



Probabilities of compound events

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the compound events lesson, closing G7 Statistics and the whole G6–G7 inventory. 5–8 min, independent. Items 3 and 4 are the same bag and the same question, differing only in replacement.

Timing: 5–8 min · independent. $\frac{3}{5} \times \frac{2}{5}$ on item 4 is the diagnostic — the numerator changed and the denominator did not.

1. P(both heads)? **FLUENCY**

Answer: $\frac{1}{4}$

1. $P(\text{head}) = \frac{1}{2}$ on each coin, and one coin does not affect the other.
2. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
3. Check by listing: HH, HT, TH, TT — four equally likely outcomes, one of which is HH.

If wrong: Answered $\frac{1}{2}$ by adding the probabilities, or $\frac{1}{3}$ by listing only HH, HT and TT.

2. P(6 and heads)? **APPLICATION**

Answer: $\frac{1}{12}$

1. $P(6) = \frac{1}{6}$ and $P(\text{head}) = \frac{1}{2}$. The die does not affect the coin.
2. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$.
3. Check: $6 \times 2 = 12$ equally likely outcomes, one of which is (6, H).

If wrong: Added to get $\frac{2}{3}$, or used $\frac{1}{8}$ by counting 6 faces plus 2 sides as 8 outcomes. The outcomes multiply, not add.

3. Two reds, WITH replacement APPLICATION

Answer: $\frac{9}{2^5}$

1. First draw: 3 red out of 5, so $\frac{3}{5}$.
2. The marble goes back, so the bag is unchanged: second draw is also $\frac{3}{5}$.
3. $\frac{3}{5} \times \frac{3}{5} = \frac{9}{2^5}$.

If wrong: Changed the second fraction anyway. With replacement the bag is restored, so nothing changes.

4. Two reds, WITHOUT replacement REASONING

Answer: $\frac{3}{10}$

1. First draw: $\frac{3}{5}$, same as before.
2. Now one red marble is gone. Two reds remain, and only 4 marbles in total.
3. Second draw: $\frac{2}{4}$.
4. $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.
5. Why it differs from question 3: $\frac{3}{10} = 0.30$ against $\frac{9}{2^5} = 0.36$, so it is LESS likely.
6. That makes sense — having removed a red, there are proportionally fewer reds left, so the second red is harder to get.
7. The commonest error is changing only the top number, writing $\frac{3}{5} \times \frac{2}{5}$. Both change: one marble left the bag, so the total drops too.

If wrong: Wrote $\frac{3}{5} \times \frac{2}{5}$, reducing the reds but not the total. The bag has 4 marbles for the second draw, not 5.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{4}$
2	application	$\frac{1}{12}$
3	application	$\frac{9}{2^5}$
4	reasoning	$\frac{3}{10}$

Practice Worksheet — answer key

Use: During the lesson after the deck. Items 7–9 are the payoff: two different-looking calculations giving the same answer, for a reason students can state.

Timing: 15–25 min · pairs or independent · circulate on 6 and 9. Real marbles or counters in a bag settle the denominator question faster than any explanation.

1. P(both tails)? **FLUENCY**

Answer: $\frac{1}{4}$

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

If wrong: Added to get 1.

2. P(at least one head)? **REASONING**

Answer: $\frac{3}{4}$

1. 'At least one head' is everything except both tails.
2. $P(\text{both tails}) = \frac{1}{4}$, so $P(\text{at least one head}) = 1 - \frac{1}{4} = \frac{3}{4}$.
3. Check by listing: HH, HT, TH, TT — three of the four contain a head.
4. Using the complement is much faster than adding up the three separate cases.

If wrong: Answered $\frac{1}{2}$ by reasoning that each coin gives a head half the time. That is the chance for ONE coin, not for at least one of two.

3. P(even and tails)? **APPLICATION**

Answer: $\frac{1}{4}$

1. $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$, since 2, 4 and 6 are even.
2. $P(\text{tails}) = \frac{1}{2}$. The die does not affect the coin.
3. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

If wrong: Used $\frac{1}{6}$ for the die, treating 'even' as a single outcome. Three faces are even.

4. Two blues WITH replacement **APPLICATION**

Answer: $\frac{4}{2^5}$

1. $\frac{2}{2^5}$ each time, since the marble is returned.
2. $\frac{2}{2^5} \times \frac{2}{2^5} = \frac{4}{2^5}$.

If wrong: Changed the second fraction despite the replacement.

5. Two blues WITHOUT replacement APPLICATION

Answer: $\frac{1}{10}$

1. First: $\frac{2}{5}$.
2. One blue is now gone: 1 blue left out of 4 marbles.
3. Second: $\frac{1}{4}$.
4. $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$.

If wrong: Wrote $\frac{2}{5} \times \frac{1}{5}$, keeping the denominator at 5.

6. Which is smaller, and why? REASONING

Answer: **Without replacement, $\frac{1}{10}$ against $\frac{4}{25}$**

1. $\frac{1}{10} = 0.10$ and $\frac{4}{25} = 0.16$, so without replacement is smaller.
2. It makes sense because there are only 2 blue marbles. Removing one leaves just a single blue in a bag of 4.
3. With replacement the bag is restored each time, so the second blue is as likely as the first.
4. In general, drawing the same colour twice is always LESS likely without replacement, because you have used up one of the ones you needed.
5. The effect is bigger here than with red, because blue is scarcer to begin with.

If wrong: Said they should be the same because the bag has the same marbles. The bag is the same only if you put the marble back.

7. P(red then blue), no replacement APPLICATION

Answer: $\frac{3}{10}$

1. First red: $\frac{3}{5}$.
2. A red is gone, so 4 marbles remain, of which 2 are blue.
3. Second blue: $\frac{2}{4}$.
4. $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.

If wrong: Kept the denominator at 5.

8. P(blue then red), no replacement APPLICATION

Answer: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

1. First blue: $\frac{2}{5}$.
2. A blue is gone, so 4 marbles remain, of which 3 are red.
3. Second red: $\frac{3}{4}$.
4. $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

If wrong: Kept the denominator at 5, or assumed it must differ from question 7.

9. Why are they equal? REASONING

Answer: **Because both multiply out to the same numbers, just in a different order**

1. Question 7: $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$.
2. Question 8: $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$.
3. The numerators are 2×3 in one and 3×2 in the other — the same product.
4. The denominators are 5×4 in both, because the bag shrinks from 5 to 4 whichever colour left it.
5. So the two orders give identical probabilities. Multiplication does not care about order.
6. This makes sense in the situation too: you are asking for one red and one blue in a particular order, and neither order is favoured.

If wrong: Said it must be a coincidence of these numbers. It happens whenever you draw one of each — the same two counts appear on top, and the total drops by one either way.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{4}$
2	reasoning	$\frac{3}{4}$
3	application	$\frac{1}{4}$
4	application	$\frac{4}{25}$
5	application	$\frac{1}{10}$
6	reasoning	without replacement
7	application	$\frac{3}{10}$
8	application	$\frac{3}{10}$
9	reasoning	same product

Homework — answer key

Use: Closes G7 Statistics and the full G6–G7 inventory. Item 7 rewards checking whether an event is possible before computing; item 8 states the question the whole skill turns on.

Timing: ~15 min at home · independent · review items 7 and 8 as the next-day warm-up.

1. P(both 6)? FLUENCY

Answer: $\frac{1}{36}$

1. $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. The dice do not affect each other.

If wrong: Added to get $\frac{2}{6}$.

2. P(neither shows 6)? APPLICATION

Answer: $\frac{25}{36}$

1. P(not 6) on one die = $\frac{5}{6}$.

2. $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$.

If wrong: Answered $\frac{5}{36}$ by computing $1 - \frac{1}{36}$. That is the probability of NOT BOTH being sixes, which includes rolls with exactly one six. ‘Neither shows a 6’ is a stricter event and has the smaller probability $\frac{25}{36}$.

3. Spinner green twice APPLICATION

Answer: $\frac{1}{16}$

1. $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

If wrong: Added to get $\frac{1}{2}$.

4. Three reds WITH replacement APPLICATION

Answer: $\frac{27}{125}$

1. $\frac{3}{5}$ three times: $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$.

2. The bag is restored each time, so the fraction never changes.

If wrong: Multiplied only twice, forgetting the third draw.

5. Three reds **WITHOUT** replacement REASONING

Answer: $\frac{1}{10}$

1. First: $\frac{3}{5}$. Second: 2 reds left of 4 marbles, so $\frac{2}{4}$. Third: 1 red left of 3 marbles, so $\frac{1}{3}$.
2. $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{6}{60} = \frac{1}{10}$.
3. Both numbers shrink at every step, because a marble leaves the bag each time.
4. This draws every red in the bag, which is why the last fraction has a 1 on top.

If wrong: Kept the denominator at 5 throughout, or reduced only the numerators.

6. Which is smaller, and by how much? REASONING

Answer: **Without replacement, $\frac{1}{10}$ against $\frac{27}{125}$**

1. $\frac{1}{10} = 0.10$ and $\frac{27}{125} = 0.216$.
2. So without replacement is smaller — roughly half as likely.
3. The gap is wider than it was for two draws, where the figures were 0.30 and 0.36.
4. That is because each draw without replacement removes another red, and with only 3 reds in the bag the shortage compounds quickly.
5. The more draws you make, the more the two situations diverge.

If wrong: Said they are about the same. One is more than twice the other, and the reason — compounding scarcity — is the point.

7. P(all four red), no replacement REASONING

Answer: **0 — there are only 3 red marbles**

1. The bag contains just 3 red marbles.
2. Drawing 4 reds without replacement would require a 4th red that does not exist.
3. So the event is impossible and its probability is 0.
4. You can see this without calculating: after three draws every red is gone, and the fourth marble must be blue.
5. If you did calculate, the fourth fraction would be $\frac{0}{2}$, and anything multiplied by 0 is 0 — so the arithmetic agrees.

If wrong: Started multiplying and produced a small non-zero answer. Checking whether the event is even possible comes before the arithmetic.

8. The one question to ask first REASONING

Answer: **Does the first event change the second?**

1. If NO — coins, dice, or drawing with replacement — the probabilities are unchanged and you multiply them as they are.
2. If YES — drawing without replacement — the second probability must be recalculated for the new situation, with both the numerator and the denominator adjusted.
3. Asking this before any arithmetic prevents the commonest error in the topic, which is reducing the top number and forgetting the bottom.
4. A useful physical check: picture the bag before the second draw. How many marbles are in it, and how many are the colour you want?
5. Those two counts ARE the second fraction.

If wrong: Answered 'do I multiply or add?'. That matters too, but the replacement question is the one that decides what you multiply.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{36}$
2	application	$\frac{25}{36}$
3	application	$\frac{1}{16}$
4	application	$\frac{27}{125}$
5	reasoning	$\frac{1}{10}$
6	reasoning	without, about half
7	reasoning	0 — impossible
8	reasoning	does the first change the second?

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Closes G7 Statistics and the whole G6–G7 inventory. An actual bag of counters makes slide 5 undeniable.

Slide	~Time	Note
1/8	0:00	Frame it: compound means two things at once, and the probabilities multiply.
2/8	1:00	Hook — poll whether replacement matters. Do NOT resolve. Many say it makes no difference.
3/8	2:30	Establish multiplication with independent events first, where nothing changes.
4/8	4:30	WITH replacement. Show the bag being restored; the second fraction is visibly identical.
5/8	6:00	

		WE — WITHOUT replacement. Physically remove a counter and have students COUNT what remains. Both numbers change.
6/8	8:30	CFU — all-student response. $\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$. Watch for a denominator of 5.
7/8	10:30	The single question that decides everything. The recount-the-bag instruction is what students should carry away.
8/8	13:00	Launch the in-class practice sheet. This closes the G6–G7 Math inventory.