



## Displaying data in box plots

**GRADE 6 MATH**

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

### Exit Ticket — answer key

**Use:** End of the box plot lesson, closing the three display skills. 5–8 min, independent. Item 4 is the misconception the whole lesson exists to prevent.

**Timing:** 5–8 min · independent. A ‘yes’ on item 4 means the 25%-per-region idea has not landed, regardless of how clean items 1–3 are.

#### 1. Minimum and maximum? **FLUENCY**

Answer: **Minimum 2, maximum 20**

1. The data is already in order, so read the ends: 2 and 20.
2. These are the two whisker tips on the box plot.

**If wrong:** Gave the first and last values without checking the data was sorted. Always sort first.

#### 2. Median? **FLUENCY**

Answer: **9**

1. Ten values, so average the 5th and 6th: 8 and 10.
2.  $(8 + 10) \div 2 = 9$ .
3. The median is not a value in this data set, which is normal when  $n$  is even.

**If wrong:** Picked 8 or 10 rather than averaging the middle pair.

### 3. Q1, Q3 and the IQR? APPLICATION

Answer: **Q1 = 5, Q3 = 15, IQR = 10**

1. Split at the median into two halves of five values each.
2. Lower half 2, 4, 5, 7, 8 → its middle value is Q1 = 5.
3. Upper half 10, 12, 15, 18, 20 → its middle value is Q3 = 15.
4.  $IQR = Q3 - Q1 = 15 - 5 = 10$ . That is the width of the box, the middle half of the data.

**If wrong:** Found the median of the whole set again instead of the median of each half.

### 4. Does the longer whisker hold more values? REASONING

Answer: **No — each whisker holds 25% of the data, here 2.5 values' worth**

1. The five-number summary cuts the data into four quarters, and each quarter holds one quarter of the values.
2. With 10 values, each quarter accounts for  $10 \div 4 = 2.5$  values' worth.
3. The upper whisker is longer only because those top values are more spread out: 15 to 20 covers 5 units, while 2 to 5 covers 3.
4. Length measures SPREAD, not COUNT. A long whisker means the values out there are scattered; a short one means they are bunched.
5. So the longer whisker holds the same share of the data, thinly spread.

**If wrong:** Answered yes, reading width as amount. This is the central misconception in box plots and it recurs on every assessment.

**Quick answers** for fast checking:

| # | Level       | Answer                          |
|---|-------------|---------------------------------|
| 1 | fluency     | <b>2 and 20</b>                 |
| 2 | fluency     | <b>9</b>                        |
| 3 | application | <b>Q1 5, Q3 15, IQR 10</b>      |
| 4 | reasoning   | <b>no — each quarter is 25%</b> |

## Practice Worksheet — answer key

**Use:** During the lesson after the deck. Items 8 and 9 are two faces of the same misconception — that size on the page means amount of data.

**Timing:** 15–25 min · pairs or independent · circulate on 8 and 9. Having students physically count values into four groups of equal size settles it faster than explaining.

### 1. Minimum? FLUENCY

Answer: **3**

1. The data is sorted; the first value is 3.

**If wrong:** Read an unsorted list.

### 2. Maximum? FLUENCY

Answer: **16**

1. The last value is 16.

**If wrong:** Read the second-largest.

### 3. Median? FLUENCY

Answer:  **$8\frac{1}{2}$**

1. Eight values, so average the 4th and 5th: 8 and 9.
2.  $(8 + 9) \div 2 = 8\frac{1}{2}$ .

**If wrong:** Picked 8 or 9 without averaging.

### 4. Q1? APPLICATION

Answer:  **$5\frac{1}{2}$**

1. With n even, split into two halves of four: 3, 5, 6, 8 and 9, 11, 14, 16.
2. Q1 is the median of the lower half:  $(5 + 6) \div 2 = 5\frac{1}{2}$ .

**If wrong:** Answered 5 by taking a value rather than averaging the middle pair of the half.

### 5. Q3? APPLICATION

Answer:  **$12\frac{1}{2}$**

1. Q3 is the median of the upper half 9, 11, 14, 16.
2.  $(11 + 14) \div 2 = 12\frac{1}{2}$ .

**If wrong:** Answered 14.

## 6. IQR? APPLICATION

Answer: **7**

1.  $IQR = Q3 - Q1 = 12\frac{1}{2} - 5\frac{1}{2} = 7$ .
2. This is the width of the box — the spread of the middle half.

**If wrong:** Added Q1 and Q3 instead of subtracting.

## 7. Range, and how it differs from the IQR? REASONING

Answer: **Range 13; it uses the extremes, the IQR uses the middle half**

1.  $Range = 16 - 3 = 13$ .
2. The range measures the full width of the data, from the smallest value to the largest.
3. The IQR of 7 measures only the middle 50%, from Q1 to Q3.
4. Because the range depends on the two most extreme values, a single outlier can inflate it. The IQR ignores the outer quarters entirely, so it is far more stable.
5. The IQR is always less than or equal to the range.

**If wrong:** Treated them as two names for the same thing. They measure spread over different portions of the data.

## 8. Lower whisker 3 units, upper whisker 5 units — what does that tell you?

### REASONING

Answer: **The top quarter is more spread out than the bottom quarter**

1. Both whiskers hold 25% of the values — that is fixed by the construction.
2. The upper whisker covers 5 units (15 to 20) while the lower covers 3 (2 to 5).
3. So the highest quarter of the values is stretched over more ground than the lowest quarter.
4. That is a sign the data trails off to the right — consistent with being skewed right.
5. It says nothing about how MANY values are up there; that number is the same at both ends.

**If wrong:** Concluded there are more high values than low values. The counts are equal by definition; only the spacing differs.

## 9. 'The box is where most of the data is' — right? REASONING

Answer: **Half in the box, half in the whiskers — the student is only half right**

1. The box spans Q1 to Q3, which is the middle 50% of the values.
2. Each whisker holds 25%, so together they hold the other 50%.
3. So the box holds exactly half the data, not most of it.
4. The whiskers matter a great deal: they show the range, reveal outliers, and their relative lengths tell you about skew.
5. Ignoring the whiskers means ignoring half your data and everything about the extremes.

**If wrong:** Agreed, because the box looks solid and the whiskers look thin. Visual weight is not data volume — the box is drawn as a rectangle simply to mark the middle half.

**Quick answers** for fast checking:

| # | Level       | Answer                                       |
|---|-------------|----------------------------------------------|
| 1 | fluency     | <b>3</b>                                     |
| 2 | fluency     | <b>16</b>                                    |
| 3 | fluency     | <b>8½</b>                                    |
| 4 | application | <b>5½</b>                                    |
| 5 | application | <b>12½</b>                                   |
| 6 | application | <b>7</b>                                     |
| 7 | reasoning   | <b>range 13 vs IQR 7</b>                     |
| 8 | reasoning   | <b>top quarter more spread out</b>           |
| 9 | reasoning   | <b>half in the box, half in the whiskers</b> |

## Homework — answer key

**Use:** Parallels the practice sheet and closes the three display skills. Item 6 restates the width-versus-count idea inside the box, where it is harder to see than in the whiskers.

**Timing:** ~15 min at home · independent · review item 6 as the next-day warm-up, then move to SP-07.

### 1. Left edge of the box? FLUENCY

Answer: **Q1 = 5**

1. The box starts at Q1, the median of the lower half 2, 4, 5, 7, 8.

**If wrong:** Gave the minimum, 2. That is the whisker tip, not the box edge.

## 2. Right edge of the box? FLUENCY

Answer: **Q3 = 15**

1. The box ends at Q3, the median of the upper half 10, 12, 15, 18, 20.

**If wrong:** Gave the maximum, 20.

## 3. Is the median inside the box? Why always? APPLICATION

Answer: **Yes — 9, and it must be**

1. The median is 9, and the box runs from 5 to 15, so 9 sits inside it.
2. Q1 is the median of the lower half, so it is always below the overall median.
3. Q3 is the median of the upper half, so it is always above it.
4. Therefore  $Q1 \leq \text{median} \leq Q3$  for every data set — the median line can never fall outside the box.

**If wrong:** Answered yes for this data set alone without giving a reason that holds generally.

## 4. Width of the left half of the box? APPLICATION

Answer: **4**

1. From  $Q1 = 5$  to the median = 9:  $9 - 5 = 4$ .

**If wrong:** Subtracted in the wrong order and gave  $-4$ . A width is never negative.

## 5. Width of the right half of the box? APPLICATION

Answer: **6**

1. From the median = 9 to  $Q3 = 15$ :  $15 - 9 = 6$ .

**If wrong:** Used the maximum instead of Q3, giving 11.

## 6. Different widths — does one half hold more values? REASONING

Answer: **No — each holds 25% of the data**

1. The left half of the box is 4 units wide, the right half is 6 units wide.
2. But Q1 to the median is one quarter of the data, and the median to Q3 is another quarter.
3. So both halves of the box hold the same share, 25% each, or 2.5 values' worth out of 10.
4. The right half is wider because those values are more spread out, not because there are more of them.
5. This is the same principle as the whiskers: width is spread, not count.

**If wrong:** Answered that the wider half holds more. Every region of a box plot holds 25% by construction — the count is fixed and only the spacing varies.

## 7. What does the box width measure? REASONING

Answer: **The spread of the middle half — it is the IQR**

1. The box runs from  $Q1 = 5$  to  $Q3 = 15$ , so its width is  $15 - 5 = 10$ .
2. That is the interquartile range, the IQR.
3. It measures how spread out the middle 50% of the data is.
4. A narrow box means the middle half is tightly bunched; a wide box means it is scattered.
5. Unlike the range, it is unaffected by outliers, because the outer quarters lie outside the box.

**If wrong:** Called it the range. The range is 18 here, measured from 2 to 20; the box width is the IQR of 10.

## 8. Why do five numbers create four regions? REASONING

Answer: **Because the regions are the GAPS between the marks, and five marks have four gaps between them**

1. The five numbers are positions: minimum, Q1, median, Q3, maximum.
2. The regions are the spaces BETWEEN consecutive positions: min to Q1, Q1 to median, median to Q3, Q3 to max.
3. Five marks in a row always have four gaps between them, one fewer than the number of marks.
4. Each of those four gaps holds one quarter of the data, and  $4 \times 25\% = 100\%$ , so all the data is accounted for.

**If wrong:** Answered five regions by counting the marks themselves. The marks are boundaries, not regions — the same reason a fence with five posts has four panels.

**Quick answers** for fast checking:

| # | Level       | Answer                       |
|---|-------------|------------------------------|
| 1 | fluency     | <b>Q1 = 5</b>                |
| 2 | fluency     | <b>Q3 = 15</b>               |
| 3 | application | <b>yes, always</b>           |
| 4 | application | <b>4</b>                     |
| 5 | application | <b>6</b>                     |
| 6 | reasoning   | <b>no — 25% each</b>         |
| 7 | reasoning   | <b>the IQR</b>               |
| 8 | reasoning   | <b>five marks, four gaps</b> |

## Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Needs the median from SP-03. Slide 5 is the load-bearing one.

| Slide | ~Time | Note                                                                                                              |
|-------|-------|-------------------------------------------------------------------------------------------------------------------|
| 1/8   | 0:00  | Frame it: a box plot compresses a whole data set into five numbers.                                               |
| 2/8   | 1:00  | Hook — poll on the longer whisker. Do NOT resolve. Nearly everyone says yes; the picture invites it.              |
| 3/8   | 2:30  | Build the summary with them. Emphasise finding the median of each HALF, not of the whole set again.               |
| 4/8   | 5:00  | Draw the plot to scale on a number line. To scale matters — a schematic sketch hides the width point coming next. |

|     |       |                                                                                                                                                |
|-----|-------|------------------------------------------------------------------------------------------------------------------------------------------------|
| 5/8 | 7:00  | WE — resolve the hook. If you can, have ten students stand and physically split into four groups; the equal counts become obvious.             |
| 6/8 | 10:00 | CFU — all-student response. $Q1 = 5\frac{1}{2}$ , $Q3 = 12\frac{1}{2}$ . Watch for whole numbers, which mean the middle pair was not averaged. |
| 7/8 | 12:00 | Close on the 50/50 split and the median-inside-the-box guarantee. Then flag SP-07 as where all of this gets used in context.                   |
| 8/8 | 14:00 | Launch the in-class practice sheet.                                                                                                            |