



Drawing inferences from random samples

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the inference lesson. 5–8 min, independent. Item 4 is the standard's core: sample statistics estimate population statistics and are not the same thing.

Timing: 5–8 min · independent. Watch for the word ‘estimate’ being used; students who write ‘300 students own pets’ have missed the point even with correct arithmetic.

1. 12 of 20? **FLUENCY**

Answer: **60%**

1. $12 \div 20 = 0.6 = 60\%$.
2. Or $12/20$ simplifies to $3/5$.

If wrong: Answered 12%. The 12 is a count out of 20, not out of 100.

2. Estimate for 500 students **APPLICATION**

Answer: **300 students**

1. $60\% \text{ of } 500 = 0.6 \times 500 = 300 \text{ students}$.
2. Or scale directly: $500 \div 20 = 25 \text{ samples' worth}$, and $12 \times 25 = 300$.
3. Say ESTIMATE, not ‘300 students own pets’ — you surveyed 20, not 500.

If wrong: Answered 12, forgetting to scale up, or 500×12 without dividing by 20.

3. Five samples: 12, 9, 11, 14, 10 APPLICATION

Answer: **Mean 11.2; spread 5**

1. Sum: $12 + 9 + 11 + 14 + 10 = 56$. Mean: $56 \div 5 = 11.2$.
2. Highest 14, lowest 9, so the spread is $14 - 9 = 5$.
3. Five samples of the same size, taken the same way, gave five different answers.

If wrong: Averaged only the first and last, or reported the spread as the list of values. Spread is a single number.

4. Which estimate, and why is the spread useful? REASONING

Answer: **Use the combined result, about 280; the spread shows how much a single sample can wander**

1. Pooling all five: 56 pet owners out of 100 students surveyed, which is 56%.
2. Estimate for the school: 56% of 500 = 280 students.
3. This is better than any single sample because it rests on 100 students rather than 20.
4. Why the spread helps: samples ranged from 9 to 14, which scales to estimates from 225 to 350. That is a wide gap.
5. Knowing this tells you a single sample of 20 could easily be 50 students off, so you should report roughly 280 rather than exactly 280.
6. Without repeating the sample you would have had no way to know how shaky one estimate is. The variation is information, not error.

If wrong: Picked the sample closest to the others, or averaged the five estimates without noticing that pooling is cleaner. The deeper error is treating the spread as a problem to eliminate rather than as a measure of uncertainty.

Quick answers for fast checking:

#	Level	Answer
1	fluency	60%
2	application	300 students
3	application	mean 11.2, spread 5
4	reasoning	about 280; spread shows uncertainty

Practice Worksheet — answer key

Use: During the lesson after the deck. Items 4–6 show two legitimate samples producing estimates 125 students apart, which is the most persuasive argument for repeating a sample.

Timing: 15–25 min · pairs or independent · circulate on 6 and 9. If you can, have the class draw real samples from a bag of counters; the variation is far more convincing when they generate it.

1. 9 of 20? **FLUENCY**

Answer: **45%**

1. $9 \div 20 = 0.45 = 45\%$.

If wrong: Answered 9%.

2. 14 of 20? **FLUENCY**

Answer: **70%**

1. $14 \div 20 = 0.70 = 70\%$.

If wrong: Answered 14%.

3. How far apart? **FLUENCY**

Answer: **25 percentage points**

1. $70 - 45 = 25$.

If wrong: Answered 5, the difference in counts rather than percentages.

4. Estimate from 9 of 20, school of 500 **APPLICATION**

Answer: **225 students**

1. $45\% \text{ of } 500 = 0.45 \times 500 = 225 \text{ students}$.

If wrong: Answered 9, or 500×9 .

5. Estimate from 14 of 20 **APPLICATION**

Answer: **350 students**

1. $70\% \text{ of } 500 = 350 \text{ students}$.

If wrong: Answered 14.

6. How far apart, and what does it tell you? REASONING

Answer: **125 students apart — a single sample of 20 is unreliable**

1. $350 - 225 = 125$ students.
2. Both came from correctly taken random samples of the same size, and they disagree by 125 students out of 500.
3. That is a quarter of the school, from nothing more than which 20 students happened to be picked.
4. So a single sample of 20 gives a rough indication, not a dependable figure.
5. This is exactly why you take several samples: one alone gives you a number with no sense of how much it might be off.

If wrong: Concluded one of the samples must have been taken wrongly. Both were fine; this much variation is normal at $n = 20$.

7. 56 of 100? APPLICATION

Answer: **56%**

1. $56 \div 100 = 56\%$.

If wrong: Answered 56 students.

8. Pooled estimate for 500 APPLICATION

Answer: **280 students**

1. 56% of 500 = 280 students.

If wrong: Answered 56.

9. Why more trustworthy, and why not a fact? REASONING

Answer: **It rests on 100 students, so it varies less — but it is still an estimate**

1. The pooled figure uses 100 students instead of 20, five times as much information.
2. Larger samples vary less from one to the next, so 280 is much less likely to be far off than 225 or 350 were.
3. Notice 280 sits between the two extreme single-sample estimates, as you would expect.
4. Why it is still not a fact: 400 students were never asked. The true number could be 270 or 295.
5. The only way to know exactly would be to ask all 500 — at which point it is a census, not a sample.
6. An estimate with a known amount of uncertainty is the honest output of sampling, and reporting it as exact is the error.

If wrong: Said the pooled estimate is now the true value. More data narrows the uncertainty; it never removes it.

Quick answers for fast checking:

#	Level	Answer
1	fluency	45%
2	fluency	70%
3	fluency	25 points
4	application	225 students
5	application	350 students
6	reasoning	125 apart; unreliable
7	application	56%
8	application	280 students
9	reasoning	less variable, still an estimate

Homework — answer key

Use: Parallels the practice sheet. Item 8 is about the SENTENCE rather than the arithmetic, which is where sampling actually goes wrong in the world.

Timing: ~15 min at home · independent · read two or three item 8 rewrites aloud as the next-day warm-up and check each names the sample.

1. 11 of 20? FLUENCY

Answer: **55%**

1. $11 \div 20 = 55\%$.

If wrong: Answered 11%.

2. 10 of 20? FLUENCY

Answer: **50%**

1. $10 \div 20 = 50\%$.

If wrong: Answered 10%.

3. Estimate from 11, school of 400 APPLICATION

Answer: **220 students**

1. $55\% \text{ of } 400 = 220$.

If wrong: Answered 11.

4. Estimate from 10 APPLICATION

Answer: **200 students**

1. $50\% \text{ of } 400 = 200$.

If wrong: Answered 10.

5. How far apart? FLUENCY

Answer: **20 students**

1. $220 - 200 = 20$.

If wrong: Answered 1, the count difference.

6. Pooled: 21 of 40 APPLICATION

Answer: **52.5%; estimate 210 students**

1. $21 \div 40 = 0.525 = 52.5\%$.
2. 52.5% of $400 = 210$ students.

If wrong: Averaged the two percentages without checking; here it happens to agree, because both samples were the same size.

7. Why between, and always? REASONING

Answer: **Because pooling averages the two — and yes, when the samples are the same size**

1. Pooling combines all 40 students, so the result is pulled toward both samples.
2. 210 lies between 200 and 220, as it must: a combined figure cannot be more extreme than both of its parts.
3. This will always happen — a pooled proportion is a weighted average of the individual ones, and an average always lies between the smallest and largest.
4. When the samples are the SAME size the weights are equal, so pooling matches averaging the percentages: $(55 + 50) \div 2 = 52.5\%$.
5. With DIFFERENT sizes they differ, and pooling is the correct method, because a sample of 60 should count for more than a sample of 20.

If wrong: Said it is a coincidence. A combined value always lies between its parts; only the weighting changes.

8. Rewrite '210 students walk to school' REASONING

Answer: **e.g. "Based on a random sample of 40, we estimate about 210 of the 400 students walk to school."**

1. The original states a fact about all 400 students, but only 40 were asked.
2. An honest version does three things: names the sample, uses the word ESTIMATE, and signals imprecision with 'about'.
3. Better still, it can indicate the uncertainty: single samples of 20 gave 200 and 220, so something like 'roughly 210, likely within about 20 either way' is defensible.
4. Saying '210 students walk' claims a precision the data cannot support, even though 210 is the best available figure.
5. The number is right; the sentence is wrong.

If wrong: Changed 210 to a different number. The figure is fine — what needed fixing was the claim being made about it.

Quick answers for fast checking:

#	Level	Answer
1	fluency	55%
2	fluency	50%
3	application	220 students
4	application	200 students
5	fluency	20 students
6	application	52.5%; 210 students
7	reasoning	always, when sizes match
8	reasoning	name the sample, say estimate

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. If you can spare ten minutes, have groups draw real samples from a bag of counters — self-generated variation is far more convincing than quoted numbers.

Slide	~Time	Note
1/8	0:00	Frame it: sampling gives you an estimate with uncertainty attached, and both halves matter.
2/8	1:00	Hook — ask for the number AND their confidence. Most give 300 with high confidence, which slide 4 undermines.
3/8	3:00	The scaling method. Insist on the proportion as an intermediate step; scaling counts directly invites errors.
4/8	5:00	WE1 — the four extra samples. Let the range 225 to 350 sit visibly before commenting.
5/8	7:30	WE2 — reframe the variation as information rather than as a fault in the method.
6/8	9:30	CFU — all-student response. 280 students. Watch for students scaling 56 by 500 without dividing by 100.
7/8	11:30	The honest sentence. Mark for this wording on the exit ticket; students calibrate to what is marked.
8/8	13:30	Launch the in-class practice sheet.