

GRADE 7 · STATISTICS & PROBABILITY · ANSWER KEY

PROBABILITY FROM EXPERIMENTAL DATA

Today's goal: I can predict the approximate relative frequency given the probability and understand that more trials of a chance experiment will increase the accuracy of the probability estimate.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Item 8 is the gambler's fallacy, and its final point is worth drawing out: a long run of no reds is better evidence against the MODEL than for red being due. **Timing:** 15–25 min · pairs or independent · circulate on 8 and 9. If you can, run a live class experiment and pool results — watching the pooled frequency settle is the argument.

- 1 $\frac{3}{10}$ 2 $\frac{6}{25}$ 3 $\frac{3}{50}$, or 0.06 4 15 predicted, 12 observed 5 120 reds
 6 It matches exactly — but that proves nothing 7 Ten trials is far too few to establish anything
 8 The spinner has no memory — the next spin is still $\frac{3}{10}$

1 **MILD** 6 of 20

 $\frac{3}{10}$

1. $\frac{6}{20} = \frac{3}{10}$.

Watch for: Answered 6, giving a count.

2 **MILD** 12 of 50

 $\frac{6}{25}$

1. $\frac{12}{50} = \frac{6}{25} = 0.24$.

Watch for: Answered 12.

3 **MEDIUM** Distance from $\frac{3}{10}$
 $\frac{3}{50}$, or 0.06

1. $0.30 - 0.24 = 0.06 = \frac{3}{50}$.

2. The observed frequency is BELOW the theory this time; the gap can fall either side.

Watch for: Gave the difference in counts rather than in frequencies.

4 **MEDIUM** Predict for 50; compare with 12

15 predicted, 12 observed

1. $\frac{3}{10}$ of 50 = 15 reds predicted.

2. 12 were observed, 3 fewer than predicted.

3. A gap of 3 in 50 spins is unremarkable.

Watch for: Predicted 12 because that is what happened. The prediction comes from the probability, before you look.

5 **MEDIUM** Predict for 400

120 reds

1. $\frac{3}{10}$ of 400 = 120.

Watch for: Multiplied by 3 without dividing by 10.

6 **SPICY** Compare with theory; does a match prove fairness?

It matches exactly – but that proves nothing

1. $\frac{3}{10}$ observed, $\frac{3}{10}$ theoretical, so they agree exactly.
2. That is a pleasing result and it is weak evidence.
3. With only 20 spins, several nearby counts are all quite likely, and 6 happens to be one of them.
4. A biased spinner could easily produce 6 reds in 20 spins too, so this outcome does not distinguish fair from biased.
5. Agreement over 500 spins would be far stronger evidence. Agreement over 20 is close to none.

Watch for: Said the match confirms the theory. One agreeing result never confirms a model – the same logic as testing an algebraic identity at a single value.

7 **SPICY** '3 of 10 proves the probability is $\frac{3}{10}$ '

Ten trials is far too few to establish anything

1. The observed frequency is $\frac{3}{10}$, which does match the theory.
2. But with 10 spins, the possible counts are 0 through 10, and several of them are reasonably likely.
3. Getting 3 is consistent with a probability of $\frac{3}{10}$ – and also with 0.2, or 0.4, or a range of other values.
4. So the result does not single out $\frac{3}{10}$; it merely fails to contradict it.
5. Evidence strengthens with trials. Ten spins narrow the possibilities barely at all; a thousand narrow them a great deal.
6. A single small experiment can rule things out, but it cannot prove a probability.

Watch for: Agreed, because the numbers matched. Matching in a small sample is common enough to carry almost no information.

8 **EXTRA SPICY** 'Red is due after 99 spins with none'

The spinner has no memory – the next spin is still $\frac{3}{10}$

1. Each spin is independent: the spinner does not know or record what came before.
2. So the probability of red on spin 100 is $\frac{3}{10}$, exactly as it was on spin 1.
3. Nothing is 'owed' or 'due'. This belief is common enough to have a name – the gambler's fallacy.
4. What IS true is that over many more spins the overall relative frequency will drift back toward $\frac{3}{10}$. That happens by the later spins swamping the early run, not by red becoming more likely.
5. Worth adding: 99 spins with no reds would be extraordinarily unlikely if the probability really were $\frac{3}{10}$. The sensible conclusion is not that red is due, but that the spinner may not have three red sectors after all.

Watch for: Agreed, reasoning that things even out. They even out by dilution over many trials, never by a change in the odds of the next one.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Item 7 states the distinction the whole skill rests on, and it is the one students carry furthest beyond school. **Timing:** ~15 min at home · independent · review item 7 as the next-day warm-up before starting probability models.

1 $\frac{11}{40}$ **2** $\frac{1}{40}$, or 0.025**3** 24 predicted, 22 observed**4** 180 reds**5** $\frac{7}{10}$, which is $\frac{1}{5}$ from $\frac{1}{2}$ **6** No — and the answer is to flip it many more times**7** The first is true; the second is false**1** **MILD** 22 of 80 $\frac{11}{40}$ 1. $\frac{22}{80} = \frac{11}{40} = 0.275$.**Watch for:** Answered 22.**2** **MEDIUM** Distance from $\frac{1}{40}$ $\frac{1}{40}$, or 0.0251. $0.300 - 0.275 = 0.025 = \frac{1}{40}$.**Watch for:** Subtracted the counts, $24 - 22 = 2$.**3** **MEDIUM** Predict for 80; compare with 22**24 predicted, 22 observed**

- $\frac{24}{80}$ of 80 = 24 reds predicted.
- 22 observed, 2 fewer.
- A gap of 2 in 80 spins is very small.

Watch for: Predicted 22 by copying the observation.**4** **MEDIUM** Predict for 600**180 reds**1. $\frac{1}{5}$ of 600 = 180.**Watch for:** Answered 60.**5** **MEDIUM** 7 heads in 10 flips $\frac{7}{10}$, which is $\frac{1}{5}$ from $\frac{1}{2}$

- Relative frequency: $\frac{7}{10} = 0.7$.
- Theory for a fair coin: $\frac{1}{2} = 0.5$.
- Distance: $0.7 - 0.5 = 0.2 = \frac{1}{5}$.

Watch for: Said the distance is 2, comparing 7 heads with 5 expected.**6** **SPICY** Does 7 of 10 show bias? What would you do?**No — and the answer is to flip it many more times**

- A gap of 0.2 sounds large, but 10 flips is a tiny experiment.
- Getting 7 heads in 10 flips of a FAIR coin is not at all unusual; it happens frequently.
- So this result is consistent with a fair coin and does not show bias.
- What to do: flip it many more times — 200, or 1000. With a fair coin the relative frequency will settle near 0.5; with a biased one it will settle somewhere else.
- The point is that the gap shrinks with trials for a fair coin, so a persistent gap over many trials is the evidence, not a large gap over few.
- One short experiment cannot distinguish an unusual run from a genuine bias. Only more data can.

Watch for: Concluded the coin is biased because 7 is well above 5. In 10 flips that is ordinary; the sample is far too small to support the claim.**7** **SPICY** 'Better estimate' versus 'more predictable outcome'**The first is true; the second is false**

- More trials make the ESTIMATE better: the overall relative frequency closes in on the true probability, so you learn the probability more accurately.
- More trials do NOT make the next OUTCOME more predictable. Each spin or flip is independent, so the chance on the next trial is unchanged no matter how many came before.
- After 1000 flips of a fair coin you would know the probability is very close to 0.5 — but you would still have no idea whether the next flip lands heads.
- Knowing the probability precisely and predicting a single outcome are different things, and probability only ever offers the first.
- This is why the gambler's fallacy is wrong: it confuses long-run stability of the AVERAGE with predictability of the NEXT result.

Watch for: Said both are true because more information always helps. It helps you know the probability; it never helps you know the next outcome.

Exit Ticket

4 problems

Use: End of the experimental probability lesson. 5–8 min, independent. The three trial counts are chosen so the gaps shrink cleanly: 0.100, 0.040, 0.016. **Timing:** 5–8 min · independent. Any answer declaring the spinner unfair from 20 trials is the misconception this lesson exists for.

1 2^5 , which is $\frac{1}{10}$ from $\frac{3}{10}$ **2** 17^5_0 , which is $\frac{1}{2^5}$ from $\frac{3}{10}$ **3** $79^5_{250} = 0.316$, a gap of 0.016 — the gaps shrink

4 About 60 reds; an exact 60 would be unremarkable but is not expected

1 **MEDIUM** 8 of 20

2^5 , which is $\frac{1}{10}$ from $\frac{3}{10}$

1. Relative frequency: $\frac{8}{20} = 2^5 = \frac{4}{10}$.
2. Distance from theory: $\frac{4}{10} - \frac{3}{10} = \frac{1}{10}$.
3. Close to $\frac{3}{10}$ but not equal, which is exactly what 20 trials should give.

Watch for: Concluded the spinner is unfair. A gap this size after only 20 spins is entirely ordinary.

2 **MEDIUM** 34 of 100

17^5_0 , which is $\frac{1}{2^5}$ from $\frac{3}{10}$

1. Relative frequency: $\frac{34}{100} = 0.34$.
2. Theory is $\frac{3}{10} = 0.30$, so the gap is $0.04 = \frac{1}{2^5}$.
3. Smaller than the gap after 20 spins.

Watch for: Said the spinner has changed. It is the same spinner; more trials simply give a steadier estimate.

3 **SPICY** 158 of 500, and the pattern

$79^5_{250} = 0.316$, a gap of 0.016 — the gaps shrink

1. Relative frequency: $\frac{158}{500} = 0.316$.
2. Gap: $0.316 - 0.300 = 0.016$.
3. Across the three: 20 spins gave a gap of 0.100, 100 spins gave 0.040, 500 spins gave 0.016.
4. The pattern is that the gap SHRINKS as trials increase — the relative frequency closes in on the theoretical probability.
5. It does not reach it and stop. With more spins it would keep hovering nearby, getting gradually closer on average.

Watch for: Said 500 spins gave the ‘right’ answer. It gave a better estimate; 0.316 is still not 0.300.

4 **SPICY** Predict for 200 spins; surprised by an exact match?

About 60 reds; an exact 60 would be unremarkable but is not expected

1. Prediction: $\frac{3}{10}$ of 200 = 60 reds.
2. Would exactly 60 be surprising? Not at all — it is the single most likely count.
3. But it is not what you should EXPECT in the sense of predicting it confidently. Results near 60, such as 55 or 64, are each perfectly ordinary, and together they are far more likely than 60 exactly.
4. So the honest prediction is ‘about 60’, not ‘60’.
5. Getting exactly 60 is neither evidence that the theory is right nor a sign that something was rigged. It is one ordinary outcome among many.

Watch for: Said you should get exactly 60 and that anything else means the spinner is faulty. A prediction from probability is a centre, not a target.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: theory predicts, experiment observes, and they agree only approximately.
2	1:00	Hook — poll fair or unfair. Do NOT resolve. A gap of 0.10 looks damning until you see 500 spins.
3	2:30	Define relative frequency and compute the first gap.
4	4:30	WE1 — the shrinking gaps. Write all three side by side: 0.100, 0.040, 0.016.
5	7:00	WE2 — predicting a count, and the honest wording ‘about 60’.

6	10:30	The gambler's fallacy, plus the sharper point: a long run with no reds is evidence against the MODEL, not for red being due.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	9:00	CFU — all-student response. 0.24, a gap of 0.06. Watch for counts instead of frequencies. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #6 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:30	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.