



Probability as a number from 0 to 1

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the probability scale lesson. 5–8 min, independent. Item 4 establishes the two ends of the scale, which every later probability skill depends on.

Timing: 5–8 min · independent. A whole number answer on item 1 means ‘probability as a count’ and needs correcting before SP-06.

1. P(red)? **FLUENCY**

Answer: $\frac{3}{10}$

1. There are 3 red marbles out of 10 in total.
2. $P(\text{red}) = \frac{3}{10}$, which is already in lowest terms.

If wrong: Answered 3. A probability is a fraction of the whole, so it needs the total underneath.

2. P(blue), and which is most likely? **APPLICATION**

Answer: $\frac{1}{2}$; **blue is most likely**

1. 5 blue out of 10, so $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$.
2. Compare: $P(\text{red}) = \frac{3}{10}$, $P(\text{green}) = \frac{2}{10} = \frac{1}{5}$.
3. $\frac{1}{2}$ is the largest of the three, so blue is most likely.
4. You know from the numbers because a bigger probability means a more likely outcome — that is what the scale measures.

If wrong: Said blue because there are more of them, without converting to a probability. True here, but the comparison must be between probabilities when totals differ.

3. P(not red), and how it fits REASONING

Answer: $\frac{7}{10}$

1. Not red means blue or green: $5 + 2 = 7$ marbles out of 10.
2. $P(\text{not red}) = \frac{7}{10}$.
3. It fits because $\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1$.
4. Either the marble is red or it is not — one of those must happen, so the two probabilities add to 1.
5. This gives a shortcut: $P(\text{not } A) = 1 - P(A)$, so you can find it without recounting.

If wrong: Answered $\frac{7}{3}$ by comparing red with non-red. A probability's denominator is always the TOTAL, not the other group.

4. P(yellow) and P(some colour) REASONING

Answer: **0 and 1**

1. There are no yellow marbles, so $P(\text{yellow}) = \frac{0}{10} = 0$.
2. Every marble has some colour, so $P(\text{some colour}) = \frac{10}{10} = 1$.
3. On the scale, 0 means IMPOSSIBLE — it cannot happen at all.
4. 1 means CERTAIN — it must happen.
5. These are the two ends of the scale, and every other probability lies between them. No probability is ever negative or greater than 1.
6. Note that 0 is a genuine probability, not the absence of one.

If wrong: Said $P(\text{yellow})$ has no answer because there are no yellow marbles. Zero IS the answer, and it means impossible.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{3}{10}$
2	application	$\frac{1}{2}$; blue
3	reasoning	$\frac{7}{10}$
4	reasoning	0 and 1

Practice Worksheet — answer key

Use: During the lesson after the deck. Items 8 and 9 are the heart of the standard: the number of outcomes and their likelihoods are independent questions.

Timing: 15–25 min · pairs or independent · circulate on 8 and 9. Collect item 9 examples on the board; the range students produce is itself the lesson.

1. P(green)? FLUENCY

Answer: $\frac{2}{5}$

1. 2 green out of 10: $\frac{2}{10} = \frac{2}{5}$.

If wrong: Answered 2.

2. P(not green)? FLUENCY

Answer: $\frac{3}{5}$

1. 8 marbles are not green: $\frac{8}{10} = \frac{4}{5}$.

2. Or $1 - \frac{2}{5} = \frac{3}{5}$.

If wrong: Answered $\frac{2}{5}$ by comparing green with non-green.

3. Sum, and why must it be that? REASONING

Answer: 1

1. $\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$.

2. It must be 1 because the marble is either green or it is not — there is no third option.

3. One of the two is certain to happen, and certainty is 1.

4. This is the complement rule: $P(A) + P(\text{not } A) = 1$ always.

If wrong: Said it is a coincidence of these numbers. It holds for every event and its complement.

4. P(red or green)? APPLICATION

Answer: $\frac{1}{2}$

1. Red or green: $3 + 2 = 5$ marbles out of 10.

2. $P = \frac{5}{10} = \frac{1}{2}$.

If wrong: Added the probabilities' denominators as well, giving $\frac{5}{20}$. Only the numerators combine when the events cannot both happen.

5. Compare with P(blue) REASONING

Answer: **Both are $\frac{1}{2}$ — because red-or-green is exactly ‘not blue’**

1. $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$, and $P(\text{red or green}) = \frac{1}{2}$.
2. They are equal because the bag has only three colours, so ‘red or green’ and ‘not blue’ describe exactly the same set of marbles.
3. By the complement rule, $P(\text{not blue}) = 1 - \frac{1}{2} = \frac{1}{2}$.
4. So this is not a coincidence — with 5 blue out of 10, blue and not-blue must split evenly.

If wrong: Said it is a coincidence. Once you notice red-or-green IS not-blue, the equality is forced.

6. Red or green — which is less likely? APPLICATION

Answer: **Green**

1. $P(\text{red}) = \frac{3}{10}$ and $P(\text{green}) = \frac{2}{10}$.
2. $\frac{2}{10} < \frac{3}{10}$, so green is less likely.
3. Smaller probability means less likely — that is what the scale means.

If wrong: Compared 2 and 3 as counts. Valid here because the totals match, but comparing probabilities is the habit that generalises.

7. An event with probability 0 and one with probability 1 APPLICATION

Answer: **e.g. drawing yellow (0); drawing red, blue or green (1)**

1. Probability 0: drawing a yellow marble, or a black one — there are none, so it is impossible.
2. Probability 1: drawing a marble that is red, blue or green, since every marble is one of those.
3. Also acceptable for 1: drawing a marble at all.
4. 0 and 1 are the two ends of the scale, and every real probability sits between them.

If wrong: Gave ‘drawing green’ for probability 0 because green is rarest. Rare is not impossible — green has probability $\frac{1}{5}$.

8. 'Either red or not, so $P(\text{red}) = \frac{1}{2}$ ' REASONING

Answer: **Two outcomes does not make them equally likely**

1. It is true that the marble is either red or not red — those are the only two possibilities.
2. But they are not equally likely: 3 marbles are red and 7 are not.
3. $P(\text{red}) = \frac{3}{10}$ and $P(\text{not red}) = \frac{7}{10}$. They add to 1, as they must, but they are not equal.
4. Equal likelihood has to come from the SITUATION — here, from the counts — not from the number of possible outcomes.
5. Counting outcomes tells you how many there are. It says nothing about how likely each one is.

If wrong: Agreed, because splitting into two cases feels symmetric. Two cases can have any pair of probabilities that add to 1.

9. Two outcomes, clearly not equally likely REASONING

Answer: **e.g. whether it snows in London tomorrow in July**

1. Two outcomes: it snows, or it does not. Only two possibilities, so the '50-50' reasoning would give $\frac{1}{2}$ each.
2. But snow in London in July is extremely unlikely — the probability is close to 0, and no-snow is close to 1.
3. Other good examples: whether a randomly chosen person was born on a Tuesday ($\frac{1}{7}$ against $\frac{6}{7}$), or whether a thrown die shows a six ($\frac{1}{6}$ against $\frac{5}{6}$).
4. In each case there are exactly two outcomes and they are nowhere near even.
5. The lesson: how many outcomes there are and how likely each one is are separate questions.

If wrong: Offered a coin flip. That has two outcomes which ARE equally likely, so it illustrates the student's claim rather than refuting it.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{4^5}$
2	fluency	$\frac{1}{4^5}$
3	reasoning	1, by the complement rule
4	application	$\frac{1}{2}$
5	reasoning	equal — it is 'not blue'
6	application	green

7	application	yellow (0); any colour (1)
8	reasoning	two outcomes $\neq$ even
9	reasoning	e.g. snow in July

Homework — answer key

Use: Parallels the practice sheet. Item 8 asks students to DERIVE the 0-to-1 range from the definition, which makes it reconstructable rather than memorised.

Timing: ~15 min at home · independent · review item 7 as the next-day warm-up; the instant out-of-range test is worth making automatic.

1. P(blue)? FLUENCY

Answer: $\frac{1}{2}$

1. 5 out of 10 = $\frac{1}{2}$.

If wrong: Answered 5.

2. P(not blue)? FLUENCY

Answer: $\frac{1}{2}$

1. 5 marbles are not blue: $\frac{5}{10} = \frac{1}{2}$. Or $1 - \frac{1}{2}$.

If wrong: Answered $\frac{5}{5} = 1$.

3. Why is blue exactly even? REASONING

Answer: **Because exactly half the marbles are blue**

1. There are 5 blue and 5 not-blue, out of 10.
2. So the two groups are the same size, and their probabilities are both $\frac{1}{2}$.
3. This is what ‘even chance’ actually means — not that there are two outcomes, but that the two outcomes have equal probability.
4. Blue is even; red is not, because $3 \neq 7$.

If wrong: Said it is even because there are two options, blue and not blue. That is true of red too, and red is not even.

4. P(red or blue)? APPLICATION

Answer: $\frac{8}{10}$

1. $3 + 5 = 8$ out of $10 = \frac{8}{10}$.

If wrong: Added the fractions' denominators.

5. Why does it equal P(not green)? REASONING

Answer: **Because red-or-blue and not-green are the same marbles**

1. The bag has only three colours, so a marble that is not green must be red or blue.
2. $P(\text{not green}) = 1 - \frac{2}{10} = \frac{8}{10}$, matching question 4.
3. It must be equal because the two descriptions pick out exactly the same 8 marbles.
4. Different words, same event, so necessarily the same probability.

If wrong: Verified the arithmetic without explaining why it had to work. The reason is that the two events are identical.

6. Order on the 0 to 1 scale APPLICATION

Answer: **$P(\text{green}) = \frac{2}{10}$, $P(\text{red}) = \frac{3}{10}$, $P(\text{blue}) = \frac{5}{10}$, $P(\text{not green}) = \frac{8}{10}$**

1. As tenths: $\frac{2}{10}$, $\frac{3}{10}$, $\frac{5}{10}$, $\frac{8}{10}$.
2. So the order from least to most likely is green, red, blue, not-green.
3. Converting to a common denominator is what makes them directly comparable.

If wrong: Ordered by the numerators alone, without a common denominator. $\frac{2}{10}$ and $\frac{3}{10}$ cannot be compared by their tops.

7. P(red) = 3 and P(blue) = 5 REASONING

Answer: **Probabilities must lie between 0 and 1, and these are counts**

1. Both numbers exceed 1, so neither can be a probability.
2. What the student wrote are the COUNTS of marbles, not the fractions of the whole.
3. Correct: $P(\text{red}) = \frac{3}{10}$ and $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$.
4. How you can tell instantly: any probability above 1 or below 0 is impossible, no matter what the situation is. You do not need to know anything about the bag.
5. A second tell: their two values sum to 8, and probabilities of different outcomes in one situation can never sum above 1.

If wrong: Corrected the numbers without naming the general rule. The instant test — is it between 0 and 1 — is the transferable part.

8. Why can no probability exceed 1? REASONING

Answer: **Because it is favourable outcomes divided by total outcomes**

1. $P = \text{favourable} \div \text{total}$.
2. The favourable outcomes are a subset of all the outcomes, so the numerator can never be larger than the denominator.
3. The largest it can be is all of them, giving $\text{total} \div \text{total} = 1$, which is certainty.
4. The smallest is none of them, giving $0 \div \text{total} = 0$, which is impossibility.
5. So every probability lies between 0 and 1 inclusive, by the definition itself rather than by convention.
6. There is nothing more likely than certain, which is what a probability above 1 would have to mean.

If wrong: Said it is a rule you have to remember. It follows from the definition, which makes it something you can reconstruct rather than recall.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{2}$
2	fluency	$\frac{1}{2}$
3	reasoning	5 blue, 5 not blue
4	application	$\frac{4}{5}$
5	reasoning	same event
6	application	green, red, blue, not-green
7	reasoning	above 1 is impossible
8	reasoning	follows from the definition

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Slide 4's distinction — number of outcomes versus their likelihood — is what the rest of the probability strand rests on.

Slide	~Time	Note
1/8	0:00	Frame it: probability puts likelihood on a fixed scale so different situations can be compared.
2/8	1:00	Hook — poll yes or no. Do NOT resolve. The two-outcomes-so-even argument is very persuasive to students.
3/8	2:30	Establish the scale and the formula. Draw the 0-to-1 line and keep it visible.
4/8	4:30	

		WE1 — resolve the hook with the actual counts. Say the key line: counting outcomes is not measuring likelihood.
5/8	7:00	WE2 — the sum to 1 and the complement rule. Derive it from ‘something must happen’.
6/8	9:00	CFU — all-student response. $\frac{1}{4}^5$ and $\frac{3}{4}^5$, summing to 1. Watch for counts.
7/8	10:30	The out-of-range check. It costs nothing and catches the commonest error in the topic.
8/8	13:00	Launch the in-class practice sheet.