



Volume with fractional edge lengths

GRADE 6 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the volume lesson. 5–8 min, independent. Item 4 is the packing-equals-multiplying idea, which is the heart of the standard.

Timing: 5–8 min · independent. If item 4 answers 36 cubic units, the count/measure distinction needs another pass.

1. Prism $3 \times 4 \times 5$? **FLUENCY**

Answer: **60 cubic units**

1. $V = l \times w \times h = 3 \times 4 \times 5 = 60$ cubic units.

If wrong: Added the edges (12) or wrote square units instead of cubic units.

2. Cube with edge $\frac{1}{2}$? **FLUENCY**

Answer: **$\frac{1}{8}$ cubic unit**

1. $V = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ cubic unit.

2. It is smaller than 1 because the cube is smaller than a unit cube.

If wrong: Answered $1\frac{1}{2}$ by adding the edges, or $\frac{3}{2}$; by multiplying $\frac{1}{2}$ by 3.

3. Prism $2\frac{1}{2} \times 3 \times 2$? **APPLICATION**

Answer: **15 cubic units**

1. Write $2\frac{1}{2}$ as $\frac{5}{2}$.

2. $\frac{5}{2} \times 3 \times 2 = \frac{5}{2} \times 6 = \frac{6}{2} \times 5 = 15$ cubic units.

If wrong: Dropped the fraction and used 2, giving 12.

4. $1\frac{1}{2} \times 1\frac{1}{2} \times 2$ — volume, and count of $\frac{1}{2}$ -cubes? REASONING

Answer: **$4\frac{1}{2}$ cubic units, packed by 36 half-cubes**

1. Formula: $\frac{3}{2} \times \frac{3}{2} \times 2 = \frac{9}{2} = 4\frac{1}{2}$ cubic units.
2. Packing: along each edge, $1\frac{1}{2} \div \frac{1}{2} = 3$ cubes, 3 cubes and $2 \div \frac{1}{2} = 4$ cubes $\rightarrow 3 \times 3 \times 4 = 36$ cubes.
3. Each half-cube holds $\frac{1}{8}$ cubic unit, so $36 \times \frac{1}{8} = \frac{6}{8} = 4\frac{1}{2}$ cubic units.
4. The two agree: multiplying edge lengths is a shortcut for counting the cubes that pack the box.

If wrong: Reported 36 as the volume. 36 is a COUNT of small cubes, not a measure in cubic units — each cube is only $\frac{1}{8}$ of a unit.

Quick answers for fast checking:

#	Level	Answer
1	fluency	60 cubic units
2	fluency	$\frac{1}{8}$ cubic unit
3	application	15 cubic units
4	reasoning	$4\frac{1}{2}$ cubic units; 36 cubes

Practice Worksheet — answer key

Use: During the lesson after the deck. Item 7 is the packing-versus-measuring distinction; item 9 separates volume from surface area and sets up G-05.

Timing: 15–25 min · pairs or independent · circulate on 7 and 9. Require cubic units throughout — the unit is the fastest tell for item 9.

1. $2 \times 3 \times 4$? FLUENCY

Answer: **24 cubic units**

1. $2 \times 3 \times 4 = 24$.

If wrong: Added to get 9.

2. Cube edge $\frac{1}{3}$? FLUENCY

Answer: **$\frac{1}{27}$ cubic unit**

1. $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$.

If wrong: Answered 1 by cancelling, or $\frac{1}{3}$ by multiplying only once.

3. $5 \times 2 \times 3$? FLUENCY

Answer: **30 cubic units**

1. $5 \times 2 \times 3 = 30$.

If wrong: Used square units.

4. $3\frac{1}{2} \times 2 \times 4$? APPLICATION

Answer: **28 cubic units**

1. $3\frac{1}{2} = \text{VIII}$. $\text{VIII} \times 8 = 56/2 = 28$ cubic units.

If wrong: Rounded $3\frac{1}{2}$ down to 3, giving 24.

5. $\frac{3}{4} \times 2 \times 8$? APPLICATION

Answer: **12 cubic units**

1. $\frac{3}{4} \times 16 = \frac{64}{4}$... more simply, $2 \times 8 = 16$, and $\frac{3}{4}$ of $16 = 12$ cubic units.

2. The volume is LESS than 16 because one edge is less than one unit.

If wrong: Answered 16 by ignoring the $\frac{3}{4}$, or 48 by multiplying by 4 instead of $\frac{3}{4}$.

6. $2\frac{1}{2} \times 1\frac{1}{2} \times 4$? APPLICATION

Answer: **15 cubic units**

1. $\frac{5}{8} \times \frac{3}{2} = \frac{15}{4}$. Then $\frac{15}{4} \times 4 = 15$ cubic units.

If wrong: Multiplied $2 \times 1 \times 4 = 8$ by ignoring both fractions.

7. Half-cubes in a $2 \times 1\frac{1}{2} \times 1$ box? REASONING

Answer: **24 cubes**

1. Along the length: $2 \div \frac{1}{2} = 4$ cubes.

2. Along the width: $1\frac{1}{2} \div \frac{1}{2} = 3$ cubes.

3. Along the height: $1 \div \frac{1}{2} = 2$ cubes.

4. $4 \times 3 \times 2 = 24$ cubes.

5. Check: the volume is $2 \times \frac{3}{2} \times 1 = 3$ cubic units, and $24 \times \frac{1}{8} = 3$. They agree.

If wrong: Answered 3 — that is the volume in cubic units, not the number of half-cubes. Each half-cube is only $\frac{1}{8}$ of a cubic unit, so there are eight times as many.

8. $V = 24$, $w = 2$, $h = 3$ — find l ? REASONINGAnswer: **4 units**

1. $V = l \times w \times h$, so $l = V \div (w \times h)$.
2. $w \times h = 2 \times 3 = 6$.
3. $l = 24 \div 6 = 4$ units.
4. Check: $4 \times 2 \times 3 = 24$.

If wrong: Divided 24 by 2 and by 3 separately in the wrong order, or subtracted.**9. Cube edge 2 — is 8 volume or surface area?** REASONINGAnswer: **8 is the volume; 24 is the surface area**

1. Volume fills the inside: $2 \times 2 \times 2 = 8$ cubic units.
2. Surface area covers the outside: a cube has 6 faces, each $2 \times 2 = 4$ square units, so $6 \times 4 = 24$ square units.
3. Tell them apart by the units: volume is CUBIC units, surface area is SQUARE units.
4. Both students computed correctly — they answered different questions.

If wrong: Called one of them simply wrong. Neither is wrong; they measure different things.**Quick answers** for fast checking:

#	Level	Answer
1	fluency	24 cubic units
2	fluency	$\sqrt[3]{27}$ cubic unit
3	fluency	30 cubic units
4	application	28 cubic units
5	application	12 cubic units
6	application	15 cubic units
7	reasoning	24 cubes
8	reasoning	4 units
9	reasoning	8 = volume, 24 = surface area

Homework — answer key

Use: Parallels the practice sheet. Item 7 repeats the packing idea with third-cubes so the pattern is not memorised from halves alone.

Timing: ~15 min at home · independent · review item 7 as the next-day warm-up.

1. $4 \times 5 \times 2$? FLUENCY

Answer: **40 cubic units**

1. $4 \times 5 \times 2 = 40$.

If wrong: Added to get 11.

2. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$? FLUENCY

Answer: **$\frac{1}{24}$ cubic unit**

1. Multiply across: $(1 \times 1 \times 1) \div (2 \times 3 \times 4) = \frac{1}{24}$.

If wrong: Added the fractions. Volume always multiplies.

3. $6 \times 3 \times 2$? FLUENCY

Answer: **36 cubic units**

1. $6 \times 3 \times 2 = 36$.

If wrong: Used square units.

4. $4\frac{1}{2} \times 2 \times 3$? APPLICATION

Answer: **27 cubic units**

1. $4\frac{1}{2} = \frac{9}{2}$. $\frac{9}{2} \times 6 = 27$ cubic units.

If wrong: Rounded to 4, giving 24.

5. $\frac{2}{3} \times 3 \times 5$? APPLICATION

Answer: **10 cubic units**

1. $3 \times 5 = 15$. $\frac{2}{3}$ of 15 = 10 cubic units.

2. Less than 15, as expected, since one edge is under a unit.

If wrong: Answered 15 by ignoring the $\frac{2}{3}$.

6. $3\frac{1}{2} \times 2\frac{1}{2} \times 4$? APPLICATION

Answer: **35 cubic units**

1. $3\frac{1}{2} \times \frac{5}{2} = 3\frac{5}{4}$. Then $3\frac{5}{4} \times 4 = 35$ cubic units.

If wrong: Used $3 \times 2 \times 4 = 24$ by dropping both fractions.

7. Third-cubes in a $1 \times \frac{2}{3} \times 1$ box? REASONING

Answer: **18 cubes**

1. Along the length: $1 \div \frac{1}{3} = 3$ cubes.
2. Along the width: $\frac{2}{3} \div \frac{1}{3} = 2$ cubes.
3. Along the height: $1 \div \frac{1}{3} = 3$ cubes.
4. $3 \times 2 \times 3 = 18$ cubes.
5. Check: volume = $1 \times \frac{2}{3} \times 1 = \frac{2}{3}$ cubic unit, and $18 \times \frac{1}{27} = \frac{18}{27} = \frac{2}{3}$. They agree.

If wrong: Answered $\frac{2}{3}$ — that is the volume, not the cube count.

8. $V = 45$, $l = 5$, $h = 3$ — find w ? REASONING

Answer: **3 units**

1. $w = V \div (l \times h)$.
2. $l \times h = 5 \times 3 = 15$.
3. $w = 45 \div 15 = 3$ units.
4. Check: $5 \times 3 \times 3 = 45$.

If wrong: Subtracted $45 - 5 - 3$.

Quick answers for fast checking:

#	Level	Answer
1	fluency	40 cubic units
2	fluency	$\frac{1}{24}$ cubic unit
3	fluency	36 cubic units
4	application	27 cubic units
5	application	10 cubic units
6	application	35 cubic units
7	reasoning	18 cubes
8	reasoning	3 units

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Slide 5 carries the standard's core claim; do not rush it.

Slide	~Time	Note
1/8	0:00	Frame it: volume answers 'how much fits inside'.

2/8	1:00	Hook — poll bigger/smaller than 1. Do NOT resolve. Many students expect $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ to be large.
3/8	3:00	Establish the formula and tie the word CUBIC to the three factors.
4/8	5:00	WE1 — resolve the hook. If you have unit cubes, show eight half-cubes filling one.
5/8	7:00	WE2, the heart of the standard. Say the count/measure line aloud twice: 36 cubes, $4\frac{1}{2}$ cubic units.
6/8	10:00	CFU — all-student response. Answer 15 cubic units. Watch for 12 from dropping the half.
7/8	12:00	Pre-empt the confusion that G-05 will meet head on. The unit is the tell.
8/8	14:00	Launch the in-class practice sheet.