

GRADE 7 · THE NUMBER SYSTEM · ANSWER KEY

MULTIPLYING & DIVIDING RATIONAL NUMBERS

Today's goal: I can multiply rational numbers, divide rational numbers and multiply and divide rational numbers.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra** big brain.

Practice

9 problems

Use: During the lesson after the deck. Items 1–4 are the core contrast; item 9 generalises to any number of factors and is worth the time. **Timing:** 15–25 min · pairs or independent · circulate on 4 and 9. Require the sign written down before the size on every item.

1 -21 2 21 3 -10 4 -4 5 4 6 1^5 7 $-3/2$

8 Because multiplying and adding are different operations with different sign rules

9 -24 — an ODD number of negatives gives a negative product

1 **MILD** $-7 \times 3?$

-21

1. Different signs → negative. $7 \times 3 = 21 \rightarrow -21$.

Watch for: Answered 21.

2 **MILD** $-7 \times (-3)?$

21

1. Same signs → positive. 21.

Watch for: Answered -21.

3 **MILD** $-7 + (-3)?$

-10

1. Both moves go left → -10.

Watch for: Answered 10, applying the multiplication rule.

4 **MEDIUM** $-36 \div 9?$

-4

1. Different signs → negative → -4.

Watch for: Answered 4.

5 **MEDIUM** $-36 \div (-9)?$

4

1. Same signs → positive → 4.

Watch for: Answered -4.

6 **MEDIUM** $-1/2 \times (-2^5)?$

1^5

1. Same signs → positive.

2. Sizes: $1/2 \times 2^5 = 2^4 = 1^5$.

3. Answer 1^5 , positive.

Watch for: Kept the answer negative out of habit because both factors looked negative.

7 **MEDIUM** $-\frac{3}{4} \div \frac{1}{2}$?

$-\frac{3}{2}$

1. Different signs $\rightarrow$ negative.
2. Keep $\cdot$ Change $\cdot$ Flip: $\frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$.
3. Answer $-\frac{3}{2}$, or $-1\frac{1}{2}$.
4. Bigger than $\frac{3}{4}$ in size, because dividing by $\frac{1}{2}$ doubles.

Watch for: Answered $-\frac{3}{8}$ by multiplying instead of dividing.

8 **SPICY** Same numbers, different signs — why?

Because multiplying and adding are different operations with different sign rules

1. $-7 \times (-3) = 21$, positive. $-7 + (-3) = -10$, negative.
2. Multiplying by a negative reverses direction, and reversing twice returns you to positive.
3. Adding a negative moves left, and moving left twice takes you further left.
4. So the same pair of numbers gives opposite signs depending on the operation.
5. The rule 'two negatives make a positive' belongs to multiplication and division. It says nothing about addition.

Watch for: Said one of the two must be wrong. Both are right; the operation decides.

9 **SPICY** $(-2) \times (-3) \times (-4)$, and the rule

-24 — an ODD number of negatives gives a negative product

1. Work in pairs: $(-2) \times (-3) = 6$, positive.
2. Then $6 \times (-4) = -24$, negative.
3. Three negatives here, and the product is negative.
4. Rule: count the negatives. An EVEN count gives a positive product, because they pair off and each pair cancels. An ODD count leaves one unpaired, so the product is negative.
5. This works for any number of factors and saves computing sign changes one at a time.

Watch for: Answered 24, assuming any collection of negatives gives a positive. Two cancel; a third flips it back.

Homework

8 problems

Use: Parallels the practice sheet. Items 1–3 repeat the multiply/add contrast with new numbers, since one exposure rarely dislodges the chant. **Timing:** ~15 min at home · independent · review items 2 and 3 side by side as the next-day warm-up.

1 -40

2 40

3 -13

4 -5

5 5

6 $-\frac{1}{3}$

7 -2

8 1 — four negatives is an even count, so the product is positive

1 **MILD** -8×5 ?

-40

1. Different signs $\rightarrow$ negative $\rightarrow -40$.

Watch for: Answered 40.

2 **MILD** $-8 \times (-5)$?

40

1. Same signs $\rightarrow$ positive $\rightarrow 40$.

Watch for: Answered -40.

3 **MILD** $-8 + (-5)$?

-13

1. Both moves go left $\rightarrow -13$.
2. Compare question 2: same numbers, opposite sign, because this is addition.

Watch for: Answered 13, applying the multiplication rule to an addition.

4 **MEDIUM** $-45 \div 9$?

-5

1. Different signs $\rightarrow$ negative $\rightarrow -5$.

Watch for: Answered 5.

5 **MEDIUM** $-45 \div (-9)?$

5

1. Same signs $\rightarrow$ positive $\rightarrow 5$.

Watch for: Answered -5 .

6 **MEDIUM** $-2^5 \times \frac{5}{6}?$

$-\frac{1}{3}$

1. Different signs $\rightarrow$ negative.
2. Sizes: $2^5 \times \frac{5}{6} = \frac{10}{30} = \frac{1}{3}$. The 5s cancel.
3. Answer $-\frac{1}{3}$.

Watch for: Did not cancel and left $-\frac{10}{30}$ unsimplified.

7 **MEDIUM** $\frac{1}{2} \div (-\frac{1}{4})?$

-2

1. Different signs $\rightarrow$ negative.
2. Keep $\cdot$ Change $\cdot$ Flip: $\frac{1}{2} \times \frac{4}{1} = 2$.
3. Answer -2 . Bigger in size than $\frac{1}{2}$, because dividing by a quarter quadruples.

Watch for: Answered $-\frac{1}{8}$ by multiplying instead of dividing.

8 **SPICY** $(-1)^4$; and the counting rule

1 – four negatives is an even count, so the product is positive

1. Pair them: $(-1) \times (-1) = 1$, and $(-1) \times (-1) = 1$.
2. Then $1 \times 1 = 1$.
3. Four negatives is an EVEN count, so they pair off completely with none left over, and the product is positive.
4. Had there been five, one would be unpaired and the product would be -1 .
5. The size is 1 throughout because each factor has size 1 – only the sign was ever in question.

Watch for: Answered -1 by assuming more negatives means more negative. What matters is odd or even, not how many.

Exit Ticket

4 problems

Use: End of the multiply/divide lesson. 5–8 min, independent. Item 3 is the whole point: the rule and its boundary taught together. **Timing:** 5–8 min · independent. Answering 24 for both parts of item 3 is the chant error and is the thing to reteach.

1 MILD $-6 \times 4?$ **-24**

1. Different signs, so the product is negative.
2. $6 \times 4 = 24$, so the answer is -24.

Watch for: Answered 24 by ignoring the sign. Decide the sign first, then the size.

2 MEDIUM $-20 \div 5$ and $-20 \div (-5)$ **-4 and 4**

1. $-20 \div 5$: different signs $\rightarrow$ negative $\rightarrow -4$.
2. $-20 \div (-5)$: same signs $\rightarrow$ positive $\rightarrow 4$.
3. Division follows exactly the same sign rule as multiplication, because dividing is multiplying by the reciprocal.

Watch for: Gave the same sign for both. Changing the sign of the divisor flips the sign of the answer.

3 SPICY $-6 \times (-4)$ and $-6 + (-4)$ **24 and -10 — different operations, different rules**

1. Multiplying: same signs give a positive, so $-6 \times -4 = 24$.
2. Adding: both moves go LEFT on the number line, so $-6 + (-4) = -10$.
3. The signs differ because the operations mean different things. Multiplying by a negative REVERSES direction; adding a negative just keeps going left.
4. “Two negatives make a positive” is a statement about multiplication and division ONLY.
5. A quick sanity test: owing \$6 four times over is a bigger debt, not a credit.

Watch for: Answered 24 for both, applying the chant to the addition. The debt check catches it instantly.

4 SPICY $-\frac{2}{3} \times \frac{3}{4}$, three ways **$-\frac{1}{2}$, which can be written $-\frac{1}{2}$, $\frac{1}{-2}$ or $-(\frac{1}{2})$**

1. Different signs $\rightarrow$ negative. Sizes: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
2. So the answer is $-\frac{1}{2}$.
3. Three equivalent placements: $-\frac{1}{2}$ (sign on the whole fraction), $\frac{-1}{2}$ (sign on the numerator), and $\frac{1}{-2}$ (sign on the denominator).
4. All three name the same number, because a negative divided by a positive and a positive divided by a negative both give a negative.
5. Convention: write it out front as $-\frac{1}{2}$. The others are correct but harder to read.

Watch for: Wrote $-\frac{1}{2}$, applying the sign twice. One negative, placed once — doubling it makes the number positive.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: the sign rule is easy. Knowing which operation it applies to is the lesson.
2	1:00	Hook — poll same or different. Do NOT resolve. Many students will say the same, which is exactly the misconception.
3	3:00	State the rule for multiplying and dividing. Note that division inherits it via the reciprocal, so it is one rule, not two.
4	5:00	WE1 — resolve the hook. Put both answers side by side and leave them up for the rest of the lesson.
5	7:00	WE2 — the reason. The debt sentence does more work than the direction language for most students; use whichever lands.
6	11:30	The counting rule. Derive it from pairing rather than announcing it, then test with four factors.
8	9:30	CFU — all-student response. -4 and 4. Watch for the same sign given twice. Whiteboards: everyone tries Mild, then climbs. These are #1, #4, #9 on the practice sheet.
9		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
10	13:30	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.