

GRADE 5 • GEOMETRY • ANSWER KEY

THE COORDINATE PLANE

Today's goal: I can use a coordinate system; graph points in the first quadrant and represent real-world problems by graphing points in the first quadrant.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems + Mr DJ's worked example

Use: In-class practice after the deck. Items 1 and 8 are the sheet's real content, and neither is about geometry: a point on the diagonal is one where the correct method and the reversed one AGREE, so it cannot tell them apart. Grade 5 makes this argument three times — the “Relating two number patterns” sheet (Grade 5)’s one-pair rule, the “Line plots with fractions” sheet (Grade 5)’s Sara, and here — and it is worth naming as one idea when it comes up the third time. Item 6 supplies a general move that will outlast this topic: test a method on the simplest case, because a method that puts the origin off the grid is broken regardless of what it does elsewhere. Item 5 links directly back to the “Relating two number patterns” sheet (Grade 5) — three points at a constant rate on a line through the origin is the same picture as two number patterns from 0 — and item 3 is the argument for real-world contexts: a reversed reading becomes audibly absurd (12 hours to go 3 miles) in a way a bare grid can never show. **Timing:** 20–25 min · pairs · squared paper required · items 3, 6 and 8 on the board.

1 (2, 5) **2** (2,7) 2 right 7 up; (7,2) 7 right 2 up; (4,4) on the diagonal; (0,3) on the y-axis; (3,0) on the x-axis

3 1 hour and 4 miles; 3 hours and 12 miles; 6 hours and 24 miles — all at 4 miles per hour

4 (6, 6) and (9, 9) — the ones whose two coordinates are equal

5 because 0 across and 3 up puts a point on the y-axis, while 3 across and 0 up puts it on the x-axis — two perpendicular directions

6 12 hours and 3 miles — a quarter of a mile per hour, which is far slower than walking, so it is not a sensible journey for this graph

7 counting lines outwards gives every coordinate one too small, and applied to the origin it gives (-1, -1), which is not on the grid at all

8 because a mnemonic is one more thing to remember and can itself be misremembered, while the axis labels are printed on the graph and can be read at the moment you need them

9 because swapping its coordinates gives the same point, so the right method and the reversed one agree — being correct about (4, 4) does not show you have understood the order

1 **MILD** The same numbers, the other way

(2, 5)

1. Start at the origin.
2. 2 right, then 5 up, so (2, 5).
3. Compare with the item before: the same two numbers name a DIFFERENT point, which is why the order is not a convention to be casual about.

Watch for: Wrote (5, 2), either reversing this pair or copying the previous answer. Plotting both shows they are different points.

2 **MEDIUM** Five points

(2,7) 2 right 7 up; (7,2) 7 right 2 up; (4,4) on the diagonal; (0,3) on the y-axis; (3,0) on the x-axis

1. Start at the origin every time.
2. (2, 7): 2 spaces right, 7 up. (7, 2): 7 right, 2 up.
3. (4, 4): 4 right, 4 up — on the diagonal.
4. (0, 3): stay put across, 3 up. (3, 0): 3 right, stay put up.
5. Read each back off the grid before moving on.

Watch for: Placed (0, 3) and (3, 0) at the same point, or on the wrong axes. 0 across keeps you on the y-axis; 0 up keeps you on the x-axis, and they are perpendicular.

3 **MEDIUM** Three points with units

1 hour and 4 miles; 3 hours and 12 miles; 6 hours and 24 miles — all at 4 miles per hour

1. (1, 4): 1 hour, 4 miles. (3, 12): 3 hours, 12 miles. (6, 24): 6 hours, 24 miles.
2. $4 \div 1 = 4$, $12 \div 3 = 4$, $24 \div 6 = 4$ miles per hour.
3. So all three describe the same steady speed.
4. They therefore lie on one straight line through the origin — the same picture as the “Relating two number patterns” sheet (Grade 5)’s ordered pairs from two patterns.

Watch for: Gave the readings without units, or divided the wrong way round to get 0.25. The units decide which number is the time and which the distance, so dropping them makes the division a guess.

4 **SPICY** Which survive swapping

(6, 6) and (9, 9) — the ones whose two coordinates are equal

1. Swapping (6, 6) gives (6, 6), and swapping (9, 9) gives (9, 9). Both unchanged.
2. Swapping (3, 5) gives (5, 3), a different point.
3. Swapping (0, 2) gives (2, 0), also different — one is on the y-axis and the other on the x-axis.
4. So only points with equal coordinates survive, and they all sit on the diagonal through the origin.

Watch for: Included (0, 2), on the grounds that “a 0 makes no difference”. (0, 2) and (2, 0) are as different as any other pair — they are on perpendicular axes.

5**SPICY**

(0, 3) against (3, 0)

because 0 across and 3 up puts a point on the y-axis, while 3 across and 0 up puts it on the x-axis — two perpendicular directions

1. (0, 3): move 0 spaces across, so stay on the vertical axis, then 3 up.
2. (3, 0): move 3 across, then 0 up, so stay on the horizontal axis.
3. The two points are the same distance from the origin and in directions at right angles to each other.
4. So a 0 does not mean “nothing happens” — it means “no movement in THIS direction”, and which direction it applies to is what makes the two points different.

Watch for: Said “they are both 3 from the origin”. True, and it is why they are easy to confuse rather than why they differ — the direction is what distinguishes them.

6**SPICY**

What (12, 3) would mean

12 hours and 3 miles — a quarter of a mile per hour, which is far slower than walking, so it is not a sensible journey for this graph

1. The first coordinate is hours, so 12 hours; the second is miles, so 3 miles.
2. $3 \div 12 = 0.25$ miles per hour.
3. That is slower than a person strolls, so nothing on this graph should look like it.
4. It shows that reversing a pair does not just move a point — it changes what the point CLAIMS, and a real-world context makes the absurdity audible.

Watch for: Said it is “the same as (3, 12) backwards” without interpreting it. The value of a real-world context is that a reversed reading becomes obviously silly, which a bare grid cannot show.

7**SPICY**

Test the method on the origin

counting lines outwards gives every coordinate one too small, and applied to the origin it gives (-1, -1), which is not on the grid at all

1. The child calls the origin's own line “1”, so every line is labelled one more than its true distance.
2. To reach their “4” they stop 3 spaces out, and to reach their “6” they stop 5 — which is why (4, 6) landed at (3, 5).
3. Applied to the origin: their labels begin at 1, so there is no line labelled 0 anywhere on their grid, and the origin cannot be named at all.
4. A method with no 0 in it cannot describe the one point every coordinate pair is measured from.
5. Testing a method on the easiest case is a good general habit: if it cannot handle the origin, it cannot be trusted anywhere.

Watch for: Recalculated (4, 6) correctly instead of testing the origin. The origin test is the point of the item — a method that breaks on the simplest case is broken, and that argument needs no correct answer to compare with.

8



SPICY

Mnemonic against axis labels

because a mnemonic is one more thing to remember and can itself be misremembered, while the axis labels are printed on the graph and can be read at the moment you need them

1. “Along the corridor, up the stairs” only helps if you remember it the right way round, and children reverse it as often as they reverse the coordinates.
2. It also says nothing when the axes are labelled with something else, like hours and miles.
3. The labels are on the page: find the axis marked x , and the first number is measured along it.
4. So the order is looked up rather than recalled, and looking up does not fail under pressure.

Watch for: Said “mnemonics are for babies” or “you should just know it”. The specific point is that a mnemonic is a second thing that can go wrong, while the axis labels are evidence available at the moment of use.

9



EXTRA SPICY

Why $(4, 4)$ is a bad check

because swapping its coordinates gives the same point, so the right method and the reversed one agree — being correct about $(4, 4)$ does not show you have understood the order

1. $(4, 4)$ swapped is $(4, 4)$.
2. So a child who reads coordinates backwards plots it in exactly the right square.
3. A test that both the right and the wrong method pass tells you nothing about which one is being used.
4. To check understanding you need a point where the two methods DISAGREE — any point off the diagonal, such as $(3, 5)$.
5. This is the same idea as testing a rule on a second, different example rather than the one that suggested it.

Watch for: Said “because it is too easy”. It is not about difficulty: $(4, 4)$ is a case where the wrong method gives the right answer, so it cannot distinguish the two — the same evidence argument as the “Relating two number patterns” sheet (Grade 5) and the “Line plots with fractions” sheet (Grade 5).

Exit Ticket

4 problems

Use: End of the coordinate-plane lesson. Item 3's second half is the one to mark carefully, and it is an evidence question rather than a geometry one: $(4, 4)$ reversed is $(4, 4)$, so the wrong method and the right one agree on every diagonal point. A child who has only ever been tested there has not been tested. That is the same argument as the “Relating two number patterns” sheet (Grade 5)'s one-pair rule and the “Line plots with fractions” sheet (Grade 5)'s Sara — three banks in Grade 5 where a confirming case proves nothing. Item 4 is the off-by-one thread's LAST appearance in K-5, and its purest: the origin is the ruler's 0, exactly as in an earlier lesson. The check offered there is worth teaching as a general move — apply the method to the easiest case and see whether it survives, since Sy's puts the origin off the grid. Note that both items ask for a REVERSAL-PROOF check rather than a mnemonic; the axes are labelled, so the order can be looked up on the page instead of recalled. **Timing:** 10 min · independent · squared paper required.

1 A is 2 across and 7 up; B is 7 across and 2 up; C is on the y-axis 5 up; D is on the x-axis 5 across

2 6 hours and 24 miles — which is 4 miles per hour

3 she has swapped the two numbers, plotting the first one up and the second across; testing on (4, 4) would not find it because reversing (4, 4) gives (4, 4) again — the same point

4 he counted the grid lines starting from the origin's own line, so every coordinate comes out one short; his method would put the origin at (-1, -1), which is not on the grid at all

1  MEDIUM Four points

A is 2 across and 7 up; B is 7 across and 2 up; C is on the y-axis 5 up; D is on the x-axis 5 across

1. A(2, 7): from the origin, 2 spaces right, then 7 up.
2. B(7, 2): 7 spaces right, then 2 up. A and B are quite different places.
3. C(0, 5): 0 across — stay on the y-axis — then 5 up.
4. D(5, 0): 5 across, then 0 up, so it sits on the x-axis.
5. Read each one back off the grid to check.

Watch for: Plotted A and B in each other's positions, or placed C and D at the same point because “the 0 does not do anything”. 0 across and 0 up are different instructions and give different points.

2  MEDIUM Reading (6, 24)

6 hours and 24 miles — which is 4 miles per hour

1. The first coordinate is on the hours axis: 6 hours.
2. The second is on the miles axis: 24 miles.
3. 24 miles in 6 hours is $24 \div 6 = 4$ miles per hour.
4. Without the units the answer is just “6 and 24”, which does not say what was travelled or for how long.

Watch for: Answered “6 and 24” with no units. On a real-world graph the units are what the point MEANS — the same discipline as the “Line plots with fractions” sheet (Grade 5)’s dots against litres.

3

SPICY

Ola's reversal

she has swapped the two numbers, plotting the first one up and the second across; testing on (4, 4) would not find it because reversing (4, 4) gives (4, 4) again — the same point

1. In (3, 5) the 3 is measured along the x-axis, so it is 3 across and 5 up.
2. Ola went 5 across and 3 up, which is the point (5, 3) — a different place, and not a near miss.
3. (4, 4) has both coordinates equal, so swapping them changes nothing: the wrong method and the right one land in the same square.
4. Every point on the diagonal behaves that way, so a test made of them cannot tell the two methods apart.
5. To catch the error, test on a point whose coordinates differ — (3, 5) or (2, 7).
6. The reliable fix is to read the axis LABELS: the first number belongs to the axis marked x.

Watch for: Said Ola “mixed up x and y” and stopped. The second half of the item is the valuable one and it is about EVIDENCE: a diagonal point cannot distinguish the two methods, so being right about (4, 4) is not being right — the same argument as the “Relating two number patterns” sheet (Grade 5)’s one-pair rule.

4

SPICY

Sy's line-counting

he counted the grid lines starting from the origin's own line, so every coordinate comes out one short; his method would put the origin at (-1, -1), which is not on the grid at all

1. Coordinates count the SPACES travelled from the origin, not the lines passed.
2. Sy called the origin's own line “1”, so the line one space out is his “2”, and to reach his “3” he stopped only 2 spaces out. He landed at (2, 4).
3. Every coordinate is exactly one too small, which is the signature of this error.
4. The check: his numbering starts at 1, so it has no 0 in it at all — and therefore no way to name the origin, which is the point (0, 0).
5. A method that cannot name the simplest point on the grid is wrong before any other point is tried. The origin is the ruler's 0, not its 1.

Watch for: Said Sy “counted wrong” or “started in the wrong place”. Both are true and neither is checkable; applying the method to the origin is, and it fails immediately. This is the off-by-one thread's last K-5 appearance — the same error as starting at the ruler's 1 (an earlier lesson).

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Establish the whole difficulty in the first minute: the two numbers look identical in kind, so nothing in the notation prevents the reversal. It is a convention, and a convention has to be known rather than worked out.
2	0:40	Build the plane as TWO NUMBER LINES, which is the standard's own wording. Children have used number lines since Grade 1, and the coordinate plane is not a new object.
3	1:20	THE slide. Do not teach a mnemonic. The axes are labelled, so the first number belongs to the x-axis and can be identified by looking. A rhyme is one more thing to misremember; a label is on the page.
4	2:10	The important warning, and it is about ASSESSMENT: (4, 4) and every diagonal point look right either way, so testing on them proves nothing. Choose (3, 5) or (2, 7).
5	2:55	The off-by-one, which is the thread's last K-5 appearance. Move the finger and count SPACES aloud. Then show that counting lines would put the origin at (-1, -1), which is off the grid — the fastest way to catch the method.
6	3:50	Zero as an instruction, not an absence. (0, 4) and (4, 0) on the board together, so the difference is visible.
7	4:40	The real-world half of the standard. Insist on the units in every reading: “3 hours, 12 miles”, never “3 and 12”. Then reverse it and let the class hear how absurd 12 hours for 3 miles sounds.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.

10	6:00	Four points including both axis cases, then a reading with units. The habit to install: plot it, then read it back off the grid. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #4 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.