

GRADE 7 · RATIOS & PROPORTIONAL RELATIONSHIPS · ANSWER KEY

MULTI-STEP PERCENT PROBLEMS

Today's goal: I can solve multi-step percent problems, including by using proportional relationships.


How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Items 4–7 walk students into the round-trip error deliberately; item 8 resolves it with the reciprocal. **Timing:** 15–25 min · pairs or independent · circulate on 7 and 9. Item 8 is hard; the check on 200 is what makes it convincing.

1 15 **2** 20 **3** 63 **4** 300 **5** 150 **6** –25%, a 25% decrease

7 Because they were taken of different wholes **8** A 33⅓% decrease

1 **MILD** 30% of 50?

15

1. $0.3 \times 50 = 15$.

Watch for: Answered 30.

2 **MILD** 8% of 250?

20

1. $0.08 \times 250 = 20$.

Watch for: Answered 8, or 2 by misplacing the decimal.

3 **MEDIUM** Decrease 90 by 30%?

63

1. $100\% - 30\% = 70\%$, so $90 \times 0.7 = 63$.

2. Or 30% of 90 = 27, then $90 - 27 = 63$.

Watch for: Answered 27, the reduction rather than the result.

4 **MEDIUM** Increase 200 by 50%?

300

1. $200 \times 1.5 = 300$.

Watch for: Answered 250 by adding 50.

5 **MEDIUM** Decrease 300 by 50%?

150

1. $300 \times 0.5 = 150$.

Watch for: Answered 200, assuming it returns to the start.

6 **MEDIUM** Percent change from 200 to 150?

–25%, a 25% decrease

1. Change: $150 - 200 = -50$.

2. Relative to the ORIGINAL: $-50 \div 200 = -0.25 = -25\%$.

3. Always divide by the starting value, not the ending one.

Watch for: Divided by 150 to get –33%. Percent change is always measured against where you started.

7 **SPICY** Why did the two 50% changes not cancel?

Because they were taken of different wholes

1. The 50% increase was 50% of 200, which is 100.
2. The 50% decrease was 50% of 300, which is 150 — half as much again.
3. A bigger fall than rise leaves you below the start: $200 \rightarrow 300 \rightarrow 150$.
4. Percents only cancel when applied to the same whole. Applying the second to a larger amount makes it worth more.
5. The bigger the first change, the further the round trip misses.

Watch for: Said the arithmetic must be wrong. The arithmetic is right; the expectation is wrong.

8 **SPICY** What decrease undoes a 50% increase?

A $33\frac{1}{3}\%$ decrease

1. A 50% increase multiplies by 1.5, so undoing it means dividing by 1.5, i.e. multiplying by $\frac{2}{3}$.
2. Multiplying by $\frac{2}{3}$ is a decrease of $\frac{1}{3}$, which is $33\frac{1}{3}\%$.
3. Check on 200: $200 \times 1.5 = 300$, then $300 \times \frac{2}{3} = 200$. Back to the start.
4. So a 50% rise is undone by a $33\frac{1}{3}\%$ fall, NOT a 50% fall — the two percents are of different sizes because they are of different wholes.
5. General rule: to undo a multiplier, use its reciprocal, not its mirror image.

Watch for: Answered 50%, which is the whole misconception restated. The check on 200 is what settles it.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet, and deliberately inverts it: here UNEQUAL percents cancel exactly. A student who memorised 'round trips always lose' is caught by item 6. **Timing:** ~15 min at home · independent · review item 6 beside the exit-ticket item 4 as the next-day warm-up. The pair is the whole idea.

1 27

2 100

3 80

4 +25%

5 -20%

6 Back at 80, which is surprising because the percents differ

7 Because each is measured against a different starting value

1 **MILD** 60% of 45?

27

1. $0.6 \times 45 = 27$.

Watch for: Answered 60.

2 **MEDIUM** Increase 80 by 25%?

100

1. $80 \times 1.25 = 100$. Or 25% of 80 = 20, then $80 + 20 = 100$.

Watch for: Answered 20, the increase rather than the total.

3 **MEDIUM** Decrease 100 by 20%?

80

1. $100 \times 0.8 = 80$. Or 20% of 100 = 20, then $100 - 20 = 80$.

Watch for: Answered 20, the reduction rather than the result.

4 **MEDIUM** Percent change from 80 to 100?

+25%

1. Change: $100 - 80 = 20$. Divide by the start: $20 \div 80 = 0.25 = 25\%$.

Watch for: Divided by 100 to get 20%.

5 **MEDIUM** Percent change from 100 to 80?

-20%

1. Change: $80 - 100 = -20$. Divide by the start: $-20 \div 100 = -0.2 = -20\%$.

Watch for: Divided by 80 to get -25%.

6  **SPICY** Started at 80 — where did you finish?

Back at 80, which is surprising because the percents differ

1. $80 \rightarrow 100 \rightarrow 80$. Exactly back to the start.
2. Surprising because the two percents were NOT equal: up 25%, then down 20%.
3. On the exit ticket, equal percents (up 20%, down 20%) did NOT return to the start.
4. So unequal percents can cancel while equal ones do not — which is the opposite of what intuition suggests.
5. The reason is the whole: 25% of 80 is 20, and 20% of 100 is also 20. Same amount, different percents, because different wholes.

Watch for: Expected to finish below 80, generalising from the exit ticket that round trips always lose. What matters is the amounts, not the percents.

7  **SPICY** Same \$20 gap, different percents — why?

Because each is measured against a different starting value

1. Going up, the \$20 is compared with 80: $20 \div 80 = 25\%$.
2. Coming down, the same \$20 is compared with 100: $20 \div 100 = 20\%$.
3. The gap is identical; the BASE is not, and percent change always divides by where you started.
4. What that means for undoing: a p% increase is never undone by a p% decrease. It is undone by a smaller percent, because the decrease is taken of a larger amount.
5. Practically: to reverse an increase, work with the multiplier and use its reciprocal — 1.25 is undone by dividing by 1.25, which is a 20% cut, not a 25% one.

Watch for: Said one of the two must be wrong. Both are correct; they answer different questions because they have different bases.

Exit Ticket

4 problems

Use: End of the percent lesson, closing G7 Ratios. 5–8 min, independent. Item 4 is the misconception the lesson exists for.

Timing: 5–8 min · independent. An answer of \$100 on item 4 is near-universal before teaching and is the thing to reteach.

1 MILD 20% of 60?

12

1. $20\% = \frac{20}{100} = \frac{1}{5}$. $\frac{1}{5}$ of 60 = 12.
2. Or $0.2 \times 60 = 12$.

Watch for: Answered 20, copying the percent. A percent is an instruction, not the answer.

2 MEDIUM \$40 meal, 15% tip — total?

\$46

1. Tip: $15\% \text{ of } 40 = 0.15 \times 40 = \$6$.
2. Total: $\$40 + \$6 = \$46$.
3. Faster: multiply by 1.15 once $\rightarrow 40 \times 1.15 = \$46$.

Watch for: Answered \$6, giving the tip rather than the total. Read what the question asked for.

3 MEDIUM \$80 jacket, 25% off — sale price?

\$60

1. Saving: $25\% \text{ of } 80 = \$20$. Price: $\$80 - \$20 = \$60$.
2. Faster: $100\% - 25\% = 75\%$, so $80 \times 0.75 = \$60$.

Watch for: Answered \$20, the discount rather than the price.

4 SPICY \$100 up 20%, then down 20% — back to \$100?

No — \$96

1. Up 20%: $100 \times 1.2 = \$120$.
2. Down 20%: $120 \times 0.8 = \$96$.
3. Not \$100, because the two percents are taken of DIFFERENT wholes.
4. The 20% rise was 20% of \$100, worth \$20. The 20% fall was 20% of \$120, worth \$24 — a bigger amount.
5. A larger fall than rise leaves you below where you started. The net change is -4%.
6. This holds for any percent: up then down by the same percent always finishes below the start.

Watch for: Answered \$100, reasoning that the percents cancel. Percents only cancel when they are taken of the same whole, and here the second whole is bigger.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: every percent question hides the words 'of what?' — that is the whole lesson.
2	1:00	Hook — poll back-at-\$100 or not. Do NOT resolve. Nearly every hand says yes.
3	2:30	The single-multiplier method. It is faster and it makes slide 7 possible.
4	4:30	WE1 — resolve the hook. Let the \$96 sit for a moment before explaining.
5	6:00	WE2 — the reason. Compare the two dollar amounts, \$20 against \$24. The dollars persuade where the percents do not.
6	10:00	The reciprocal. Harder; check on 200 live so they see it land exactly back.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	8:00	CFU — all-student response. $200 \rightarrow 300 \rightarrow 150$. The loss is dramatic, which helps it stick. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:00	Launch the in-class practice sheet. Note the homework inverts this — unequal percents that DO cancel. Launch the practice sheet. Exit ticket in the last 5–8 minutes.