



Multiplying fractions

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the multiplying-fractions lesson. Item 3 is the diagnostic and its whole point is INTERFERENCE: Sam's conversion to twelfths is correct, and it is the child who has just mastered the “Adding and subtracting unlike denominators” sheet (Grade 5) who reaches for it. So a class that did well on adding is at MORE risk here, not less — the same relationship the “Decimal arithmetic” sheet (Grade 5) has with lining up decimal points, and the two banks are worth teaching as a pair. The answer to look for distinguishes the two operations by what they DO: adding counts pieces, so they must be the same size; multiplying makes a new size, so the bottoms multiply. Item 4 asks what Ada missed rather than what she should have done — naming the two cross terms is the transferable part, because the identical omission drops two terms of a bracket expansion in algebra. Note item 1's second part: a child who answers 3 has done the first of two steps, which is an unfinished method rather than a wrong one and should be marked that way.

Timing: 5–10 min · independent · fraction strips available.

1. A product and a fraction of a whole **FLUENCY**

Answer: **3/10 and 15**

1. $\frac{3}{4} \times \frac{2}{5}$: tops $3 \times 2 = 6$, bottoms $4 \times 5 = 20$, so $\frac{6}{20} = \frac{3}{10}$.
2. Check: both factors are under 1, so the answer is under both — $\frac{3}{10}$ is less than $\frac{3}{4}$ and less than $\frac{2}{5}$.
3. $\frac{5}{6}$ of 18: $18 \div 6 = 3$, then five of them = 15.
4. Check: $\frac{5}{6}$ is nearly 1, so the answer should be a little under 18, and 15 is.

If wrong: Answered 3 for the second, having divided by 6 and stopped. That is the first of two steps — an unfinished method rather than a wrong one, and worth marking as such.

2. Two and a half times one and a half FLUENCY

Answer: $3\frac{3}{4}$

1. Convert: $2\frac{1}{2} = \frac{5}{2}$ and $1\frac{1}{2} = \frac{3}{2}$.
2. Tops $5 \times 3 = 15$, bottoms $2 \times 2 = 4$, so $15/4$.
3. $15/4 = 3\frac{3}{4}$.
4. Check: multiplying $2\frac{1}{2}$ by more than one must make it bigger, and $3\frac{3}{4}$ is bigger than $2\frac{1}{2}$.

If wrong: Answered $2\frac{1}{4}$ by multiplying the wholes and the fractions separately. It is SMALLER than $2\frac{1}{2}$, which is impossible when multiplying by $1\frac{1}{2}$ — more than one lot of something exceeds it.

3. Sam's answer of 2 REASONING

Answer: **because both numbers are less than one, so the answer must be smaller than both, and 2 is bigger than both; a common denominator is for adding — multiplying just needs tops times tops and bottoms times bottoms, giving $2/12 = 1/6$**

1. $2/3$ and $1/4$ are both under 1. Multiplying by a number under one makes something smaller, so the answer must be under $1/4$.
2. Sam's 2 is bigger than either factor, so it is impossible before any method is checked. It is twelve times the right answer.
3. His conversion was correct and irrelevant: a common denominator is needed to ADD, because you cannot count pieces of different sizes. Multiplying does not count anything.
4. $2/3 \times 1/4$ means a quarter OF two thirds: cut two thirds into four and keep one piece, giving $2/12 = 1/6$.
5. The twelfths arise from the cutting — a third cut into four IS a twelfth — which is why the bottoms multiply rather than being made equal first.

If wrong: Said Sam “does not need a common denominator for multiplying”. True and it is the rule without the reason, so it will not survive the next mixed-up moment. The reason is that adding COUNTS pieces and so needs them the same size, while multiplying MAKES a new size.

4. Ada's missing pieces REASONING

Answer: **she has missed $2 \times \frac{1}{2}$ and $1 \times \frac{1}{2}$ — that is 1 and $\frac{1}{2}$, which together with her $2\frac{1}{4}$ gives $3\frac{3}{4}$**

1. $2\frac{1}{2} \times 1\frac{1}{2}$ means every part of the first multiplied by every part of the second.
2. Ada has $2 \times 1 = 2$ and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, giving $2\frac{1}{4}$.
3. She has not done $2 \times \frac{1}{2} = 1$, nor $1 \times \frac{1}{2} = \frac{1}{2}$.
4. Those two come to $1\frac{1}{2}$, and $2\frac{1}{4} + 1\frac{1}{2} = 3\frac{3}{4}$ — the right answer.
5. Converting to $\frac{5}{2} \times \frac{3}{2}$ avoids the whole problem, because there are then no parts to forget.

If wrong: Said Ada “should have used improper fractions”. That is the safer method and it does not answer the item, which asks what she MISSED. Naming the two cross terms is what makes the error visible — and the same omission loses two terms of a bracket expansion later.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$3/10$ and 15
2	fluency	$3\frac{3}{4}$
3	reasoning	under both factors; adding counts pieces, multiplying makes a new size
4	reasoning	$2 \times \frac{1}{2}$ and $1 \times \frac{1}{2}$, which are $1\frac{1}{2}$

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 9 is the sheet's central item and its answer is the same one as the “Decimal arithmetic” sheet (Grade 5) item 9 for decimals: adding COUNTS, so units must match beforehand; multiplying CUTS, so the unit comes out of the operation. A child who holds that needs neither rule memorised. Item 2 is worth marking as a set — 4, 5 and 3 is one unfinished method applied three times, not three errors, and item 6 asks the child to name what was already right. Item 4's drawing is where the bottoms-multiply rule becomes visible: 4 columns times 5 rows IS the 20 in the denominator. Item 7 borrows the “Multiplication as scaling” sheet (Grade 5)'s content, which is the next bank, and children who can answer it without calculating are ready for it.

Timing: 20–25 min · pairs · fraction strips and squared paper · items 5, 7 and 9 on the board.

1. A half of a third FLUENCY

Answer: **1/6**

1. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ — tops multiplied, bottoms multiplied.
2. Smaller than both factors, because both are under 1.
3. In words: half of a third is a sixth.

If wrong: Answered 2 or similar, from converting to a common denominator and keeping it. The answer is bigger than both factors, which cannot happen when both are under one.

2. A product that simplifies FLUENCY

Answer: **1/6**

1. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$, which simplifies to $\frac{1}{6}$.
2. Smaller than both factors.
3. Note it equals item 1's answer by a different route — worth pointing out.

If wrong: Left it as $\frac{2}{12}$, which is correct but unsimplified; or answered bigger than $\frac{2}{3}$, which is impossible here.

3. Neither is a unit fraction FLUENCY

Answer: **3/10**

1. $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$, which simplifies to $\frac{3}{10}$.
2. Smaller than both factors.
3. This is the one where cancelling first is quicker: the 2 and the 4 share a factor.

If wrong: Answered $\frac{6}{20}$ unsimplified, or added the fractions instead. $\frac{3}{10}$ is less than both $\frac{3}{4}$ and $\frac{2}{5}$, which is the size check.

4. Three fractions of wholes FLUENCY

Answer: **8, 15 and 15**

1. $\frac{2}{3}$ of 12: $12 \div 3 = 4$, times 2 = 8.
2. $\frac{3}{4}$ of 20: $20 \div 4 = 5$, times 3 = 15.
3. $\frac{5}{6}$ of 18: $18 \div 6 = 3$, times 5 = 15.
4. Each is less than the whole number, since each fraction is under 1.

If wrong: Answered 4, 5 and 3 — divided by the bottom and stopped, on all three. One consistent unfinished method rather than three slips.

5. A mixed number by a mixed number APPLICATION

Answer: $3\frac{3}{4}$

1. Convert both: $\frac{5}{2} \times \frac{3}{2} = \frac{15}{4} = 3\frac{3}{4}$.
2. Bigger than $2\frac{1}{2}$, because it is multiplied by more than one.
3. Converting first is what avoids the missed cross-pieces — see the exit ticket.

If wrong: Answered $2\frac{1}{4}$, multiplying the wholes and the fractions separately and missing the two cross-pieces. That is a real and common method applied incompletely, not a slip.

6. A mixed number by a whole APPLICATION

Answer: **7**

1. $\frac{7}{2} \times 2 = \frac{14}{2} = 7$.
2. Or: $3\frac{1}{2}$ doubled is 7.
3. Bigger than $3\frac{1}{2}$, as multiplying by 2 must be.

If wrong: Answered 6, doubling only the whole part and dropping the half. Doubling $3\frac{1}{2}$ is worth checking mentally — two halves make another whole.

7. A rectangle $\frac{3}{4}$ by $\frac{2}{5}$ APPLICATION

Answer: **$\frac{3}{10}$ of a square metre**

1. Area is length times width: $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$.
2. Picture a square metre cut into 4 columns and 5 rows, giving 20 cells, each $\frac{1}{20}$ of a square metre.
3. The rectangle covers 3 columns and 2 rows, so 6 of those cells.
4. $\frac{6}{20} = \frac{3}{10}$ of a square metre — and this picture is why the bottoms multiply: 4×5 gives the number of cells.

If wrong: Answered $\frac{6}{9}$ or $1\frac{1}{6}$, adding somewhere in the process. Area is a multiplication, and the drawing settles it: 20 cells, 6 covered.

8. Why sixths REASONING

Answer: **because a third cut in half is a sixth — the new unit is made by the cutting, so it need not appear in either fraction**

1. $\frac{1}{2} \times \frac{1}{3}$ means half OF a third.
2. Cutting a third in half makes pieces of which six fit in a whole, so each is a sixth.
3. So the answer is one sixth — and the sixth was created by the cutting, not taken from either fraction.
4. That is why the denominators multiply: halves of thirds are sixths, and $2 \times 3 = 6$.

If wrong: Said “because $2 \times 3 = 6$ ”. That is the rule and the item asks why it works; the cutting argument is the reason, and it is what makes the rule memorable rather than arbitrary.

9. $\frac{2}{3}$ of 12 answered 4 REASONING

Answer: **they divided 12 by 3 correctly, which is the first step; they still need to multiply by 2, giving 8**

1. $\frac{2}{3}$ of 12 means splitting 12 into 3 equal parts and taking 2 of them.
2. $12 \div 3 = 4$ finds ONE third. That step is right.
3. Taking two of them gives 8.
4. So the method is unfinished rather than wrong — and the check confirms it: $\frac{2}{3}$ is more than a half, so the answer should be more than 6, and 4 is not.

If wrong: Said they “did it wrong”. They did half of it right, and saying which half is what tells the child where to resume — a much more useful piece of feedback than a cross.

10. $\frac{2}{3} \times 6$ or $\frac{2}{3} \times \frac{1}{6}$ REASONING

Answer: **$\frac{2}{3} \times 6$ is bigger, because 6 is more than 1 so it makes $\frac{2}{3}$ grow, while $\frac{1}{6}$ is less than 1 so it makes $\frac{2}{3}$ shrink**

1. Multiplying by more than 1 gives more than you started with: $\frac{2}{3} \times 6$ is more than $\frac{2}{3}$.
2. Multiplying by less than 1 gives less: $\frac{2}{3} \times \frac{1}{6}$ is less than $\frac{2}{3}$.
3. So one is above $\frac{2}{3}$ and the other below it, and the first is bigger.
4. No calculation needed, though checking gives 4 and $\frac{1}{9}$.

If wrong: Calculated both and compared. Correct, and the comparison follows from whether each multiplier is above or below 1 — which is the “Multiplication as scaling” sheet (Grade 5)’s content and works for any pair.

11. Flour for 1 1/2 batches APPLICATION

Answer: **1 cup**

1. $\frac{2}{3} \times 1\frac{1}{2} = \frac{2}{3} \times \frac{3}{2}$.
2. Tops $2 \times 3 = 6$, bottoms $3 \times 2 = 6$, so $\frac{6}{6} = 1$.
3. Exactly 1 cup.
4. Check: one batch needs $\frac{2}{3}$, and half a batch needs $\frac{1}{3}$, and $\frac{2}{3} + \frac{1}{3} = 1$. The two routes agree.

If wrong: Answered $\frac{2}{3} \times 1 = \frac{2}{3}$ and ignored the half batch, or added instead of multiplying. The second route in the steps — $\frac{2}{3}$ plus half of $\frac{2}{3}$ — is a good independent check.

12. Two operations, two rules REASONING

Answer: **because adding COUNTS pieces together, and you can only count pieces that are the same size; multiplying does not count anything — it cuts one amount by the other, and the cutting makes a new size of piece**

1. $\frac{1}{2} + \frac{1}{3}$: halves and thirds cannot be counted together, so both are recut into sixths first. The sixths are AGREED in advance.
2. $\frac{1}{2} \times \frac{1}{3}$: half of a third is a single act of cutting, and the sixth it produces is the RESULT.
3. So one rule makes the units match before starting, and the other lets the units come out of the operation.
4. This is why a child who has just mastered adding is at more risk here: the common-denominator step feels like the responsible thing to do, and it is answering a question that was not asked.

If wrong: Said “because they are different operations”. The distinction to reach for is counting against cutting: adding needs matching units up front, multiplying manufactures the unit — the same explanation as the “Decimal arithmetic” sheet (Grade 5) item 9 for decimals.

Quick answers for fast checking:

#	Level	Answer
1	fluency	1/6
2	fluency	1/6
3	fluency	3/10
4	fluency	8, 15, 15
5	application	3¾
6	application	7

7	application	3/10 of a square metre; 6 of 20 cells
8	reasoning	a third halved is a sixth; the unit is made by cutting
9	reasoning	dividing by 3 is step one; multiply by 2 for 8
10	reasoning	6 is over 1 so it grows; 1/6 is under 1 so it shrinks
11	application	1 cup
12	reasoning	adding counts and needs matching units; multiplying cuts and makes one

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Say the headline in the first minute: the common-denominator rule is for adding and does not belong here. Children who have just mastered the “Adding and subtracting unlike denominators” sheet (Grade 5) are the ones at risk, so framing this as a WHICH-RULE lesson rather than a new procedure is the point.
2/8	0:40	Do the wrong method in full and get 2. Do not correct it yet. Two numbers under one produced an answer of 2, and the class should feel that before hearing why.
3/8	1:30	The check, borrowed from the “Multiplication as scaling” sheet (Grade 5) a lesson early because it is needed here: multiplying by less than one makes things smaller, so the answer is under both factors. That rejects 2 with no method at all.
4/8	2:20	THE slide. Read $\times$ as “of” and do the cutting: two thirds, cut into four, keep one. A bar model or folded paper strip makes this concrete in seconds.
5/8	3:20	Where the twelfths came from, and this is the reason the bottoms multiply. The new unit is MADE by the cutting — nobody agreed it in advance, which is precisely the difference from adding.
6/8	4:20	A fraction of a whole number. The child who answers 4 has done the first of two steps, so say that: divide by the bottom, then multiply by the top. Not a wrong method — an unfinished one.
7/8	5:20	Mixed numbers. Convert to improper fractions and the difficulty disappears. Show the part-by-part answer and name the two missing pieces — this is the distributive law, and the same omission loses two terms of a bracket in algebra.
8/8	6:45	Three products, with the bigger-or-smaller judgement first each time. Exit ticket after.