the lie. This is the “the diagram means nothing until you know its unit” thread from an earlier lesson onwards.

8



SPICY

Equally spaced labels

it makes the three values look evenly spread when they are not — $1/8$ and $1/2$ are three eighths apart while $1/2$ and $7/8$ are also three eighths apart, but the plot loses the four missing marks and so hides where nothing was measured

1. On a real eighths axis there are marks at $2/8$, $3/8$, $5/8$ and $6/8$ with no dots.
2. Drawing only the three values with data spaces them equally by accident of how many there are, not by their sizes.
3. So the reader cannot see that four possible measurements never occurred.
4. A plot with an unequal or incomplete axis cannot be read at all — the position of a dot no longer means anything, which is the point the whole thread makes.

Watch for: Said the picture is “still correct because the values are right”. The values are; their POSITIONS are what a line plot communicates, and once the spacing is arbitrary the positions say nothing.

9



EXTRA SPICY

Eight dots, eight litres

the number of measurements has been confused with the amount measured — 8 dots means 8 beakers, and the total is 5 litres

1. Each dot stands for one beaker, so the dots count beakers.
2. How much is in each beaker is given by its position on the axis, not by the dot itself.
3. Total: $2 \times 1/4 + 1/2 + 4 \times 3/4 + 1 = 5$ litres.
4. So 8 is the count and 5 is the amount, and they are different kinds of number — one has no unit, the other is in litres.

Watch for: Said “8 is the number of beakers” without naming the total. The two numbers being different KINDS — a count against a volume — is what makes the confusion checkable, and it is the same units point as the “Dividing with unit fractions” sheet (Grade 5).

Exit Ticket

4 problems

Use: End of the line-plots lesson. Item 3 is the diagnostic and it is unusual: it hands the child a CORRECT final answer and asks why the method is still wrong. Sara halved the total and halved the count, and the two errors cancelled — so $5/8$ is luck. That makes this the cleanest instance in Grade 5 of a claim three banks make: a right answer does not validate a method (see also the “Rounding decimals” sheet (Grade 5)’s staged rounding and the “Relating two number patterns” sheet (Grade 5)’s one-pair rule). The item names the second data set where the two routes give $1/2$ and $3/8$, so the coincidence cannot be leaned on. Note that item 1 catches Sara’s error EVEN WHEN item 2 comes out right, which is why the total is asked for separately — without it the mistake is invisible. Item 4 continues the “Line plots with fractions” sheet (Grade 4)’s axis rule and belongs to two named threads: the diagram means nothing until you know its unit, and $1/8$ before $1/2$ despite $8 > 2$. A child who plots only the three values with data has drawn a list, and the list hides the clustering that is the plot’s whole point.

Timing: 10 min · independent · squared paper and fraction strips available.

1 (a) 8 beakers (b) 4 different amounts (c) 5 litres

2 $5/8$ of a litre each

3 her total ignores the dot counts — four beakers held $3/4$ and only one of them was counted, so the real total is 5, not $2\frac{1}{2}$; she still got $5/8$ because she halved the total AND the count, and the two halvings cancelled

4 an axis in eighths from 0 to $7/8$: 0, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$ — and $2/8$, $3/8$, $5/8$ and $6/8$ have no dots

1 **MEDIUM** Beakers, amounts, total

(a) 8 beakers (b) 4 different amounts (c) 5 litres

1. The dots count the beakers: $2 + 1 + 4 + 1 = 8$.
2. The labels count the different amounts: $1/4$, $1/2$, $3/4$ and 1 — four of them.
3. Total: $2 \times 1/4 = 1/2$; one $1/2$; $4 \times 3/4 = 3$; one 1. So $1/2 + 1/2 + 3 + 1 = 5$ litres.
4. The four beakers at $3/4$ hold 3 litres between them, which is more than half the total.

Watch for: Answered $21/2$ for the total — the four labels added once each, with the dot counts ignored. Note this is wrong even though the redistribution question that follows can still come out right.

2 **MEDIUM** Redistributed equally

$5/8$ of a litre each

1. The total is 5 litres and there are 8 beakers.
2. $5 \div 8 = 5/8$ of a litre each.
3. Check: 8 lots of $5/8$ is $40/8 = 5$ litres. Correct.
4. $5/8$ is not one of the amounts on the plot, and it does not need to be — a shared-out amount need not be one that occurred.

Watch for: Divided by 4 instead of 8, using the number of LABELS. On this data that still gives $5/8$ by coincidence; a child who wrote “ $\div 4$ ” should be marked on the reasoning even though the answer matches.

3 **SPICY** Sara's total

her total ignores the dot counts — four beakers held $3/4$ and only one of them was counted, so the real total is 5, not $2\frac{1}{2}$; she still got $5/8$ because she halved the total AND the count, and the two halvings cancelled

1. The plot records 8 measurements. Sara's sum has four terms, so five of the eight beakers were left out.
2. Counting every dot: $2 \times 1/4 + 1/2 + 4 \times 3/4 + 1 = 5$ litres. Her $21/2$ is exactly half of it.
3. She then divided by 4 rather than 8 — also exactly half.
4. Halving the top and the bottom of a fraction leaves it unchanged, so $21/2 \div 4$ and $5 \div 8$ are the same number: $5/8$.
5. So the right answer came from two errors cancelling, which is luck. On a different data set they will not cancel: for $1/8$ three times, $1/2$ twice and $7/8$ once, her method gives $1/2$ and the right answer is $3/8$.

Watch for: Said Sara “got the right answer so her method is fine”, or corrected the total without explaining the coincidence. The coincidence IS the item: a right answer does not validate a method, which is the same argument as the “Rounding decimals” sheet (Grade 5)’s staging check and the “Relating two number patterns” sheet (Grade 5)’s one-pair rule.

4 **SPICY** The axis for the six cups

an axis in eighths from 0 to $7/8$: 0, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$ — and $2/8$, $3/8$, $5/8$ and $6/8$ have no dots

1. The smallest unit in the data is an eighth, so the axis runs in eighths.
2. It must reach the largest value, $7/8$.
3. Marks: 0, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$ — equally spaced.
4. $1/2$ is the same as $4/8$, so the two dots at a half sit at the fourth mark.
5. The marks with no dots are $2/8$, $3/8$, $5/8$ and $6/8$. They still belong on the axis, because it is a number line and the gaps are part of what the plot shows.

Watch for: Drew only the three values that have data — $1/8$, $1/2$, $7/8$ — equally spaced. That is a list, not a number line: it hides that $7/8$ is seven times $1/8$, and it makes the data look evenly spread when it is clustered at the bottom.

Slide	Time	What to do
1	0:00	This continues the “Line plots with fractions” sheet (Grade 4). The axis rules are the same; what is new is USING the plot — totalling and redistributing with fraction arithmetic — which is where the dots-against-labels error lives.
2	0:35	Restate the axis rule from Grade 4: equal steps of the smallest unit, empty marks included. A child who omits the empty marks has drawn a list, not a number line.
3	1:10	The inverse relationship, quickly — it should be familiar by now from six earlier appearances. $1/2$ is $4/8$, so it sits four steps along.
4	1:50	Put the data up and ask two questions: how many beakers, and how many different amounts. Getting 8 and 4 as separate answers is what the next slide depends on.
5	2:35	THE slide. Total by counting every dot. Write the $2\times$ and $4\times$ explicitly, because that multiplication is the thing children omit.
6	3:30	Redistribute: $5 \div 8$. Note that $5/8$ is not on the plot — children look for it and conclude they are wrong. An average need not be a value that occurred.
7	4:20	The most valuable slide, and it takes nerve to teach: the WRONG method gets $5/8$ on this data, because the total and the count were both halved. Say plainly that a right answer does not prove a method — the same lesson as the “Rounding decimals” sheet (Grade 5) and the “Relating two number patterns” sheet (Grade 5).
9	5:45	The second data set exists so the coincidence cannot be leaned on: here the two methods give $3/8$ and $1/2$. Let them find the disagreement. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #5 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11		Launch the practice sheet. Exit ticket in the last 5–8 minutes.