

GRADE 7 · GEOMETRY · ANSWER KEY

AREA OF A CIRCLE

Today's goal: I can find the area of a circle to solve problems, including by using the formula for area of a circle and give an informal derivation of the relationship between the circumference and area of a circle.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: During the lesson after the deck. Items 5–9 exploit the genuine coincidence at $r = 2$ and then break it — a compact lesson in why one example proves nothing. **Timing:** 15–25 min · pairs or independent · circulate on 6 and 8. Require units on every answer; they are the fastest tell for the area/circumference confusion.

- 1 $49\pi \text{ cm}^2$ 2 $4\pi \text{ cm}^2$ 3 $16\pi \text{ cm}^2$ 4 $49\pi \text{ cm}^2$ 5 $4\pi \text{ cm}$ 6 Because they are the same circle
7 Four times 8 No — a coincidence at $r = 2$ only 9 Area $9\pi \text{ cm}^2$, circumference $6\pi \text{ cm}$ — different

1 **MILD** Radius 7 cm?

$49\pi \text{ cm}^2$

1. $A = \pi(7)^2 = 49\pi \text{ cm}^2$.

Watch for: Answered 14π , the circumference.

2 **MILD** Radius 2 cm?

$4\pi \text{ cm}^2$

1. $A = \pi(2)^2 = 4\pi \text{ cm}^2$.

Watch for: Answered 2π , forgetting to square.

3 **MILD** Radius 4 cm?

$16\pi \text{ cm}^2$

1. $A = \pi(4)^2 = 16\pi \text{ cm}^2$.

Watch for: Answered 8π .

4 **MEDIUM** Diameter 14 cm?

$49\pi \text{ cm}^2$

1. Halve first: $r = 7$. Then $A = \pi(7)^2 = 49\pi \text{ cm}^2$.

Watch for: Answered 196π by squaring 14 — four times too big.

5 **MEDIUM** Circumference, radius 2 cm?

$4\pi \text{ cm}$

1. $C = 2\pi(2) = 4\pi \text{ cm}$.

Watch for: Answered $4\pi \text{ cm}^2$, using area units for a length.

6 **SPICY** Why the same?

Because they are the same circle

1. Radius 7 and diameter 14 describe one circle, since $d = 2r$.

2. Same circle, same area.

3. The two questions differ only in which measurement they hand you.

Watch for: Said it was a coincidence. It is the same circle, so it must give the same area.

7 **SPICY** How many times bigger, and why?

Four times

- $16\pi \div 4\pi = 4$.
- The radius doubled from 2 to 4, and the formula squares it: $(2r)^2 = 4r^2$.
- So doubling the radius always multiplies the area by 4, whatever the starting radius.
- The square in the formula is doing this — it is the same reason area scales by k^2 in a scale drawing.

Watch for: Answered twice, reading the radius change instead of the area change.

8 **SPICY** Both 4π — the same thing?

No — a coincidence at $r = 2$ only

- Area is $4\pi \text{ cm}^2$; circumference is $4\pi \text{ cm}$. The NUMBERS match but the quantities do not.
- The units differ: cm^2 against cm , because one is a covering and the other a distance.
- They coincide only because $\pi r^2 = 2\pi r$ exactly when $r = 2$. At any other radius they differ.
- This is a genuine numerical coincidence at $r = 2$, and a good reminder that matching numbers do not mean matching quantities.

Watch for: Concluded the formulas are equivalent. They agree at exactly one radius, which is the opposite of equivalent.

9 **EXTRA SPICY** Radius 3 — show it

Area $9\pi \text{ cm}^2$, circumference $6\pi \text{ cm}$ — different

- $A = \pi(3)^2 = 9\pi \text{ cm}^2$. $C = 2\pi(3) = 6\pi \text{ cm}$.
- $9\pi \neq 6\pi$, so at $r = 3$ the two quantities give different numbers.
- That confirms question 8: the agreement at $r = 2$ was a coincidence, not a rule.
- Testing a second value is what turns a suspicion into a conclusion — one case can never establish a general claim, but one counterexample can destroy it.

Watch for: Recomputed $r = 2$ rather than testing a new radius. A second value is the whole point of the question.

Homework

8 problems

Use: Closes G7 Geometry batch 1. Item 8 is the payoff for both this skill and the “Scale drawings” lesson — the squaring rule with money attached, which students remember. **Timing:** ~15 min at home · independent · review item 8 as the next-day warm-up; it is worth the full five minutes.

1 $81\pi \text{ cm}^2$

2 $81\pi \text{ cm}^2$

3 $1/4\pi \text{ m}^2$

4 6 cm

5 $12.5\pi \text{ cm}^2$

6 Area $100\pi \text{ cm}^2$, circumference $20\pi \text{ cm}$

7 The units — area is in cm^2

8 The 24-inch — four times the pizza for barely over three times the price

1 **MILD** Radius 9 cm?

$81\pi \text{ cm}^2$

- $A = \pi(9)^2 = 81\pi \text{ cm}^2$.

Watch for: Answered 18π , the circumference.

2 **MEDIUM** Diameter 18 cm?

$81\pi \text{ cm}^2$

- Halve first: $r = 9$. $A = \pi(9)^2 = 81\pi \text{ cm}^2$.
- Same circle as question 1.

Watch for: Answered 324π by squaring 18.

3 **MEDIUM** Radius $1/2 \text{ m}$?

$1/4\pi \text{ m}^2$

- $A = \pi(1/2)^2 = \pi \times 1/4 = 1/4\pi \text{ m}^2$.
- Squaring a fraction below 1 makes it smaller.

Watch for: Answered $1/2\pi$ by forgetting to square.

4 **MEDIUM** Area $36\pi \text{ cm}^2$ — radius?

6 cm

- $\pi r^2 = 36\pi$, so $r^2 = 36$.
- The number that times itself gives 36 is 6, so $r = 6 \text{ cm}$.
- Check: $\pi(6)^2 = 36\pi$.

Watch for: Answered 36, or 18 by halving. Undoing the formula means undoing the square.

5 **MEDIUM** Semicircle, radius 5 cm?

$12.5\pi \text{ cm}^2$

1. Full circle: $\pi(5)^2 = 25\pi \text{ cm}^2$.
2. Half of that: $12.5\pi \text{ cm}^2$.
3. Unlike perimeter, area needs no extra piece — the straight edge has no area.

Watch for: Added something for the diameter. A straight edge is a length and contributes no area.

6 **MEDIUM** Radius 10 — area and circumference

Area $100\pi \text{ cm}^2$, circumference $20\pi \text{ cm}$

1. $A = \pi(10)^2 = 100\pi \text{ cm}^2$. $C = 2\pi(10) = 20\pi \text{ cm}$.

Watch for: Gave both as 100π or both as 20π .

7 **SPICY** How to tell which is the area?

The units — area is in cm^2

1. Area is measured in square units, cm^2 , because it comes from multiplying two lengths.
2. Circumference is a distance, measured in plain cm.
3. So the units alone identify the quantity, even before you check the arithmetic.
4. A second tell at this radius: 100π is much larger than 20π , and for any radius above 2 the area number exceeds the circumference number.
5. Writing units every time turns this into an automatic check.

Watch for: Said the bigger number is the area. True above $r = 2$, false below it, and equal at exactly $r = 2$ — the units are the reliable test.

8 **SPICY** 12-inch pizza \$9 vs 24-inch pizza \$30

The 24-inch — four times the pizza for barely over three times the price

1. 12-inch pizza: $r = 6$, area = 36π square inches, costing \$9.
2. 24-inch pizza: $r = 12$, area = 144π square inches, costing \$30.
3. The diameter doubled, so the AREA quadrupled: $144\pi \div 36\pi = 4$.
4. Price only rose from \$9 to \$30, which is about 3.3 times.
5. Four times the food for 3.3 times the price makes the large better value.
6. Per square inch: $\$9 \div 36\pi \approx \$0.080$, against $\$30 \div 144\pi \approx \$0.066$.
7. The trap is comparing diameters — 24 is twice 12, so it looks like twice the pizza. It is four times.

Watch for: Judged by diameter and concluded the small was better value since \$30 is more than twice \$9. Pizza is sold by area, and area scales by the square.

Exit Ticket

4 problems

Use: End of the area lesson, closing G7 Geometry batch 1. 5–8 min, independent. Item 4 links back to the “Scale drawings” lesson’s squaring rule, now in a circular setting. **Timing:** 5–8 min · independent. An answer of 100π on item 2 is exactly four times too big — a recognisable fingerprint.

1 **MILD** Radius 5 cm? **$25\pi \text{ cm}^2$**

1. $A = \pi r^2 = \pi \times 5^2 = 25\pi \text{ cm}^2$.
2. Square the radius first, then multiply by π .

Watch for: Answered 10π by computing $2\pi r$ — that is the circumference.

2 **SPICY** Diameter 10 cm — why not 100π ? **$25\pi \text{ cm}^2$**

1. Halve the diameter first: $r = 5$.
2. $A = \pi \times 5^2 = 25\pi \text{ cm}^2$.
3. Squaring the diameter gives $\pi \times 10^2 = 100\pi$, which is four times too large.
4. Why four times: the diameter is twice the radius, and squaring doubles that doubling — $2^2 = 4$.
5. So the error is not small. It multiplies the answer by 4 every time.

Watch for: Answered 100π by squaring the diameter. The formula names the radius, and the size of the error (exactly $4\times$) is a useful fingerprint.

3 **SPICY** Circumference too, and the units**Area $25\pi \text{ cm}^2$; circumference $10\pi \text{ cm}$**

1. Area: $\pi r^2 = 25\pi \text{ cm}^2$. Circumference: $2\pi r = 10\pi \text{ cm}$.
2. Circumference is a DISTANCE around the edge, measured in cm.
3. Area is a COVERING of the inside, measured in cm^2 — square units, because it comes from multiplying two lengths.
4. The square in πr^2 is exactly why the units are squared, and it is the only visible difference between the formulas.
5. If your two answers are the same number, check which formula you used twice.

Watch for: Gave both as 25π , or used the same units for each. The unit is the fastest way to tell which quantity you computed.

4 **EXTRA SPICY** Radius 3 vs radius 6 **9π and 36π — B is four times bigger**

1. Circle A: $\pi(3)^2 = 9\pi \text{ cm}^2$. Circle B: $\pi(6)^2 = 36\pi \text{ cm}^2$.
2. $36\pi \div 9\pi = 4$, so B has four times the area.
3. The radius doubled, but the area quadrupled, because area depends on r SQUARED.
4. This is the same rule as scale drawings in the “Scale drawings” lesson: lengths $\times k$ gives area $\times k^2$, and here $k = 2$.
5. Circumference behaves differently — it would only double, since it depends on r directly.

Watch for: Answered twice, applying the length rule to area. Doubling a radius always quadruples the area.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: area covers the inside. Circumference goes around the edge. Different questions.
2	1:00	Hook — poll 25π vs 100π . Do NOT resolve. 100π is chosen whenever the diameter is given.
3	2:30	WE1 — halve, then square, in that order. Write the two steps separately.
4	4:30	Name the fingerprint: exactly four times too big. Students can self-diagnose from it.
5	6:30	WE2 — the two formulas side by side, with units. The units are the practical test.
6	10:30	Doubling and quadrupling. Trail the pizza problem here; it lands better after the rule.
8	8:30	CFU — all-student response. $49\pi \text{ cm}^2$. Watch for 14π , the circumference. Whiteboards: everyone tries Mild, then climbs. These are #1, #4, #6 on the practice sheet.

9		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
10	13:00	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.
