

GRADE 5 · MEASUREMENT & DATA · ANSWER KEY

VOLUME AND CUBIC UNITS

Today's goal: I can recognize volume as an attribute of solid figures; understand cubic units and measure volumes by counting unit cubes.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

11 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 6 is the sheet's most useful item and it is about METHOD rather than volume: a $2 \times 2 \times 2$ cannot hide a cube, because hiding one needs a neighbour on both sides in all three directions and so needs 3 in each. That is why demonstrating this topic on a small example teaches nothing — the problem does not exist yet. Item 2 catches the child who has memorised “one hidden cube” from the deck: at $4 \times 4 \times 4$ it is eight, forming a block, and at $5 \times 5 \times 5$ it is 27. Item 3 gives them a free check they can use forever — slice the solid a different way and see whether you agree with yourself — and three disagreeing counts mean at least two are wrong, so choosing between them is not an option. Item 4's pair is worth a board discussion: for a $6 \times 6 \times 6$ the volume and surface are both 216, which does not make them the same measurement. **Timing:** 20–25 min · pairs · unit cubes and squared paper · items 3, 6 and 9 on the board.

1 24 cubic units

2 24 cubic units

3 27 cubic units

4 64 cubic units

5 neither — both hold 24 cubic units

6 8 hidden cubes, making a $2 \times 2 \times 2$ cube in the middle

7 yes, 24 cubic units every time — two layers of 12, three layers of 8, or four layers of 6

8 24 unit squares — different because volume counts the cubes filling the space while surface counts the squares covering the skin

9 because every cube in it touches an outside face — there is no room for a middle, since you need at least 3 in a direction to have anything not on an edge

10 no — a count of loosely stacked cubes does not give the volume, because volume is how many cubes fill the space with no gaps

11 because a drawing can only show the outside of a solid, so counting it always misses the cubes inside — layers count every cube whether it can be seen or not

1 MILD Layers of 12, two of them

24 cubic units

1. The base layer is $4 \times 3 = 12$ cubes.
2. Two layers, so 24 cubic units.

Watch for: Added the three dimensions instead: $4 + 3 + 2 = 9$. Volume counts cubes in a block, and a block 4 by 3 by 2 plainly holds more than 9 of them.

2 MILD Layers of 12 again

24 cubic units

1. The base layer is $6 \times 2 = 12$ cubes.
2. Two layers, so 24 cubic units.
3. Same volume as item 1 from different dimensions — worth pointing out.

Watch for: Added the dimensions: $6 + 2 + 2 = 10$.

3 MILD A 3-cube

27 cubic units

1. Layers of $3 \times 3 = 9$.
2. Three of them, so 27 cubic units.

Watch for: Answered 26, counting only the cubes with a face showing. Counting by layers does not depend on what can be seen, which is why it is the method to use.

4 MILD A 4-cube

64 cubic units

1. Layers of $4 \times 4 = 16$.
2. Four of them, so 64 cubic units.

Watch for: Answered 56, counting only visible faces. The 8 hidden interior cubes of a 4-cube are exactly what layer-counting includes and eyeballing misses.

5 MEDIUM $6 \times 2 \times 2$ against $4 \times 3 \times 2$

neither — both hold 24 cubic units

1. $6 \times 2 \times 2$: layers of 12, two layers — 24.
2. $4 \times 3 \times 2$: layers of 12, two layers — 24.
3. So they hold the same number of cubes, although they are different shapes.
4. Their surfaces are not equal, though: 56 unit squares against 52 — same volume, different skin.

Watch for: Chose the $6 \times 2 \times 2$ because “6 is the biggest number”. One dimension does not decide a volume; all three do, and here the two products are equal.

6 SPICY Hidden cubes in a $4 \times 4 \times 4$

8 hidden cubes, making a $2 \times 2 \times 2$ cube in the middle

1. A cube is hidden if it has no face on any outside surface.
2. Strip one layer of cubes off every face: that removes the outermost cube in each direction, leaving $4 - 2 = 2$ in each direction.
3. So the hidden cubes form a $2 \times 2 \times 2$ block: 8 cubes.
4. Check: 64 total, 56 with a face showing, 8 hidden.

Watch for: Answered 1, carrying over the $3 \times 3 \times 3$ result. The hidden part grows as the solid does — a $5 \times 5 \times 5$ hides 27 — so it is a block, not a single cube.

7

SPICY

Layers three ways

yes, 24 cubic units every time — two layers of 12, three layers of 8, or four layers of 6

1. Layers parallel to the 4×3 face: 12 per layer, 2 layers — 24.
2. Layers parallel to the 4×2 face: 8 per layer, 3 layers — 24.
3. Layers parallel to the 3×2 face: 6 per layer, 4 layers — 24.
4. The solid does not change when you decide how to slice it, so the count cannot either. That is a useful check: slice it a second way and see if you agree with yourself.

Watch for: Got three different answers and picked one. Three disagreeing counts mean at least two are wrong, and re-slicing is a free way to check — the agreement is the evidence, not the choosing.

8

SPICY

Volume and surface of a $2 \times 2 \times 2$

24 unit squares — different because volume counts the cubes filling the space while surface counts the squares covering the skin

1. Each face of the cube is $2 \times 2 = 4$ unit squares.
2. There are 6 faces, so $6 \times 4 = 24$ unit squares.
3. Volume is 8 cubic units: the cubes that fill it.
4. So the two numbers answer different questions — how much space is inside, and how much covering the outside needs.
5. The units differ too: cubic units against square units, which is the clearest sign they are not the same kind of measurement.

Watch for: Said one of them “must be wrong because they should match”. They measure different things and there is no reason for them to match; for a $6 \times 6 \times 6$ they happen to be equal at 216, and even then they are answers to different questions.

9

SPICY

Why a $2 \times 2 \times 2$ hides nothing

because every cube in it touches an outside face — there is no room for a middle, since you need at least 3 in a direction to have anything not on an edge

1. In a $2 \times 2 \times 2$, each of the 8 cubes is at a corner.
2. A corner cube has three faces on the outside, so none is hidden.
3. To hide a cube you need at least one cube on each side of it in every direction, which needs 3 in each direction.
4. So $3 \times 3 \times 3$ is the smallest solid with a hidden cube — and it hides exactly one.
5. This is why a small example is a bad test: the problem does not exist yet at $2 \times 2 \times 2$.

Watch for: Said “because it is small”. The specific reason is that hiding a cube needs a neighbour on both sides in all three directions, so 3 is the smallest side length that allows it — and that is worth knowing, because it is why practising on small examples teaches nothing here.

10

SPICY

Loose cubes with gaps

no — a count of loosely stacked cubes does not give the volume, because volume is how many cubes fill the space with no gaps

1. Volume asks how much space the box encloses.
2. Cubes with gaps between them leave some of that space unaccounted for, so the count is less than the space.
3. The box's volume has not changed — only the measuring has gone wrong.
4. To measure it, the cubes must be packed tight, which is what “without gaps” means in the standard.
5. It is the same requirement as tiling for area: tiles with gaps do not measure a rectangle either.

Watch for: Said “yes, because you still counted the cubes”. The cubes were counted correctly; they just were not measuring the box. What is being counted has to fill the thing being measured.

11

EXTRA SPICY

Why layers beat drawings

because a drawing can only show the outside of a solid, so counting it always misses the cubes inside — layers count every cube whether it can be seen or not

1. A flat picture shows at most three faces of a solid, so interior cubes never appear.
2. Counting the drawing therefore answers a question about the surface.
3. Layers work from the DIMENSIONS instead: how many cubes in one layer, and how many layers.
4. So the count is complete regardless of what the picture shows, and it also works for a solid described in words with no picture at all.
5. And it can be checked, by slicing the solid a different way and seeing whether the two counts agree.

Watch for: Said “because it is quicker”. Speed is not the reason — a drawing-count is not merely slow, it is systematically WRONG for any solid at least 3 across in every direction, and it gets worse as the solid grows.

Exit Ticket

4 problems

Use: End of the volume-and-cubic-units lesson. Item 3 concedes that Sam's count is CORRECT, which is the whole point: he has not miscounted, he has answered a question about the surface while being asked about the space. So “he forgot one” misses it — the answer wanted says that a flat drawing shows only an outside, and that counting by layers makes visibility irrelevant. This is the “the diagram means nothing until you know its unit” thread (an earlier lesson, an earlier lesson, the Grade 3 “Volume and mass” lesson, an earlier lesson, an earlier lesson, the “Line plots with fractions” sheet (Grade 4), the “Line plots with fractions” sheet (Grade 5)), and note what is NEW about this appearance: the earlier diagrams were mis-scaled or mislabelled, while this one is incomplete BY NATURE, since no flat picture can show an inside. Item 4 is the “packed without gaps” condition, which is an earlier lesson’s tiling rule in three dimensions, and it separates a property of the box from a description of what someone did with it. Watch the units on every item — “24 square units” loses the only word that says what was measured. **Timing:** 5–10 min · independent · unit cubes available.

1 27 cubic units, and 1 cube is completely hidden

2 24 cubic units

3 because the volume counts ALL the unit cubes and one of them is hidden from every direction — his 26 is a correct count of the cubes on the outside, which is a different question; the volume is 27 cubic units

4 the box's volume is 24 cubic units, because volume is how many cubes fill it with no gaps; 20 is how many cubes were put in, which measures nothing about the box

1 MILD A $3 \times 3 \times 3$

27 cubic units, and 1 cube is completely hidden

1. Layers of $3 \times 3 = 9$, three layers, so 27 cubic units.
2. A cube is hidden if it has no face on any outside surface.
3. In a $3 \times 3 \times 3$ that is the single cube at the very centre.
4. So 26 cubes touch the outside and 1 does not.

Watch for: Answered 0 hidden, on the grounds that “you can see them all if you turn it round”. Turning it round shows different outsides; the centre cube has no face on any of them.

2 MEDIUM A $4 \times 3 \times 2$ block

24 cubic units

1. One layer is $4 \times 3 = 12$ cubes.
2. There are 2 layers.
3. $12 + 12 = 24$ cubic units.
4. Layers could be taken the other way instead — three layers of 8, or four of 6 — and all give 24.

Watch for: Answered 24 with no unit, or “24 square units”. A volume is measured in CUBIC units, and the unit is what says whether space or skin was measured.

3 SPICY Sam's 26

because the volume counts ALL the unit cubes and one of them is hidden from every direction — his 26 is a correct count of the cubes on the outside, which is a different question; the volume is 27 cubic units

1. Sam counted cubes with a face showing. That is 26, and it is right.
2. The cube at the very centre of a $3 \times 3 \times 3$ has no face on any outside surface, so no drawing and no viewing angle can show it.
3. It is still part of the solid, so it counts towards the volume: 27 cubic units.
4. Counting by layers avoids the problem entirely: $3 \times 3 = 9$ per layer, three layers, 27. Nothing has to be visible.
5. The general point: a flat drawing shows the outside of a solid, so counting the drawing answers a question about the surface, not the space.

Watch for: Said Sam “miscounted” or “forgot one”. He did not miscount — his count was complete for what he was counting. The error is about what a DRAWING is, and the layer method is what makes visibility irrelevant.

4 SPICY 20 cubes in a 24-cube box

the box's volume is 24 cubic units, because volume is how many cubes fill it with no gaps; 20 is how many cubes were put in, which measures nothing about the box

1. The box holds 24 unit cubes packed with no gaps, so its volume is 24 cubic units.
2. Volume is a property of the BOX, and it does not change depending on how many cubes someone puts in it.
3. A loose arrangement of 20 cubes with spaces leaves the same box, still 24 cubic units.
4. This is why the standard says “packed without gaps”: a count of cubes measures a volume only when the cubes actually fill the space.
5. It is the same rule as tiling a rectangle to find area — tiles with gaps between them do not measure it (an earlier lesson).

Watch for: Answered 20, treating the count of cubes present as the volume. The volume belongs to the box; the 20 describes what someone did with it, and the “no gaps” condition is exactly what separates the two.

Slide	Time	What to do
1	0:00	Build the solids with real cubes if you have them. The hidden-cube problem is obvious with blocks in hand and almost invisible on paper, and the whole lesson is about the difference between those two.
2	0:40	Define the unit cube properly — 1 unit on every side, volume one cubic unit. Volume then needs no formula at all: it is a COUNT.
3	1:15	“No gaps” is in the standard, and it is an earlier lesson tiling rule in three dimensions. A loose pile of cubes in a box does not measure the box.
4	1:50	Draw the $3 \times 3 \times 3$ and ask for a count. Expect 9, 19 and 26 — and take 26 seriously, because it is a careful and complete count of the outside.
5	2:40	THE slide. A flat picture cannot show an inside, so the drawing is not the solid. Break a built cube open, or take the top layer off, to show the middle cube existing.
6	3:30	Layers, which is the real fix: 3×3 then three of them. Nothing needs to be seen, so the drawing's limitation stops mattering. This also sets up the “Volume by multiplication” sheet (Grade 5) without handing over the formula.
7	4:30	Volume against surface, briefly but explicitly — children conflate them and both are counts of unit things. Inside or skin.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	5:45	The $4 \times 4 \times 4$ hidden-cube question is the good one: eight hidden cubes, forming a $2 \times 2 \times 2$ in the middle. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #5, #6 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.