



GRADE 7 · RATIOS & PROPORTIONAL RELATIONSHIPS · ANSWER KEY

MULTI-STEP RATIO PROBLEMS

Today's goal: I can solve multi-step ratio problems, including by using proportional relationships.

How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Items 3–6 form a sequence: make the error, name it, then disprove it with the rate. Item 8 teaches that one check is not enough. **Timing:** 15–25 min · pairs or independent · circulate on 5, 6 and 9. Item 7 is the most transferable idea on the sheet.

- 1 \$8 2 \$72 3 30 minutes 4 \$12 5 \$21 6 They added 3 to both quantities; ratios scale by multiplying
7 24 minutes for 10 kg is 2.4 min/kg, but the machine takes 3 min/kg 8 They sum to \$33, and their ratio simplifies to 4 : 7

1 **MILD** 5 m \$40 — 1 m?

\$8

1. $\$40 \div 5 = \$8$ per metre.

Watch for: Answered \$35 by subtracting.

2 **MILD** 9 m?

\$72

1. $9 \times \$8 = \$72$.

Watch for: Added \$8 once to \$40, giving \$48.

3 **MEDIUM** 7 kg in 21 min — 10 kg?

30 minutes

- Per kg: $21 \div 7 = 3$ minutes.
- 10 kg: $10 \times 3 = 30$ minutes.
- Or scale: $10 \div 7 = \frac{10}{7}$, and $21 \times \frac{10}{7} = 30$.

Watch for: Answered 24 by adding 3 to both. See the next two questions.

4 **MEDIUM** Ratio 4 : 7, total \$33 — smaller share?

\$12

- Total parts: $4 + 7 = 11$.
- One part: $\$33 \div 11 = \$3$.
- Smaller share: $4 \times \$3 = \$12$.

Watch for: Answered \$4, treating the ratio number as the amount.

5 **MEDIUM** Larger share?

\$21

1. $7 \times \$3 = \$21$.

Watch for: Answered \$7.

6 **SPICY** The student's 24 — what did they do, and what is wrong?

They added 3 to both quantities; ratios scale by multiplying

1. They went $7 \rightarrow 10$ by adding 3, then $21 \rightarrow 24$ by adding 3.
2. Adding the same number to both keeps the DIFFERENCE the same, not the ratio.
3. The original difference is $21 - 7 = 14$, and $24 - 10 = 14$ — so they preserved exactly the wrong thing.
4. Ratios are about how many times bigger, not how much bigger.

Watch for: Said the student made an arithmetic slip. The arithmetic is correct; the operation is wrong, which is a different and more useful diagnosis.

7 **SPICY** Show 24 cannot be right, using the rate

24 minutes for 10 kg is 2.4 min/kg, but the machine takes 3 min/kg

1. The machine's rate is fixed: $21 \div 7 = 3$ minutes per kilogram.
2. If 10 kg took 24 minutes, the rate would be $24 \div 10 = 2.4$ minutes per kilogram.
3. That would mean the machine sped up simply because it was given more to do, which is not how a constant rate works.
4. The correct answer, 30 minutes, gives $30 \div 10 = 3$ minutes per kilogram — the rate is unchanged, as it must be.

Watch for: Restated that adding is wrong without showing the consequence. Computing the implied rate is what turns an assertion into a proof.

8 **EXTRA SPICY** Check your shares two ways

They sum to \$33, and their ratio simplifies to 4 : 7

1. Sum check: $\$12 + \$21 = \$33$, the full total, so nothing is missing or double counted.
2. Ratio check: $12 : 21$ divides by 3 to give $4 : 7$, the required ratio.
3. Both checks are needed. Two numbers summing to 33 could be any split; two numbers in ratio $4 : 7$ could be any size. Together they pin down one answer.
4. This is why parts-of-a-whole problems always deserve both checks.

Watch for: Did only the sum check. \$16 and \$17 also sum to \$33 but are not in the ratio $4 : 7$, so the sum alone proves little.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Item 6 catches students who work fluently in parts but forget to convert back to money.

Timing: ~15 min at home · independent · review item 7 as the next-day warm-up; it sets up the “Multi-step percent problems” lesson.

1 \$7

2 25 km

3 \$24

4 \$25

5 \$15

6 \$10

7 $25 : 15 = 5 : 3$; the total alone allows infinitely many splits

1

MILD

3 tickets \$21 — one ticket?

\$7

1. $\$21 \div 3 = \$7$.**Watch for:** Subtracted to get \$18.

2

MEDIUM

6 L for 15 km — 10 L?

25 km

1. Per litre: $15 \div 6 = 2.5$ km.2. 10 litres: $10 \times 2.5 = 25$ km.**Watch for:** Answered 19 by adding 4 to both.

3

MEDIUM

9 kg \$54 — 4 kg?

\$24

1. Per kg: $\$54 \div 9 = \$6$.2. 4 kg: $4 \times \$6 = \$24$.

3. Less than \$54, as it must be — 4 kg is less than 9 kg.

Watch for: Answered \$49 by subtracting 5. The sense check catches it: \$49 for less rice than \$54 bought is barely a saving.

4

MEDIUM

\$40 in ratio 5 : 3 — larger share?

\$25

1. Total parts: $5 + 3 = 8$. One part: $\$40 \div 8 = \$5$.2. Larger: $5 \times \$5 = \$25$.**Watch for:** Answered \$5, giving the ratio number rather than the money.

5

MEDIUM

Smaller share?

\$15

1. $3 \times \$5 = \$15$.**Watch for:** Answered \$3.

6

MEDIUM

How much more?

\$10

1. $\$25 - \$15 = \$10$.2. Or: the shares differ by $5 - 3 = 2$ parts, and $2 \times \$5 = \$10$.**Watch for:** Answered \$2, the difference in parts rather than in money.

7

SPICY

Show the ratio, and why the total alone is not enough

 $25 : 15 = 5 : 3$; the total alone allows infinitely many splits1. Divide both by 5: $25 : 15$ becomes $5 : 3$, as required.2. The sum check gives $\$25 + \$15 = \$40$, which is correct but weak.

3. \$20 and \$20 also total \$40. So does \$39 and \$1. The total alone permits infinitely many splits and singles out none of them.

4. Only the two conditions together — right total AND right ratio — identify one answer.

5. This is the same structure as the parts-of-a-whole check on the practice sheet, and it is worth carrying into percent problems next.

Watch for: Checked the ratio but did not explain why the total is insufficient. The explanation is the question.

Exit Ticket

4 problems

Use: End of the multi-step ratio lesson. 5–8 min, independent. Item 4 is the parts-of-a-whole structure that the “Multi-step percent problems” lesson reuses for percent. **Timing:** 5–8 min · independent. An answer of \$14 on item 1 is the additive error and predicts trouble on item 4.

1 \$20

2 10 kg

3 6 : 15 — the ratio is unchanged

4 12 litres blue, 8 litres yellow

1

MILD

3 books \$12 — what do 5 cost?

\$20

1. Unit rate: $\$12 \div 3 = \$4$ per book.
2. 5 books: $5 \times \$4 = \$20$.
3. Check: $\$20 \div 5 = \$4$, the same price per book.

Watch for: Answered \$14 by adding 2 books and \$2. That makes each book cost \$2.80 instead of \$4 — the price changed, so the ratio is broken.

2

MEDIUM

4 kg fills 6 boxes — 15 boxes?

10 kg

1. Per box: $4 \div 6 = \frac{2}{3}$ kg.
2. 15 boxes: $15 \times \frac{2}{3} = \frac{30}{3} = 10$ kg.
3. Or scale: $15 \div 6 = 2\frac{1}{2}$, so $4 \times 2\frac{1}{2} = 10$ kg.

Watch for: Answered 13 by adding 9 to both. Adding the same amount to each side changes the ratio.

3

MEDIUM

Scale 2 : 5 by 3

6 : 15 — the ratio is unchanged

1. Multiply both parts: $2 \times 3 = 6$ and $5 \times 3 = 15$, giving 6 : 15.
2. Unchanged because $6 \div 15 = \frac{2}{5}$, the same as $2 \div 5$.
3. Multiplying BOTH parts by the same number is exactly what keeps a ratio equivalent.

Watch for: Multiplied only one part, giving 6 : 5 or 2 : 15. Both parts scale, or the mixture changes.

4

SPICY

Blue : yellow = 3 : 2 in 20 litres

12 litres blue, 8 litres yellow

1. Total parts: $3 + 2 = 5$.
2. Each part is $20 \div 5 = 4$ litres.
3. Blue: $3 \times 4 = 12$ litres. Yellow: $2 \times 4 = 8$ litres.
4. Check: $12 + 8 = 20$, the whole mixture.
5. The ratio 12 : 8 simplifies to 3 : 2, confirming the mix is right.

Watch for: Split 20 into 3 and 2 directly, or used thirds and halves. The 3 and 2 are PARTS, not litres — you must find what one part is worth first.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: these are ratio problems where the answer is not one step away.
2	1:00	Hook — poll \$14 vs \$20. Do NOT resolve. \$14 is chosen by more students than you expect.
3	2:30	Unit rate route first; it is the one that generalises.
4	4:30	WE1 — resolve the hook by computing the implied price, \$2.80. Showing the CONSEQUENCE beats asserting the rule.
5	6:30	WE2 — scale factor as the second route. Some students prefer it; both are correct.
6	10:30	Parts of a whole. Insist on both checks. This structure returns in the “Multi-step percent problems” lesson with percent.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	8:30	CFU — all-student response. Answer 10 kg. Watch for 13, the additive error. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:00	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.