



GRADE 5 · MEASUREMENT & DATA · ANSWER KEY

VOLUME BY MULTIPLICATION

Today's goal: I can relate volume to multiplication and addition; solve real-world problems.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

10 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 4 is a trap worth understanding before marking: the two blocks happen to form a $6 \times 3 \times 2$ box, so multiplying the outside dimensions gives the right answer — and item 5's shape is not a box, where the identical move fails. A child who gets 5 right and 6 wrong has been rewarded for the habit that then costs them. Item 2 supplies the free check this whole bank depends on: one solid, three ways to slice it, and they must agree, so a disagreement is evidence of an error and not a choice (same as the “Volume and cubic units” sheet (Grade 5) item 3). Item 3 catches a specific slip — pairs multiplying to 72 rather than to 24 — which means the height was used twice. Item 6 is the standard's own question and its answer must be about EQUAL GROUPS; “because that is the formula” leaves a child unable to say why adding is wrong rather than merely different. **Timing:** 20–25 min · pairs · unit cubes · items 2, 5 and 6 on the board.

1 24 cubic units **2** 30 cubic units **3** 24 cubic units every time: 12×2 , 8×3 , 6×4 **4** 3 layers

5 base area 24 — for example 6×4 , 8×3 , 12×2 **6** 36 cubic units **7** 30000 cubic centimetres

8 $27 + 9 = 36$ cubic units — and the join must not be counted twice because it is a surface where the pieces meet, not a layer belonging to both

9 because every layer holds the same number of cubes, so the total is that number repeated — which is multiplication; adding the three dimensions combines three lengths and counts no cubes at all

10 because a single box can be answered by putting three numbers into a formula without understanding it, while a composite solid has no single $l \times w \times h$ — so it tests whether you know the formula counts LAYERS of a base

1 **MILD** A small prism

24 cubic units

- $4 \times 3 = 12$ in the base layer.
- 2 layers, so 24 cubic units.

Watch for: Answered 9 — the dimensions added. 9 is smaller than one layer of this solid, which is the quickest way to see it.

2 **MILD** Another small prism

30 cubic units

- $5 \times 2 = 10$ in the base layer.
- 3 layers, so 30 cubic units.

Watch for: Answered 10 — the dimensions added. Again smaller than a single layer would allow.

3 **MEDIUM** Three bases, one prism

24 cubic units every time: 12×2 , 8×3 , 6×4

- $4 \times 3 = 12$ with 2 layers — 24.
- $4 \times 2 = 8$ with 3 layers — 24.
- $3 \times 2 = 6$ with 4 layers — 24.
- The solid does not change when you choose which way is up, so the answer cannot either. Disagreement between two of these means an arithmetic slip.

Watch for: Got different answers and did not treat that as a problem. Three ways of slicing one solid must agree, so a disagreement is evidence of an error rather than a choice to be made — the same check as the “Volume and cubic units” sheet (Grade 5) item 3.

4  MEDIUM Volume 60, base 5×4

3 layers

1. The base area is $5 \times 4 = 20$, so each layer holds 20 cubes.
2. $60 \div 20 = 3$ layers.
3. Check: $20 \times 3 = 60$. Correct.

Watch for: Divided 60 by 5 and then by 4 in two steps and lost track, or divided by 9 (the added dimensions). The base AREA is one number, 20, and the division is by that.

5  MEDIUM Volume 72, height 3

base area 24 — for example 6×4 , 8×3 , 12×2

1. Base area = $72 \div 3 = 24$.
2. Any two numbers multiplying to 24: 6×4 , 8×3 , 12×2 , 24×1 .
3. All four give a prism of volume 72 with 3 layers, and they are different shapes.
4. So the volume fixes the base area and says nothing about the base's shape.

Watch for: Gave 24×3 , 12×6 , 8×9 — pairs multiplying to 72 rather than to 24. The height has already been used; what is left to split is the base area.

6  MEDIUM Two blocks end to end

36 cubic units

1. $4 \times 3 \times 2 = 24$ cubic units.
2. $2 \times 3 \times 2 = 12$ cubic units.
3. $24 + 12 = 36$ cubic units.
4. In this case the two happen to form a $6 \times 3 \times 2$ box, which also gives 36 — a useful check when the pieces fit together into a box.

Watch for: Multiplied the outside dimensions of the whole arrangement without checking it was a box. Here it happens to be one, so the answer comes out right — which is exactly why it is a bad habit: item 5's shape is not a box, and the same move fails there.

7  MEDIUM The fish tank

30000 cubic centimetres

1. Base area: $40 \times 25 = 1000$ square centimetres.
2. Layers: 30.
3. $1000 \times 30 = 30000$ cubic centimetres.
4. Check with a different base: $40 \times 30 = 1200$, times 25 layers, is also 30000.
5. (30000 cubic centimetres is 30 litres, since 1000 cubic centimetres is a litre.)

Watch for: Answered 95, adding the dimensions, or lost a factor of ten in the multiplication. Re-choosing the base is the check: two routes to 30000 is strong evidence, and one route is none.

8  SPICY Block on a slab

$27 + 9 = 36$ cubic units — and the join must not be counted twice because it is a surface where the pieces meet, not a layer belonging to both

1. $3 \times 3 \times 3 = 27$ cubic units.
2. $3 \times 3 \times 1 = 9$ cubic units.
3. $27 + 9 = 36$ cubic units.
4. The two pieces do not overlap: every cube is in exactly one of them. The place they touch is a FACE, which has no volume.
5. If the bottom layer of the block were also counted as part of the slab, 9 cubes would be counted twice and the answer would be 45 — nine too many.
6. So “non-overlapping” is the condition that makes adding correct.
7. Check: the pieces stack into a $3 \times 3 \times 4$ box, which is 36.

Watch for: Answered 45, counting a shared layer in both pieces. Or answered 36 without addressing the join, which the item asks for — and the join is where a badly chosen split goes wrong.

9

SPICY

Why multiply, not add

because every layer holds the same number of cubes, so the total is that number repeated — which is multiplication; adding the three dimensions combines three lengths and counts no cubes at all

1. A $4 \times 3 \times 2$ prism has layers of 12 cubes each.
2. Two layers of 12 is $12 + 12$, which is 12×2 — multiplication is repeated addition of equal groups, and the layers ARE equal groups.
3. The 12 itself came the same way: 3 rows of 4 cubes.
4. Adding $4 + 3 + 2$ adds three lengths together, and a length is not a number of cubes — the answer 9 does not count anything in the solid.
5. And 9 is smaller than one layer, so it fails a size check immediately.

Watch for: Said “because that is the formula”. The equal-groups argument is the reason, and it is what rules out adding — a child with only the formula cannot say why $4 + 3 + 2$ is wrong rather than merely different.

10

EXTRA SPICY

Why two prisms

because a single box can be answered by putting three numbers into a formula without understanding it, while a composite solid has no single $l \times w \times h$ — so it tests whether you know the formula counts LAYERS of a base

1. For one box, $l \times w \times h$ works whether or not you know why.
2. A composite solid must first be SPLIT, and choosing where to split needs an idea of what is being counted.
3. Then each piece is a prism with its own base and layers, and the totals are added because the pieces are separate collections of cubes.
4. So the question cannot be answered by a rule alone, which is what makes it worth asking.
5. It is also the realistic case: very few actual objects are single boxes.

Watch for: Said “because it is harder”. It is harder in a specific way worth naming: no formula applies until a decision has been made about how to split the solid, and making that decision requires knowing what the formula counts.

Exit Ticket

4 problems

Use: End of the volume-by-multiplication lesson. This bank deliberately never teaches $l \times w \times h$ as a rule — every child arrives knowing it, and knowing it is the problem. It is the three-dimensional case of length-times-width before any tiling (an earlier lesson), in the same thread as underline-a-digit rounding (the “Rounding multi-digit numbers” sheet (Grade 4)) and PEMDAS as a ladder (the “Order of operations and grouping symbols” sheet (Grade 5)). So item 3 asks for an argument from CUBES and not from the formula: the answer wanted is that ONE LAYER already holds 12, so 9 is fewer cubes than a single layer. “She should have multiplied” is the rule restated, and it cannot tell a child whether an answer is slightly wrong or absurd. Item 4 is the real test of whether the formula is two THINGS (a base area and a layer count) or three empty slots — a child who cannot work backwards has the slots. Item 2 is where the formula-only child is exposed, which is exactly why the standard includes composite solids: no single $l \times w \times h$ covers the shape, and the 80 answer counts the empty space above the lower block. **Timing:** 5–10 min · independent · unit cubes available.

1 base area 24, 5 layers, volume 120 cubic units

2 32 cubic units

3 because one layer of a 4×3 prism already holds 12 cubes, and there are two layers — so 9 is fewer cubes than a single layer, which is impossible; the volume is 24 cubic units

4 base area 24 — which could be 6×4 , or 8×3 , or 12×2

1 **MEDIUM** A $6 \times 4 \times 5$ prism

base area 24, 5 layers, volume 120 cubic units

1. The base is $6 \times 4 = 24$, so one layer holds 24 cubes.
2. There are 5 layers.
3. $24 \times 5 = 120$ cubic units.
4. Check by re-choosing the base: $6 \times 5 = 30$ with 4 layers is also 120.

Watch for: Answered 15, adding $6 + 4 + 5$. One layer alone holds 24 cubes, so 15 is smaller than a single layer of the solid.

2 **MEDIUM** A composite solid

32 cubic units

1. Split it into the two blocks, which do not overlap.
2. $5 \times 4 \times 1 = 20$ cubic units.
3. $2 \times 2 \times 3 = 12$ cubic units.
4. $20 + 12 = 32$ cubic units.
5. No single $l \times w \times h$ describes this shape, which is why the two volumes are added.

Watch for: Multiplied the overall outside dimensions, treating it as one $5 \times 4 \times 4$ block and getting 80. That counts a great deal of empty space above the lower block — the shape is not a full box.

3 **SPICY** Nia's 9

because one layer of a 4×3 prism already holds 12 cubes, and there are two layers — so 9 is fewer cubes than a single layer, which is impossible; the volume is 24 cubic units

1. Picture the bottom layer: 4 cubes along by 3 across is 12 cubes, in one layer alone.
2. So the whole solid holds at least 12, and 9 is less than that.
3. There are 2 layers, so the volume is $12 \times 2 = 24$ cubic units.
4. The three numbers multiply because each layer holds the SAME number of cubes — adding them counts nothing you can point at.
5. Nia's answer would mean a block 4 cubes long held only 9 cubes altogether, and it cannot: the bottom row alone has 4.

Watch for: Said Nia “should have multiplied, not added”. That is the rule, and the item asks for an argument from CUBES — because a child who only knows which operation to use cannot tell whether 9 is wrong by a little or absurd, and 9 being smaller than one layer is what makes it absurd.

4 **SPICY** Volume 72, height 3

base area 24 — which could be 6×4 , or 8×3 , or 12×2

1. Volume = base area $\times$ number of layers, so base area = $72 \div 3 = 24$.
2. Any pair multiplying to 24 works: 6×4 , 8×3 , 12×2 , 24×1 .
3. So the base AREA is fixed at 24 and the base SHAPE is not.
4. Check one: $6 \times 4 \times 3 = 72$. Correct.

Watch for: Divided 72 by 3 and stopped at 24 without offering side lengths, or gave 24×3 as a pair. The item asks for two pairs whose PRODUCT is 24, and being able to work backwards is what shows the formula is two things rather than three slots.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Do not introduce $l \times w \times h$ as a rule. Every child will already know it, and knowing it is exactly the problem this lesson addresses — it is the three-dimensional version of length-times-width before any tiling (an earlier lesson).
2	0:35	Start from the “Volume and cubic units” sheet (Grade 5)’s layer counting, which they did last lesson. Nothing new is being taught yet, and that should be obvious to them.
3	1:15	THE slide. Name the two parts out loud: 4×3 is the area of the BASE, and 2 is the number of LAYERS. Both are things you can point at. The formula is a shortcut for the counting they can already do.
4	2:10	Answer the question children never ask: why multiply rather than add? Because every layer holds the same number. Then show $4 + 3 + 2 = 9$ and note it is smaller than one layer, which nobody with cubes in mind would accept.

5	3:00	Any face as the base, and this is a check worth insisting on — it is free, and it catches a mis-multiplication immediately.
6	3:50	Working backwards, which is what tells you whether a child has the two PARTS or just three numbers. Note the base area is determined and its shape is not.
7	4:45	The composite solid, and use an L-SHAPE rather than two blocks end to end — two blocks in a line usually form a box, so multiplying the overall dimensions works there and teaches the wrong habit. Here the overall box is $5 \times 4 \times 4 = 80$ while the solid holds 32, so 48 cubic units of it is air. Split, calculate, add — and check the pieces do not overlap.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	6:00	Both tasks demand the base area and layer count separately, which is what stops the formula being three empty slots. Exit ticket after. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #8 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.