



Relating two number patterns

GRADE 5 MATH

Answer keys for the exit ticket and practice worksheet, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the relating-two-patterns lesson. Item 3 is the diagnostic and what it is really assessing is the CHECK, not the answer — the sentence worth looking for is that a single pair of numbers fits both “6 more” and “3 times”, so testing once tests nothing. That is the load-bearing idea in a thread this bank belongs to: the same additive instinct is “4 times as many” read as “4 more” in the “Multiplicative comparison” sheet (Grade 4), adding to both parts of a fraction in the “Equivalent fractions” sheet (Grade 4), comparing ratios by subtracting in Grade 6, and calling 3-4-5 and 6-7-8 similar in the “Similarity and dilations” sheet (Grade 8). Watch also for item 2 answered as “it goes up by 9”: that is a correct description of one pattern offered in place of a comparison, and it is the most common non-answer here. Item 1 looks trivial and is not — a dropped 0 shifts every corresponding pair by one position.

Timing: 5–8 min · independent · squared paper available.

1. Both patterns **FLUENCY**

Answer: **0, 3, 6, 9, 12 and 0, 9, 18, 27, 36**

1. Start at 0 and add 3 four times: 0, 3, 6, 9, 12.
2. Start at 0 and add 9 four times: 0, 9, 18, 27, 36.
3. The starting number counts as the first term — five terms means four additions.

If wrong: Gave 3, 6, 9, 12, 15 — four terms and a fifth, with the 0 dropped because it looks like nothing has happened yet. It is the first term, and dropping it shifts every pair on the rest of the sheet.

2. Ordered pairs and the relationship **APPLICATION**

Answer: **(0,0), (3,9), (6,18), (9,27), (12,36) — the second pattern is three times the first**

1. Pair each term with the one in the same position: (0,0), (3,9), (6,18), (9,27), (12,36).
2. $9 = 3 \times 3$, $18 = 3 \times 6$, $27 = 3 \times 9$, $36 = 3 \times 12$.
3. So every term of the second is three times the matching term of the first.

If wrong: Answered “it goes up by 9”. That describes the second pattern correctly and is not a comparison between the two — the question is about corresponding terms, and this answer never looks at the first pattern at all.

3. Sam's rule REASONING

Answer: **the gaps are 0, 6, 12, 18, 24, so only one pair differs by 6; he checked his rule on a single pair, and one pair cannot tell 6 more from 3 times**

1. 6 and 18 differ by 12, not 6. 12 and 36 differ by 24.
2. Written out, the gaps are 0, 6, 12, 18, 24 — growing, not constant.
3. His rule would predict 18 for the last term, and the pattern reaches 36 — double.
4. The method error is the important one: 3 and 9 differ by 6 AND 9 is three times 3, both at once. Any single pair fits more than one rule, so a rule tested once is untested.

If wrong: Showed that 6 does not work for later pairs without naming why the check failed. The transferable lesson is about the CHECK — test on a second pair — and a child who only learns “this one was times, not more” will make the same error on the next pattern, and again on ratios in Grade 6.

4. Why three times REASONING

Answer: **because 9 is three times 3, so each step of the second pattern is three of the first pattern's steps, and after the same number of steps from 0 it has gone three times as far**

1. Both patterns start at 0, so after n steps the first has n lots of 3 and the second has n lots of 9.
2. 9 is 3×3 , so n lots of 9 is three times n lots of 3.
3. That holds for every n , which is why the multiplier is the same at every position.
4. The multiplier is $9 \div 3$ — the ratio of the step sizes — and that is where it comes from rather than being spotted.

If wrong: Said “because $9 \div 3 = 3$ ” with nothing about steps. The division is right and the argument is what makes it more than a coincidence at one position — and it also shows where the rule needs both patterns to start at 0.

Quick answers for fast checking:

#	Level	Answer
1	fluency	0, 3, 6, 9, 12 and 0, 9, 18, 27, 36
2	application	(0,0), (3,9), (6,18), (9,27), (12,36); three times
3	reasoning	gaps 0, 6, 12, 18, 24; a rule checked on one pair is unchecked
4	reasoning	each step is three steps; $9 \div 3$, and both start at 0

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 2 is worth more than it looks: the gaps for add-2 / add-8 are 0, 6, 12, 18, 24 — identical to the add-3 / add-9 gaps from the deck, for a multiplier of 4 rather than 3. So a child using differences gets the same wrong description for two different relationships, which is a demonstration

rather than a correction. Items 6 and 7 are the honest pair and are usually left out of this standard: every worked example starts at 0, which quietly makes “three times” look general. Item 8 asks for the prediction BEFORE the terms, which is the difference between knowing where the multiplier comes from and spotting it. Item 9 names the thread explicitly — take it to the board and let children find the “Multiplicative comparison” sheet (Grade 4) connection themselves.

Timing: 15–20 min · pairs · squared paper · items 2, 7 and 9 on the board.

1. Add 2 and add 8 from 0 FLUENCY

Answer: **0, 2, 4, 6, 8 and 0, 8, 16, 24, 32**

1. 0, then add 2 four times: 0, 2, 4, 6, 8.
2. 0, then add 8 four times: 0, 8, 16, 24, 32.

If wrong: Dropped the 0 from one pattern but not the other, which misaligns every pair that follows.

2. The relationship APPLICATION

Answer: **the second is four times the first**

1. $8 = 4 \times 2$. Checking a second pair: $24 = 4 \times 6$.
2. The multiplier is $8 \div 2 = 4$, the ratio of the step sizes.
3. So every term of the second is four times the matching term of the first.

If wrong: Said “6 more”. The gaps here are 0, 6, 12, 18, 24 — the same gaps as the add-3 / add-9 pair, for a completely different multiplier, which is a neat demonstration that the gap does not identify the relationship.

3. Ordered pairs and the plot APPLICATION

Answer: **(0,0), (2,8), (4,16), (6,24), (8,32) — a straight line through the origin, going up 8 for every 2 across**

1. Pair terms in matching positions: (0,0), (2,8), (4,16), (6,24), (8,32).
2. From each point to the next is 2 right and 8 up, every time.
3. Equal steps in the same direction make a straight line, and it passes through (0,0) because both patterns start at 0.

If wrong: Wrote the pairs as (8,0), (16,2) ... with the second pattern first. Order matters, and the “The coordinate plane” sheet (Grade 5) is where the same reversal shows up as a plotted point in the wrong place.

4. The differences REASONING

Answer: **0, 6, 12, 18, 24** — a growing difference means the relationship is not a difference at all, so it must be described some other way

1. The pairs are (0,0), (3,9), (6,18), (9,27), (12,36), so the differences are 0, 6, 12, 18, 24.
2. If the relationship really were “6 more”, every difference would be 6.
3. Instead each difference is 6 bigger than the last.
4. And that is exactly what tripling looks like if you insist on subtracting: three times a number, minus the number, is twice the number, which grows as the number grows.

If wrong: Listed the differences and stopped. The list is the evidence; the conclusion — a changing difference rules out a difference rule — is what the item asks for.

5. The 10th terms APPLICATION

Answer: **81**

1. Every term of the second pattern is three times the matching term of the first.
2. The 10th term of the first is 27.
3. So the 10th term of the second is $3 \times 27 = 81$.
4. This is what a relationship is FOR — it answers a question about the 10th term without generating ten of them.

If wrong: Wrote out all ten terms. That works and it is the slow route; the point of finding the relationship is that it makes listing unnecessary.

6. Starting both at 1 REASONING

Answer: **no** — **10 is not three times 4, and 19 is not three times 7; it is true only for the first term**

1. First pair: 1 and 1. Three times 1 is 1, so that one works.
2. Second pair: 4 and 10. Three times 4 is 12, not 10.
3. Third pair: 7 and 19. Three times 7 is 21, not 19.
4. So the “three times” rule fails from the second term on. What is true is that each term is three times the first, less 2.

If wrong: Answered yes after checking only the first pair — which is the same one-pair error as Sam's on the exit ticket, arriving from the opposite direction. The 1 and 1 pair fits every multiplier there is.

7. Why the 0 mattered REASONING

Answer: **because from 0 each pattern is just its step repeated, so three times the step gives three times the total; a start of 1 adds a 1 to both, and the extra 1 is not tripled**

1. From 0, after n steps the patterns are $n \times 3$ and $n \times 9$, and 9 is three times 3.
2. From 1, they are $1 + n \times 3$ and $1 + n \times 9$. Tripling the first gives $3 + n \times 9$, which is 2 more than the second.
3. So the two differ by exactly 2 wherever you look — the start was tripled when it should not have been.
4. The clean “three times” is therefore a fact about patterns that start at 0, not about the rules on their own.

If wrong: Said “because 0 times anything is 0”. True about the first term, and the question is about the rest — the useful answer is that a non-zero start gets multiplied too, and it should not be.

8. Add 5 and add 20 APPLICATION

Answer: **four times, and it checks: $20 = 4 \times 5$ and $60 = 4 \times 15$**

1. The multiplier is the ratio of the steps: $20 \div 5 = 4$.
2. So predict “four times” before writing anything.
3. Terms: 0, 5, 10, 15, 20 and 0, 20, 40, 60, 80. Check two pairs: $20 = 4 \times 5$, and $60 = 4 \times 15$.
4. Predicting first and checking second is the habit; generating first and looking for a pattern is what leads to the difference answer.

If wrong: Generated both patterns and then hunted for a relationship, arriving at “15 more” from the second pair. The step ratio gives the answer before any terms exist.

9. The same mistake as “4 more” REASONING

Answer: **both describe a multiplying relationship by subtracting, and both look right on the one example the child checked**

1. “4 times as many” read as “4 more” happens to be correct when the first number is $4 \div 3 \dots$ and in practice, whenever one example is all that is tested.
2. Here, 3 and 9 differ by 6 AND 9 is three times 3, so one pair supports both rules.
3. In each case the fix is the same: try a second, different example, and the additive rule falls apart while the multiplicative one holds.
4. This is why the check matters more than the answer — the same instinct returns for ratios, for equivalent fractions, and for similar figures.

If wrong: Said “both are wrong answers about multiplying”. The sharper connection is about EVIDENCE: in both cases a single example is consistent with both rules, so the error is invisible until a second example is tried.

Quick answers for fast checking:

#	Level	Answer
1	fluency	0, 2, 4, 6, 8 and 0, 8, 16, 24, 32
2	application	four times; $8 \div 2$
3	application	(0,0), (2,8), (4,16), (6,24), (8,32); a line through the origin
4	reasoning	0, 6, 12, 18, 24; a changing difference rules out a difference rule
5	application	81, by tripling 27
6	reasoning	no — true for the first term only
7	reasoning	a non-zero start gets multiplied too, and should not be
8	application	four times, from $20 \div 5$, predicted before generating
9	reasoning	one example fits both rules, so the error is invisible until a second

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	The question this lesson answers is narrow and worth stating: not how each pattern grows, but how the two compare AT THE SAME POSITION. Children answer the first question when asked the second more often than anything else here.
2/8	0:45	Generate both on the board in two rows, vertically aligned, so corresponding terms sit above one another. The alignment does real work — a horizontal list invites comparing neighbours within a row.

3/8	1:30	Ask for the relationship and expect “6 more”. Do not correct it. Write it down as a claim to be tested, then move to the next pair — the child should be the one who finds it fails.
4/8	2:30	THE slide. The gaps written out are the whole argument: 0, 6, 12, 18, 24. Name the general lesson explicitly, because it is the thread this belongs to — one pair of numbers can never distinguish “6 more” from “3 times”, so a rule checked once is not checked.
5/8	3:30	The multiplicative answer, and then the part worth the extra minute: the multiplier IS the ratio of the two step sizes, $9 \div 3$. That is why it holds for every pair rather than by luck.
6/8	4:30	Ordered pairs, and say aloud that the first coordinate comes from the first pattern. Plot them if there is time — the straight line through the origin is the picture of “three times”, and the “The coordinate plane” sheet (Grade 5) is where the plotting itself is taught.
7/8	5:45	The honest slide, and it is often skipped. Every textbook example starts at 0, which makes the clean “three times” look like a general truth. Start at 1 and it breaks. Children should know the 0 was a condition, not decoration.
8/8	7:00	The add-2 / add-8 pair, with the instruction to test on two pairs rather than one. Exit ticket after.