



Multiplication as scaling

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the scaling lesson, and this is the most load-bearing bank in Grade 5. The belief it targets — multiplying makes things bigger — is what makes children reject $0.3 \times 0.4 = 0.12$ in the “Decimal arithmetic” sheet (Grade 5), prefer 2 to $1/6$ in the “Multiplying fractions” sheet (Grade 5), and refuse $6 \div \frac{1}{2} = 12$ in the “Dividing with unit fractions” sheet (Grade 5). So teach this bank BEFORE or alongside those three: a child who holds the belief will quietly change correct answers until they look bigger, which means getting a question right has taught them the wrong lesson. Item 3's second half is the assessment. Zed's rule is a correct generalisation from incomplete evidence, and a child who can explain WHY it held for five years understands something better than the rule itself — and is less likely to swap it for another over-general rule. Items 1 and 4 forbid calculating on purpose: the size judgement is what CHECKS a calculation, so a child who can only reach it by calculating has no check at all. Item 2 is the error that survives agreement with everything else — ask “bigger than which?” every time.

Timing: 5–10 min · independent · no calculators, and no calculating.

1. A multiplier under one **APPLICATION**

Answer: **less than 20**

1. $7/8$ is less than 1, so multiplying by it makes 20 smaller.
2. Only the multiplier was examined — the 20 played no part in the decision.

If wrong: Calculated 17.5 and then compared. The answer is right and the item asked for a judgement WITHOUT calculating, because the judgement is what checks a calculation and so must not depend on one.

2. A multiplier of exactly one **APPLICATION**

Answer: **exactly 20**

1. 1 leaves a number unchanged, so 20×1 is 20.
2. This is the hinge of the whole idea: it is the boundary the other two sit either side of.

If wrong: Said “more”, on the belief that multiplying always increases. This is the item that names the boundary, so getting it wrong matters more than the other two.

3. A multiplier over one APPLICATION

Answer: **more than 20**

1. $9/8$ is more than 1, so multiplying by it makes 20 bigger.
2. $9/8$ is only just over 1, so the answer is only just over 20 — the direction is what was asked.

If wrong: Calculated 22.5 first; or said “less”, because $9/8$ looks like a fraction and fractions are assumed to shrink things. It is an improper fraction and it is greater than one.

4. Bigger or smaller than which REASONING

Answer: **smaller than 9, because 9 was multiplied by $3/5$ which is less than 1; bigger than $3/5$, because $3/5$ was multiplied by 9 which is more than 1**

1. Compared with 9: the multiplier acting on it is $3/5$, and $3/5 < 1$, so the answer is less than 9.
2. Compared with $3/5$: the multiplier acting on it is 9, and $9 > 1$, so the answer is more than $3/5$.
3. Both are true at once, because they are comparisons with different numbers.
4. So the size of a product relative to one factor is decided entirely by the OTHER factor.

If wrong: Answered “smaller” for both, or “bigger” for both — one factor examined and used for both comparisons. Naming the reference number each time is what prevents it.

5. Zed's rule REASONING

Answer: **$8 \times 1/2 = 4$, which is smaller; his rule worked because every multiplier he has used has been a whole number of 2 or more, and all of those do make things bigger**

1. A counter-example: $8 \times 1/2 = 4$, and 4 is less than 8. Or $20 \times 3/4 = 15$.
2. His rule is a correct summary of his evidence: 8×2 , 8×3 , 8×10 all make 8 bigger, and until this year every multiplier he met was a whole number of at least 2.
3. The dividing line is 1. Multiply by more than 1 and a number grows; by exactly 1 and it stays; by less than 1 and it shrinks.
4. He had never multiplied by anything below 1, so his evidence could not have shown him the other two cases.
5. So the rule is not careless — it is a generalisation from incomplete evidence, which is a different thing and is fixed by new evidence rather than by correction.

If wrong: Gave a counter-example and stopped. The second half is the harder and more valuable one: explaining WHY the rule survived means recognising that every multiplier so far was above 1, which is also what stops the child from simply replacing one over-general rule with another.

6. Why $8 \times \frac{3}{4} < 8$ REASONING

Answer: **because $\frac{4}{4}$ is 1, so $8 \times \frac{4}{4} = 8$ leaves it unchanged; $\frac{3}{4}$ is less than $\frac{4}{4}$, so multiplying by $\frac{3}{4}$ must give less than multiplying by $\frac{4}{4}$ — which is less than 8**

1. $\frac{4}{4} = 1$, and multiplying by 1 changes nothing, so $8 \times \frac{4}{4} = 8$.
2. $\frac{3}{4}$ is smaller than $\frac{4}{4}$ — same-sized pieces, three of them instead of four.
3. Multiplying 8 by a smaller multiplier gives a smaller answer.
4. So $8 \times \frac{3}{4} < 8 \times \frac{4}{4} = 8$.
5. No multiplication was performed, which is what the question asked for.

If wrong: Worked out 6 and observed that $6 < 8$. That answers whether the claim is true and not why it MUST be, and the argument via $\frac{4}{4}$ is the one that works for any fraction below 1 without a single calculation.

Quick answers for fast checking:

#	Level	Answer
1	application	less than 20 — $\frac{7}{8}$ is under 1
2	application	exactly 20 — the boundary
3	application	more than 20 — $\frac{9}{8}$ is over 1
4	reasoning	smaller than 9; bigger than $\frac{3}{5}$
5	reasoning	$8 \times \frac{1}{2} = 4$; every multiplier so far was above 1
6	reasoning	$\frac{4}{4} = 1$, and $\frac{3}{4} < \frac{4}{4}$

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 8 is the reason this bank exists and it should be marked hardest: the child in it has a MEANING objection, and answering with the place-counting method from the “Decimal arithmetic” sheet (Grade 5) will not move them — they will conclude the method is broken. Only the scaling argument meets what they actually said. Item 9 explains why every “do not calculate” instruction on both sheets is there, and the answer is INDEPENDENCE rather than speed: a check derived from the thing it checks cannot disagree with it, which is the same argument behind estimate-first in the “Multiplying multi-digit numbers” sheet (Grade 5) and the “Dividing by two-digit divisors” sheet (Grade 5). Item 4 is the error that survives full agreement with the scaling rule — both comparisons look identical until you notice they use different reference numbers. Item 7’s wrong answer is revealing: children offer 0 because it is the do-nothing number for ADDING, and noticing that each operation has its own is worth the detour.

Timing: 20–25 min · pairs · no calculating except items 5 and 6 · items 7, 8 and 9 on the board.

1. Four comparisons with 12 FLUENCY

Answer: **less, the same, more, less**

1. $2/3 < 1$, so less than 12.
2. $5/5 = 1$, so exactly 12.
3. $7/6 > 1$, so more than 12.
4. $1/10 < 1$, so less than 12 — much less, though the question only asked which side.

If wrong: Answered “more” for all four, on the belief that multiplying enlarges. Only one of the four multipliers is above 1, and comparing each with 1 settles all four in seconds.

2. Four products in order APPLICATION

Answer: **$20 \times 1/2$, $20 \times 3/4$, 20×1 , 20×2**

1. All four multiply the same number, 20, so the order of the products is the order of the multipliers.
2. The multipliers in order are $1/2$, $3/4$, 1, 2.
3. So the products are in that same order.
4. No multiplication is needed at all — and this is why scaling is worth understanding: it turns a calculation into a comparison.

If wrong: Calculated all four (10, 20, 15, 40) and then ordered them. Correct, and the point of the item is that a common factor makes the products' order identical to the multipliers' order.

3. Scaled by $3/4$ then $4/3$ REASONING

Answer: **12 cm again — because $3/4 \times 4/3 = 1$, so the two scalings undo each other**

1. Scaling by $3/4$ then by $4/3$ is the same as scaling once by $3/4 \times 4/3$.
2. $3/4 \times 4/3 = 12/12 = 1$.
3. Scaling by 1 changes nothing, so the photo is 12 cm.
4. Sensibly: shrinking to three quarters and then growing by a third of itself returns it exactly, which is why $3/4$ and $4/3$ are called inverses.

If wrong: Worked out 9 cm and then 12 cm. That gets there and does not answer the item, which asked for the reason — and the reason generalises to any pair of fractions that multiply to 1.

4. 1.5×0.6 , twice compared REASONING

Answer: **smaller than 1.5, because 0.6 is less than 1; bigger than 0.6, because 1.5 is more than 1**

1. Compared with 1.5: the multiplier is 0.6, which is under 1, so the answer is less than 1.5.
2. Compared with 0.6: the multiplier is 1.5, which is over 1, so the answer is more than 0.6.
3. So it lies between 0.6 and 1.5.
4. Checking (though not required): 0.9, which is indeed between them.

If wrong: Answered “smaller” to both. The second comparison uses the OTHER factor, and forgetting that is the error that survives even when the scaling rule is understood.

5. Multiplying by a half APPLICATION

Answer: **3**

1. $6 \times 1/2$ is half OF 6, which is 3.
2. The multiplier is under 1, so the answer is smaller than 6.

If wrong: Answered 12, reading it as the division. Multiplying by a half halves.

6. Dividing by a half APPLICATION

Answer: **12**

1. $6 \div 1/2$ asks how many halves fit in 6.
2. Two per whole, six wholes, so 12.

If wrong: Answered 3 — the division treated as another halving. Asking “how many halves fit in 6” makes 12 obvious, and the “Dividing with unit fractions” sheet (Grade 5) develops it.

7. Why they go opposite ways REASONING

Answer: **because multiplying by $1/2$ takes half of the number, while dividing by $1/2$ counts how many halves are in it — and pieces smaller than one whole means more of them**

1. $6 \times 1/2 = 3$: a part OF 6, so smaller.
2. $6 \div 1/2 = 12$: a COUNT of pieces, and the pieces are smaller than one, so there are more of them than there were wholes.
3. So multiplying by $1/2$ halves a number and dividing by $1/2$ doubles it — opposite effects, as they should be.
4. The general rule: with a multiplier under 1, multiplying shrinks and dividing grows.

If wrong: Said “because one is times and one is divide”, which restates the question. The reason is that one takes a part and the other counts parts.

8. A fraction between 15 and 20 APPLICATION

Answer: **any fraction between 1 and $4/3$, such as $6/5$ — because $15 \times 6/5 = 18$**

1. To exceed 15 the multiplier must be more than 1.
2. To stay under 20 it must be less than $20/15$, which is $4/3$.
3. So anything strictly between 1 and $4/3$ works: $6/5$, $5/4$, $9/8$ and so on.
4. Checking one: $15 \times 6/5 = 18$, which is between 15 and 20.

If wrong: Chose a multiplier under 1, such as $3/4$, giving 11.25 — below 15. The first condition is that the answer must GROW, so the multiplier has to be above 1 before anything else is considered.

9. Why 1 and not 0 REASONING

Answer: **because multiplying by 1 leaves a number unchanged, so 1 is where growing turns into shrinking; multiplying by 0 gives 0, which is not the turning point but the end of the line**

1. $8 \times 1 = 8$ — no change. So 1 separates the multipliers that enlarge from those that shrink.
2. $8 \times 0 = 0$, which is smaller than 8, so 0 is on the shrinking side along with everything between 0 and 1.
3. Nothing special happens AT 0 for this question; it is simply the smallest a multiplier can be here.
4. Children guess 0 because it is the boundary in ADDITION — adding 0 changes nothing — and every operation has its own do-nothing number.

If wrong: Said “because 1 is the middle”. 1 is not in the middle of anything here; it is the number that leaves things unchanged, and each operation has a different such number — 0 for adding, 1 for multiplying.

10. Answering the 1.2 claim REASONING

Answer: **0.12 is right: 0.4 is less than 1, so multiplying 0.3 by it must give less than 0.3 — and 1.2 is bigger than both numbers, which no product of two numbers under 1 can be**

1. 0.3×0.4 means four tenths OF three tenths, a part of 0.3, so it must be less than 0.3.
2. 0.12 is less than 0.3 and less than 0.4, exactly as scaling predicts.
3. 1.2 is bigger than both factors. Two numbers below 1 cannot multiply to something above either of them, let alone above 1.
4. So the “too small” objection has it backwards: 0.12 is the right size and 1.2 is the impossible one.

If wrong: Said “you have to count the decimal places”. That is the method from the “Decimal arithmetic” sheet (Grade 5) and it will not persuade someone whose objection is about meaning — they will believe the method has gone wrong. The scaling argument meets the actual objection.

11. Why compare without multiplying REASONING

Answer: **because the comparison is what CHECKS the multiplication — if you have to calculate to know the size, you have nothing independent to check the calculation against**

1. A calculation can go wrong. A size judgement made separately catches it.
2. If the judgement comes FROM the calculation, both are wrong together and the error is invisible.
3. This is the same reason the “Multiplying multi-digit numbers” sheet (Grade 5) and the “Dividing by two-digit divisors” sheet (Grade 5) ask for an estimate before dividing or multiplying — an independent expectation is the only kind that can disagree.
4. It is also faster, and it works on questions where the arithmetic would be horrible: $4783 \times 7/8$ is less than 4783, and nobody needs to multiply to know it.

If wrong: Said “because it is quicker”. Speed is a side benefit; the real reason is INDEPENDENCE — a check derived from the thing it is checking is not a check, which is the same argument as the estimate-first habit in the base-ten banks.

Quick answers for fast checking:

#	Level	Answer
1	fluency	less, same, more, less
2	application	1/2, 3/4, 1, 2 order
3	reasoning	12 cm; $3/4 \times 4/3 = 1$
4	reasoning	smaller than 1.5; bigger than 0.6
5	application	3
6	application	12

7	reasoning	one takes a part of 6; the other counts parts smaller than 1, so there are more
8	application	between 1 and $\frac{4}{3}$, e.g. $\frac{6}{5}$
9	reasoning	1 leaves a number unchanged; 0 is not a turning point
10	reasoning	$0.4 < 1$, so the product is under 0.3; 1.2 exceeds both
11	reasoning	the check must be independent of the calculation

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Teach this bank BEFORE or alongside the “Decimal arithmetic” sheet (Grade 5), the “Multiplying fractions” sheet (Grade 5) and the “Dividing with unit fractions” sheet (Grade 5). The belief it targets is what makes children reject correct answers in all three, and a child who has “fixed” a right answer to look bigger has learned the wrong lesson from getting it right.
2/8	0:40	Concede the belief properly and at length. List multiplications that DO make things bigger and say plainly: this is a correct summary of every multiplication you have met. Nothing here is about carelessness, and children who feel accused stop thinking.
3/8	1:30	Multiplying by exactly 1 is the hinge, and it is the one case children accept instantly. 1 is the dividing line — not 0, which is the other guess.
4/8	2:10	THE slide, and it is the standard's own argument. $\frac{4}{4}$ IS 1, so multiplying by it changes nothing; $\frac{3}{4}$ is less than $\frac{4}{4}$, so multiplying by it must give less. Watch for the moment children realise no arithmetic happened.
5/8	3:10	The three cases on the board, to stay there for the rest of the term. The instruction “never calculate” is not laziness — the judgement is what CHECKS a calculation, so it has to be independent of it.
6/8	4:10	The question children get wrong even after accepting everything above: bigger than WHICH? $\frac{2}{3} \times 6$ is bigger than $\frac{2}{3}$ and smaller than 6, and both are true. Insist on naming the reference number every time.
7/8	5:20	Division, as the mirror image. Ask it as “how many halves fit in 6” and the 12 stops being surprising. the “Dividing with unit fractions” sheet (Grade 5) is the full lesson.
8/8	6:30	Both prompts forbid calculating. If a child calculates and then compares, they have answered a different question — say so gently and ask again. Exit ticket after.