



Comparing two data distributions

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the comparison lesson. 5–8 min, independent. The two pairs are constructed with identical means and gaps and different spreads, so item 4 cannot be answered from the centres alone.

Timing: 5–8 min · independent. Assumes MAD from the “Measures of center and variability” sheet (Grade 6). A MAD of 0 signals that signs were kept.

1. Means and gap for A and B **APPLICATION**

Answer: **A: 20, B: 23, gap 3**

1. A: sum 100, mean $100 \div 5 = 20$.
2. B: sum 115, mean $115 \div 5 = 23$.
3. Gap: $23 - 20 = 3$.

If wrong: Compared the largest values, or gave the gap as a list of differences.

2. MAD of A, and the gap in MADs **REASONING**

Answer: **MAD 1.2; the gap is 2.5 MADs**

1. Distances from the mean of 20: 2, 1, 0, 1, 2. Sum 6, divided by 5 gives $MAD = 1.2$.
2. Gap in MADs: $3 \div 1.2 = 2.5$.
3. So the centres are two and a half typical distances apart — a substantial separation.
4. With data this tightly clustered, a gap of 3 clearly separates the groups.

If wrong: Kept the signs when finding distances, getting a MAD of 0. Deviations are always positive.

3. Means, gap and MAD for C and D APPLICATION

Answer: **C: 20, D: 23, gap 3, MAD of C is 6**

1. C: sum 100, mean 20. D: sum 115, mean 23. Gap 3 — exactly the same as before.
2. Distances from 20 in C: 10, 5, 0, 5, 10. Sum 30, divided by 5 gives MAD = 6.
3. The data is far more scattered than in Group A.

If wrong: Assumed the MAD would also be similar. Same means and same gap, very different spread.

4. Why does the first difference matter more? REASONING

Answer: **Because 3 is 2.5 MADs for A and B, but only 0.5 MADs for C and D**

1. A and B: gap 3, MAD 1.2, so the separation is $3 \div 1.2 = 2.5$ MADs.
2. C and D: gap 3, MAD 6, so the separation is $3 \div 6 = 0.5$ MADs.
3. The first pair is five times as separated as the second, despite an identical gap in means.
4. In A and B the values barely overlap: A tops out at 22, B starts at 21.
5. In C and D they overlap almost completely — C runs 10 to 30 and D runs 13 to 33, so most values could belong to either group.
6. A difference in means is only meaningful relative to the spread, which is why the standard asks for it as a multiple of the MAD.

If wrong: Said the two situations are the same because the gap is the same. The gap is the same and the meaning is entirely different, which is the whole point of the skill.

Quick answers for fast checking:

#	Level	Answer
1	application	20, 23, gap 3
2	reasoning	MAD 1.2, 2.5 MADs
3	application	20, 23, gap 3, MAD 6
4	reasoning	2.5 MADs vs 0.5 MADs

Practice Worksheet — answer key

Use: During the lesson after the deck. Questions 5 and 7 use the same gap of 3 with very different MADs; item 8 forces the comparison that makes the MAD's role visible.

Timing: 15–25 min · pairs or independent · circulate on 8 and 9. Sketching both pairs as dot plots on the same axis makes the overlap argument immediate.

1. Mean of 4, 5, 6, 7, 8? **FLUENCY**

Answer: **6**

1. Sum 30, divide by 5 $\rightarrow$ 6.

If wrong: Picked the middle value without computing; correct here by luck.

2. Mean of 7, 8, 9, 10, 11? **FLUENCY**

Answer: **9**

1. Sum 45, divide by 5 $\rightarrow$ 9.

If wrong: Same shortcut.

3. Gap? **FLUENCY**

Answer: **3**

1. $9 - 6 = 3$.

If wrong: Listed the differences pair by pair.

4. MAD of the first group? **APPLICATION**

Answer: **1.2**

1. Distances from 6: 2, 1, 0, 1, 2. Sum 6. $\text{MAD} = 6 \div 5 = 1.2$.

If wrong: Got 0 by keeping the signs.

5. Gap in MADs — strong or weak? **REASONING**

Answer: **2.5 MADs — strong**

1. $3 \div 1.2 = 2.5$.

2. The centres are two and a half typical distances apart.

3. At 2 MADs or more the groups are clearly distinct, so this is a strong difference.

4. Checking the values agrees: the first group ends at 8 and the second begins at 7, so they barely overlap.

If wrong: Reported the gap of 3 without dividing. The multiple, not the raw gap, is the comparable quantity.

6. MAD of 2, 6, 10, 14, 18? APPLICATION

Answer: **4.8**

1. Mean: $50 \div 5 = 10$.
2. Distances: 8, 4, 0, 4, 8. Sum 24. $\text{MAD} = 24 \div 5 = 4.8$.

If wrong: Used the range of 16 instead of the MAD.

7. Gap in MADs for this pair REASONING

Answer: **Gap 3; 0.625 MADs**

1. Means 10 and 13, so the gap is 3 — the same as question 3.
2. In MADs: $3 \div 4.8 = 0.625$.
3. Under 1 MAD, so the groups overlap heavily and the difference is weak.
4. The first group runs 2 to 18 and the second 5 to 21; almost every value could plausibly belong to either.

If wrong: Divided by the wrong MAD, or reported the gap of 3 and called it the same as question 5.

8. Which pair is more clearly separated? REASONING

Answer: **The first — 2.5 MADs against 0.625**

1. Question 5: 2.5 MADs. Question 7: 0.625 MADs.
2. The first pair is four times as separated, even though both pairs have a gap of exactly 3.
3. What differs is the spread: MAD 1.2 against MAD 4.8.
4. A gap only becomes meaningful when measured against how much the data varies naturally.
5. Reporting 'the means differ by 3' for both would hide the entire difference between the two situations.

If wrong: Said they are equally separated because both gaps are 3. That is the comparison the MAD exists to correct.

9. Means 50 and 53, concluded clearly different REASONING

Answer: **They have not checked the spread**

1. A gap of 3 tells you nothing on its own.
2. If the MAD is about 1, the separation is 3 MADs and the groups really are distinct.
3. If the MAD is about 10, the separation is 0.3 MADs and the groups overlap almost entirely — the gap could easily arise by chance.
4. Same means, same gap, opposite conclusions, decided entirely by the spread.
5. So the student needs the MAD of each group before any conclusion is available.
6. This is why the standard asks for the difference as a MULTIPLE of a measure of variability rather than as a raw number.

If wrong: Said they should have used more data. More data would help, but the specific omission is the spread — they cannot interpret the gap they already have.

Quick answers for fast checking:

#	Level	Answer
1	fluency	6
2	fluency	9
3	fluency	3
4	application	1.2
5	reasoning	2.5 MADs, strong
6	application	4.8
7	reasoning	0.625 MADs, weak
8	reasoning	the first pair
9	reasoning	the spread

Homework — answer key

Use: Parallels the practice sheet with a gap of 4 rather than 3, so the conclusion cannot be recalled from the lesson and has to be recomputed.

Timing: ~15 min at home · independent · review item 8 as the next-day warm-up; sketch both pairs on one axis.

1. Mean of 12–16? FLUENCY

Answer: **14**

1. Sum 70, divide by 5 $\rightarrow$ 14.

If wrong: Guessed the middle value.

2. Mean of 16–20? FLUENCY

Answer: **18**

1. Sum 90, divide by 5 $\rightarrow$ 18.

If wrong: Guessed the middle value.

3. Gap? FLUENCY

Answer: **4**

1. $18 - 14 = 4$.

If wrong: Listed differences.

4. MAD of the first group? APPLICATION

Answer: **1.2**

1. Distances from 14: 2, 1, 0, 1, 2. Sum 6. MAD = 1.2.

If wrong: Got 0 by keeping the signs.

5. Gap in MADs — strong or weak? REASONING

Answer: **3.33 MADs — strong**

1. $4 \div 1.2 = 10/3 \approx 3.33$ MADs.

2. Well above 2, so the groups are clearly distinct.

3. The values confirm it: the first group ends at 16 and the second begins at 16, so they touch at a single point and otherwise do not overlap.

If wrong: Reported 4 without dividing.

6. MAD of 5, 10, 15, 20, 25? APPLICATION

Answer: **6**

1. Mean: $75 \div 5 = 15$.
2. Distances: 10, 5, 0, 5, 10. Sum 30. MAD = 6.

If wrong: Used the range of 20.

7. Gap and MADs for this pair REASONING

Answer: **Gap 4; 0.67 MADs**

1. Means 15 and 19, so the gap is 4 — the same as question 3.
2. In MADs: $4 \div 6 = \frac{2}{3} \approx 0.67$ MADs.
3. Below 1 MAD, so the difference is weak and the groups overlap substantially.

If wrong: Used the MAD of 1.2 from the earlier group. Each pair needs its own spread.

8. Same gap of 4 — why is one convincing? REASONING

Answer: **3.33 MADs against 0.67 MADs — five times the separation**

1. First pair: gap 4, MAD 1.2, separation 3.33 MADs.
2. Second pair: gap 4, MAD 6, separation 0.67 MADs.
3. The first is five times as separated despite the identical gap.
4. In the first pair the data is tightly clustered, so a gap of 4 moves the whole group clear of the other. Values run 12–16 against 16–20, meeting only at 16.
5. In the second the data is scattered from 5 to 25 and 9 to 29, so a gap of 4 barely shifts anything — most values could belong to either group.
6. A raw gap is not comparable across data sets. Only the gap measured in MADs is.

If wrong: Concluded both are convincing because 4 is a clear difference. Whether 4 is clear depends entirely on what a typical distance from the mean happens to be.

Quick answers for fast checking:

#	Level	Answer
1	fluency	14
2	fluency	18
3	fluency	4
4	application	1.2
5	reasoning	3.33 MADs, strong

6	application	6
7	reasoning	0.67 MADs, weak
8	reasoning	five times as separated

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Assumes MAD from the “Measures of center and variability” sheet (Grade 6). Sketch both pairs as dot plots on one axis if you can — the overlap argument is visual.

Slide	~Time	Note
1/8	0:00	Frame it: comparing two groups by their means alone is the commonest mistake in statistics.
2/8	1:00	Hook — poll equally different or not. Do NOT resolve. Identical gaps make ‘equally’ the obvious answer.
3/8	3:00	Establish that both pairs have the same means and gap, so whatever distinguishes them is not the centre.
4/8	5:00	WE1 — the gap in MADs. Compute both, side by side, and let the factor of five land.
5/8	7:30	WE2 — the overlap, which is the intuition behind the arithmetic. Dot plots on a shared axis if possible.
6/8	9:30	CFU — all-student response. 2.5 MADs. Watch for the raw gap of 3.
7/8	11:00	The rough thresholds. Say clearly that these are guidance, not a rule with a sharp boundary.
8/8	13:00	Launch the in-class practice sheet.