



Comparative inferences about two populations

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the comparative inference lesson, closing the inference half of G7 Statistics. 5–8 min, independent. Both pairs share a gap of 7 and reach opposite conclusions.

Timing: 5–8 min · independent. Watch the WORDING on items 2 and 4: a conclusion about ten students is not a conclusion about two year groups.

1. Means and gap **APPLICATION**

Answer: **Year A 25, Year B 32, gap 7**

1. Year A: sum 125, mean 25 minutes.
2. Year B: sum 160, mean 32 minutes.
3. Gap: $32 - 25 = 7$ minutes.

If wrong: Compared the largest values from each sample instead of the means.

2. MAD of Year A, gap in MADs, conclusion REASONING

Answer: **MAD 1.6; gap 4.375 MADs; the populations differ**

1. Distances from 25: 3, 1, 0, 1, 3. Sum 8. $\text{MAD} = 8 \div 5 = 1.6$ minutes.
2. Gap in MADs: $7 \div 1.6 = 4.375$.
3. That is far more than 2 MADs, so the difference is much larger than the natural variation within a group.
4. Conclusion: Year B students read noticeably longer than Year A students — and because the samples were random, this supports a claim about the whole year groups, not just these ten students.
5. Note the wording: it supports the claim, it does not prove it. Samples always leave uncertainty.

If wrong: Concluded the two SAMPLES differ and stopped. Random samples let you say something about the populations behind them, which is the point of the exercise.

3. MAD of the wider Year A, and the gap in MADs APPLICATION

Answer: **MAD 8; gap 0.875 MADs**

1. The mean is still 25, since $10 + 20 + 25 + 30 + 40 = 125$ and $125 \div 5 = 25$.
2. Distances from 25: 15, 5, 0, 5, 15. Sum 40. $\text{MAD} = 40 \div 5 = 8$ minutes.
3. Gap in MADs: $7 \div 8 = 0.875$.
4. Under 1 MAD, so the gap is no larger than the ordinary spread within a single group.
5. Compare question 2, where the same gap was 4.375 MADs — five times as separated.

If wrong: Assumed the MAD would be similar to question 2. Same mean, very different spread — the mean tells you nothing about the MAD.

4. Same gap, different conclusions REASONING

Answer: **Because 7 minutes is large relative to a MAD of 1.6 and small relative to a MAD of 8**

1. First pair: $7 \div 1.6 = 4.4$ MADs. The gap dwarfs the internal variation, so it is unlikely to be an accident of which students were sampled.
2. Second pair: $7 \div 8 = 0.875$ MADs. Students within a single group already differ by about 8 minutes on average, so a 7-minute gap between group means is well inside the ordinary wobble.
3. With that much internal spread, two random samples from the SAME population could easily differ by 7 minutes.
4. So the first supports a real population difference; the second does not distinguish a genuine difference from sampling luck.
5. This is why a difference in means is never enough on its own — it must be judged against how much the data varies anyway.

If wrong: Said the second pair also shows a difference because the means still differ. The sample means do differ; the question is whether the populations do, and that gap is too small relative to the spread to tell.

Quick answers for fast checking:

#	Level	Answer
1	application	25, 32, gap 7
2	reasoning	4.375 MADs; populations differ
3	application	MAD 8; 0.875 MADs
4	reasoning	large vs small relative to spread

Practice Worksheet — answer key

Use: During the lesson after the deck. Item 7's distinction — 'no difference shown' versus 'no difference exists' — is the subtlest idea in the domain and worth naming explicitly.

Timing: 15–25 min · pairs or independent · circulate on 7 and 9. Insist every conclusion names whether it is about samples or populations.

1. Gap between 14 and 18? FLUENCY

Answer: **4**

1. $18 - 14 = 4$.

If wrong: Added the means.

2. MAD of 12, 13, 14, 15, 16? APPLICATION

Answer: **1.2**

1. Distances from 14: 2, 1, 0, 1, 2. Sum 6. MAD = 1.2.

If wrong: Got 0 by keeping the signs.

3. Gap in MADs? APPLICATION

Answer: **3.33 MADs**

1. $4 \div 1.2 = 10/3 \approx 3.33$.

If wrong: Reported the raw gap of 4.

4. Above or below 2 MADs? Conclusion? REASONING

Answer: **Above — the populations differ**

- 3.33 MADs is well above 2.
- The gap between the groups is more than three times the typical distance of a value from its own mean.
- A difference that large is unlikely to arise just from which individuals happened to be sampled.
- So this supports a conclusion about the POPULATIONS: group B is genuinely higher, not just in these samples.

If wrong: Concluded only that sample B was higher. That is an observation, not an inference — the inference is about the populations.

5. MAD of 4, 9, 14, 19, 24? APPLICATION

Answer: **6**

1. Mean is 14. Distances: 10, 5, 0, 5, 10. Sum 30. MAD = 6.

If wrong: Used the range of 20.

6. Gap of 4 in these MADs? APPLICATION

Answer: **0.67 MADs**

1. $4 \div 6 = 2/3 \approx 0.67$.

If wrong: Reused 1.2 from question 2.

7. Conclusion now, and how it differs REASONING

Answer: **No population conclusion — the gap is inside the ordinary variation**

1. 0.67 MADs is below 1, so the gap between group means is smaller than the typical distance of a single value from its own mean.
2. Individuals within one group already vary by about 6, so two random samples from the SAME population could easily show a gap of 4.
3. So this data does not distinguish a real difference from sampling luck.
4. It differs from question 4 because there the gap was 3.33 MADs — far beyond ordinary wobble — while here it is well inside it.
5. Identical means, identical gap, opposite conclusions, decided entirely by the spread.

If wrong: Said there is no difference at all. The correct statement is weaker: the data does not SHOW a difference. Absence of evidence is not evidence of absence.

8. Estimate the total for 200 APPLICATION

Answer: **2800**

1. Mean 14 per person, 200 people: $14 \times 200 = 2800$.
2. Report it as an estimate — 200 people were not measured, five were.

If wrong: Answered 14, forgetting to scale.

9. 'B had a higher mean, so population B is higher' REASONING

Answer: **The spread is missing, so there is no way to judge the gap**

1. A higher sample mean is a fact about the sample, not yet about the population.
2. Two random samples from the SAME population almost always have different means — that is ordinary sampling variation, established in SP-02.
3. So the question is whether this gap is bigger than the wobble you would expect anyway, and that requires the MAD.
4. With a small MAD the conclusion may well hold; with a large one the same gap tells you nothing.
5. What is missing: the spread of each sample, and the gap expressed as a multiple of it.
6. The reasoning is not wrong so much as unfinished — it stops at the point where the statistics begins.

If wrong: Said the student needs a bigger sample. That would help, but the specific omission is the spread — they cannot interpret the gap they already have.

Quick answers for fast checking:

#	Level	Answer
1	fluency	4
2	application	1.2
3	application	3.33 MADs
4	reasoning	above 2; populations differ
5	application	6
6	application	0.67 MADs
7	reasoning	no conclusion supported
8	application	2800
9	reasoning	the spread is missing

Homework — answer key

Use: Closes the inference half of G7 Statistics. Item 8 asks students to write the conclusion as prose, which is where the reasoning either holds or quietly collapses.

Timing: ~15 min at home · independent · read two or three item 8 pairs aloud as the next-day warm-up; check the second sentence does not accidentally assert sameness.

1. Gap between 20 and 26? FLUENCY

Answer: **6**

1. $26 - 20 = 6$.

If wrong: Added the means.

2. MAD of 18–22? APPLICATION

Answer: **1.2**

1. Distances: 2, 1, 0, 1, 2. Sum 6. MAD = 1.2.

If wrong: Kept the signs.

3. Gap in MADs? APPLICATION

Answer: **5 MADs**

1. $6 \div 1.2 = 5$.

If wrong: Gave the raw gap.

4. Conclusion, and about what? REASONING

Answer: **5 MADs — the POPULATIONS differ**

1. 5 MADs is far above the threshold of 2.
2. The gap between the groups is five times the typical distance of a value from its own mean.
3. Because the samples were random, this supports a claim about the whole populations, not just the ten individuals measured.
4. Wording that works: ‘the data supports the conclusion that population B is higher than population A’.
5. Wording that overreaches: ‘population B is higher’ stated as certain fact — samples always leave room for doubt.

If wrong: Concluded about the samples only, or stated the population claim as proven rather than supported.

5. MAD of 8, 14, 20, 26, 32? APPLICATION

Answer: **7.2**

1. Mean 20. Distances: 12, 6, 0, 6, 12. Sum 36. $MAD = 36 \div 5 = 7.2$.

If wrong: Used the range of 24.

6. Gap of 6 in these MADs? APPLICATION

Answer: **0.83 MADs**

1. $6 \div 7.2 = \frac{5}{6} \approx 0.83$.

If wrong: Reused 1.2.

7. Conclusion now — careful wording REASONING

Answer: **The data does not SHOW a difference between the populations**

1. 0.83 MADs is below 1, so the gap is inside the ordinary variation within a single group.
2. Two random samples from the same population could easily differ by this much.
3. So the correct statement is: this data does not provide evidence of a difference.
4. It is NOT correct to say the populations are the same. They might well differ — this sample simply cannot tell.
5. The distinction matters: one claim is supported by the data, the other is not supported by anything.

If wrong: Wrote ‘the populations are the same’. Failing to detect a difference is not the same as establishing there is none.

8. Two honest newspaper sentences REASONING

Answer: **One reports a difference; one reports that none was detected**

1. First pair (5 MADs): “A survey of two randomly sampled groups found group B averaged 26 against group A's 20, a difference large enough, relative to the variation within each group, to indicate a genuine difference between the populations.”
2. Second pair (0.83 MADs): “A survey found a 6-point difference between the two groups, but because individuals within each group varied by about the same amount, the survey did not establish a real difference between the populations.”
3. Both sentences report the same gap of 6 and reach different conclusions, because they account for the spread.
4. Neither claims certainty. Both name the sampling.
5. A dishonest version of the second would report the 6-point gap alone and let the reader infer a difference the data does not support — which is how most misleading statistics work.

If wrong: Wrote the same sentence twice with different numbers. The two data sets support genuinely different claims, and the sentences must differ in what they assert, not just in their figures.

Quick answers for fast checking:

#	Level	Answer
1	fluency	6
2	application	1.2
3	application	5 MADs
4	reasoning	populations differ

5	application	7.2
6	application	0.83 MADs
7	reasoning	no difference shown
8	reasoning	two different claims

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Closes the inference half of G7 Statistics, combining SP-02's sampling variation with SP-03's MAD comparison.

Slide	~Time	Note
1/8	0:00	Frame it: two samples, two populations. The samples are what you have; the populations are what you want to know about.
2/8	1:00	Hook — ask for the conclusion AND the confidence. Almost everyone says yes with high confidence.
3/8	3:00	Recall SP-02 deliberately. If samples of ONE group vary, a gap between two groups is not automatically meaningful.
4/8	5:00	WE — both MADs side by side. The identical gap of 7 with opposite conclusions is the whole lesson.
5/8	8:00	The thresholds, stated as guidance rather than as a bright line.
6/8	9:30	CFU — all-student response. 3.33 MADs; supports a population difference. Watch for conclusions about the samples only.
7/8	11:00	Precision of language. Both distinctions here cost marks and mislead readers outside school.
8/8	13:30	Launch the in-class practice sheet.