

GRADE 7 · GEOMETRY · ANSWER KEY

DRAWING SHAPES WITH GIVEN CONDITIONS

Today's goal: I can draw geometric shapes with given conditions (side lengths, angle measures, radius/diameter) and know and understand conditions that determine a unique triangle, more than one triangle, or no triangle.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Item 8 is the most valuable: it asks students to REPAIR an impossible triangle, which requires understanding the condition rather than applying it. **Timing:** 15–25 min · pairs or independent · paper strips cut to length make items 3 and 4 physical. Circulate on 7 and 9.

1 Yes

2 No

3 No

4 Yes

5 Yes — infinitely many

6 70°

7 No — they sum to 190°

8 3 short; the longest side must drop below 7, or a short side must grow

1 **◀ MILD** 6, 8, 10?

Yes

1. $6 + 8 = 14 > 10$. A triangle is possible.

Watch for: Said no because the numbers are not equal.

2 **◀ MILD** 2, 3, 6?

No

1. $2 + 3 = 5$, and $5 < 6$. The short sides cannot reach.

Watch for: Said yes without checking.

3 **◀◀ MEDIUM** 1, 1, 2?

No

1. $1 + 1 = 2$, which EQUALS the longest side.

2. The sides meet only by lying flat, giving a straight line rather than a triangle.

Watch for: Said yes because the sides reach. Reaching exactly is the degenerate case.

4 **◀◀ MEDIUM** 4, 9, 12?

Yes

1. Two shorter sides: $4 + 9 = 13$.

2. $13 > 12$, so a triangle is possible — just barely.

Watch for: Said no by comparing $4 + 9$ with 9, or by comparing the wrong pair. Always test the two SHORTEST against the LONGEST.

5 **◀◀ MEDIUM** Angles 90°, 45°, 45°?

Yes — infinitely many

1. Sum: $90 + 45 + 45 = 180^\circ$, so the angles are valid.

2. But nothing fixes the size, so there are infinitely many such triangles, all similar.

3. Each is a right-angled isosceles triangle; they differ only in scale.

Watch for: Answered one. Valid angles guarantee existence, never uniqueness.

6 **◀◀ MEDIUM** Third angle from 50° and 60°?

70°

1. $180 - 50 - 60 = 70^\circ$.

2. Two angles always determine the third, because the total is fixed.

Watch for: Added the two given angles and answered 110°, which is what remains subtracted rather than the angle itself.

7 🔥🔥🔥 **SPICY** Angles 90° , 90° , 10° ?

No — they sum to 190°

1. $90 + 90 + 10 = 190^\circ$, but a triangle's angles must total exactly 180° .
2. So no triangle has these angles.
3. Intuitively: two right angles give two sides going straight up in parallel, and parallel lines never meet, so there is no third corner.
4. The angle sum is a hard constraint, not a guideline.

Watch for: Said yes because each angle individually looks reasonable. The SUM is what matters.

8 🔥🔥🔥 **SPICY** 3, 4, 10 — by how much short?

3 short; the longest side must drop below 7, or a short side must grow

1. Two shorter sides: $3 + 4 = 7$. Longest: 10. Shortfall: $10 - 7 = 3$.
2. For a triangle to exist, the sum of the two shorter sides must EXCEED the longest.
3. Option one: shorten the long side to anything less than 7 — say 6, giving 3, 4, 6, which works since $7 > 6$.
4. Option two: lengthen a short side so the pair exceeds 10 — say 3, 8, 10, which works since $3 + 8 = 11 > 10$.
5. Either change closes the gap. What matters is the RELATIONSHIP between the sides, not their individual sizes.

Watch for: Suggested making all three sides longer. Scaling all three keeps the same relationship, so 6, 8, 20 fails for exactly the same reason.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Items 2–5 sit either side of the boundary with one unit between them, which is the cleanest way to justify the strict inequality. **Timing:** ~15 min at home · independent · review item 6 as the next-day warm-up.

- 1 Yes 2 Yes 3 No 4 Yes — infinitely many 5 60° 6 Because the condition is strictly greater than
7 Neither is mistaken — angles fix shape, not size

1 🔥 **MILD** 5, 12, 13?

Yes

1. $5 + 12 = 17 > 13$.

Watch for: Compared 5 + 13 with 12.

2 🔥 **MEDIUM** 8, 8, 15?

Yes

1. $8 + 8 = 16$, and $16 > 15$. A triangle is possible, though a very flat one.

Watch for: Said no, assuming a near-miss fails. It clears by 1, which is enough.

3 🔥 **MEDIUM** 8, 8, 16?

No

1. $8 + 8 = 16$, which EQUALS the longest side.
2. The sides lie flat along the 16, giving a straight line with no height.

Watch for: Said yes because $16 = 16$. Equality is exactly the failing case.

4 🔥 **MEDIUM** Angles 100° , 40° , 40° ?

Yes — infinitely many

1. Sum: $100 + 40 + 40 = 180^\circ$, so valid.
2. Infinitely many, since the angles fix the shape but not the size.

Watch for: Answered one.

5 🔥 **MEDIUM** Third angle from 35° and 85° ?

60°

1. $180 - 35 - 85 = 60^\circ$.

Watch for: Answered 120° , the sum of the two given.

6  **SPICY** Why does 15 work and 16 not?

Because the condition is strictly greater than

1. With 15: $8 + 8 = 16$, and $16 > 15$, so the two sides reach across with 1 unit to spare. That spare is what lets them rise into a triangle.
2. With 16: $8 + 8 = 16$ exactly. They reach but with nothing to spare, so they can only lie flat.
3. The difference of one unit is the difference between a very flat triangle and no triangle at all.
4. This is why the rule says GREATER THAN rather than greater than or equal to — the boundary itself fails.
5. At 15 the triangle is thin but real, with a genuine height and area.

Watch for: Said the difference is too small to matter. At a boundary, one unit is the whole difference.

7  **SPICY** Two different-sized triangles from the same angles

Neither is mistaken — angles fix shape, not size

1. Three angles summing to 180° determine the SHAPE of a triangle completely.
2. They say nothing about how large it is, because scaling a triangle changes no angle.
3. So both students are correct, and their triangles are SIMILAR — same shape, different size.
4. To get a unique triangle you need at least one side length; then the rest is fixed.
5. This is exactly the same idea as scale drawings in the “Scale drawings” lesson: a scaled copy has identical angles.

Watch for: Assumed one must be wrong because the answers differ. The task had infinitely many correct answers, so different results are expected.

Exit Ticket

4 problems

Use: End of the drawing-with-conditions lesson. 5–8 min, independent. Item 3 is the boundary case that justifies the strict inequality; item 4 separates shape from size. **Timing:** 5–8 min · independent · strips of paper or straws help enormously here.

1 Yes

2 No

3 No — the sides meet but lie flat

4 Infinitely many

1

MILD

Sides 3, 4, 5?

Yes

1. Check the two shorter sides against the longest: $3 + 4 = 7$.
2. $7 > 5$, so the two short sides can reach across and meet above the long one.
3. A triangle is possible — in fact exactly one, up to position.

Watch for: Said no because the numbers must be equal. Sides of different lengths are perfectly normal.

2

MEDIUM

Sides 3, 4, 10?

No

1. The two shorter sides are 3 and 4, totalling 7.
2. The longest side is 10, and $7 < 10$.
3. Laid end to end along the 10 cm side, the short sides span only 7 cm — they fall 3 cm short, so their ends can never meet.
4. No triangle is possible.

Watch for: Said yes because three lengths were given. Being given three numbers does not make them a triangle.

3

SPICY

Sides 5, 5, 10?

No — the sides meet but lie flat

1. $5 + 5 = 10$, which EQUALS the longest side rather than exceeding it.
2. The two short sides do reach across, but only by lying flat along the 10 cm side.
3. The result is a straight line, not a triangle — it has no height and no area.
4. So the condition must be strictly greater than: $3 + 4 > 5$ works, $5 + 5 > 10$ does not.
5. This boundary case is the reason the rule uses $>$ and not $\geq$.

Watch for: Said yes because the sides reach exactly. Reaching exactly gives a flat line, which is not a triangle.

4

SPICY

How many triangles with 60° , 60° , 60° ?

Infinitely many

1. The angles sum to 180° , so they are valid.
2. Every such triangle is equilateral, but nothing fixes how big it is.
3. A 1 cm equilateral triangle and a 100 cm one both have angles 60° , 60° , 60° .
4. So angles determine the SHAPE and not the SIZE — the triangles are all similar but not congruent.
5. To pin down one triangle you need at least one length.

Watch for: Answered one, thinking the angles were enough. Three angles never determine a unique triangle, because scaling changes no angle.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: given some conditions, how many triangles fit? Sometimes none, sometimes one, sometimes infinitely many.
2	1:00	Hook — poll yes or no. Do NOT resolve. Most say yes, because three numbers were given.
3	2:30	WE1 — resolve with physical strips if you have them. Watching the ends fail to meet does the teaching.
4	5:00	WE2 — the degenerate case. This is why the inequality is strict, and it is nearly always skipped.
5	7:00	Switch to angles. Establish the 180° sum before asking about uniqueness.
6	10:30	Shape versus size. Link back to the “Scale drawings” lesson — a scale drawing preserves every angle.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	9:00	CFU — all-student response. Yes: $4 + 9 = 13 > 12$. Watch for students comparing the wrong pair. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.

