



Probability models

GRADE 7 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the probability models lesson. 5–8 min, independent. Items 2 and 3 are a matched pair — the point is the SIZE of the discrepancy, not whether one exists.

Timing: 5–8 min · independent. A student rejecting the model in item 2 has the misconception; one accepting it in item 3 has the opposite problem.

1. Fair die model and expected counts **FLUENCY**

Answer: $\frac{1}{6}$ each; about 10 of each in 60 rolls

1. Six equally likely faces, so each has probability $\frac{1}{6}$.
2. Expected count: $\frac{1}{6} \times 60 = 10$ of each face.
3. Say ‘about 10’ — the model predicts a centre, not a guarantee.

If wrong: Answered 6 or 60, giving counts rather than probabilities.

2. 8, 12, 9, 11, 13, 7 — contradiction? **REASONING**

Answer: **No — these are ordinary variations**

1. Expected 10 of each; observed counts range from 7 to 13.
2. The largest deviation is 3, which is small over 60 rolls.
3. The counts sum to 60, as they must.
4. Random variation of this size is exactly what a fair die produces. Getting exactly 10 of each would in fact be unusual.
5. So the data is CONSISTENT with the model. That is not the same as proving it, but nothing here counts against it.

If wrong: Concluded the die is unfair because the counts are not all 10. A model predicts approximate frequencies, and exact agreement would be more suspicious than these deviations.

3. 2, 3, 4, 5, 6, 40 — contradiction? REASONING

Answer: **Yes — the deviation is far too large**

1. Expected 10 of each; face 6 came up 40 times, a deviation of 30.
2. Every other face came up well below 10.
3. A deviation of 30 in 60 rolls cannot plausibly be chance — a fair die would essentially never do this.
4. What is different from question 2: there the largest deviation was 3, here it is 30, ten times larger.
5. Conclusion: the fair-die model does not fit. The die is probably weighted toward 6.
6. Note the process — you do not reject a model because it disagrees, but because it disagrees by more than chance can explain.

If wrong: Said both dice are unfair because neither gave exactly 10 each. The question is always the SIZE of the discrepancy, not its existence.

4. P(7) and P(2) for two dice REASONING

Answer: **$P(7) = \frac{6}{36}$, $P(2) = \frac{1}{36}$ — 7 is six times as likely**

1. There are 36 equally likely outcomes when two dice are rolled, since each of 6 faces pairs with each of 6.
2. Total 7 happens six ways: 1+6, 2+5, 3+4, 4+3, 5+2, 6+1. So $P(7) = \frac{6}{36} = \frac{1}{6}$.
3. Total 2 happens one way: 1+1. So $P(2) = \frac{1}{36}$.
4. They differ because there are more WAYS to make 7 than to make 2.
5. This is why you cannot assume equal likelihood from being able to list the outcomes. There are 11 possible totals, and they are far from equally likely.
6. The equally likely things here are the 36 dice PAIRS, not the 11 totals.

If wrong: Answered $\frac{1}{11}$ for both, treating the 11 totals as equally likely. Listing outcomes tells you what can happen, not how often.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\frac{1}{6}$ each; about 10
2	reasoning	no — ordinary variation
3	reasoning	yes — deviation of 30
4	reasoning	$\frac{1}{6}$ and $\frac{1}{36}$

Practice Worksheet — answer key

Use: During the lesson after the deck. Item 6 is the completeness check for the model; item 9 names the real error, which is choosing the wrong level of outcome.

Timing: 15–25 min · pairs or independent · circulate on 6 and 9. A 6-by-6 grid of the 36 pairs drawn on the board makes every answer on this sheet readable at a glance.

1. Ways to total 3? FLUENCY

Answer: **2**

1. $1+2$ and $2+1$. Two ways.

If wrong: Answered 1, treating $1+2$ and $2+1$ as the same. The dice are distinguishable, so they are different outcomes.

2. Ways to total 6? FLUENCY

Answer: **5**

1. $1+5$, $2+4$, $3+3$, $4+2$, $5+1$. Five ways.

If wrong: Answered 3, missing the reversed pairs.

3. Ways to total 12? FLUENCY

Answer: **1**

1. $6+6$ only.

If wrong: Answered 2, doubling a pair that cannot be reversed.

4. P(total 6)? APPLICATION

Answer: $\frac{5}{36}$

1. 5 ways out of 36 equally likely pairs.

2. $P = \frac{5}{36}$.

If wrong: Used 11 as the denominator.

5. P(total 12)? APPLICATION

Answer: $\frac{1}{36}$

1. 1 way out of 36.

If wrong: Answered $\frac{1}{11}$.

6. Sum of all the ways REASONING

Answer: **36 — every possible pair is counted once**

1. Ways for totals 2 through 12: 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1.
2. Sum: 36.
3. It must be 36 because every one of the 36 dice pairs produces exactly one total, so counting by total counts every pair once.
4. The probabilities therefore sum to $\frac{6}{36} = 1$, as they must.
5. Notice the pattern rises to a peak at 7 and falls symmetrically — that shape is why 7 is the most likely total.

If wrong: Answered 11, counting the totals rather than the ways. The check that the model is complete is that the WAYS sum to 36.

7. Expected 7s in 72 rolls APPLICATION

Answer: **12**

1. $P(7) = \frac{6}{36} = \frac{1}{6}$.
2. $\frac{1}{6}$ of 72 = 12 rolls.

If wrong: Used $\frac{1}{11}$, giving about 6.5.

8. Expected 2s in 72 rolls APPLICATION

Answer: **2**

1. $P(2) = \frac{1}{36}$.
2. $\frac{1}{36}$ of 72 = 2 rolls.

If wrong: Used $\frac{1}{11}$, giving about 6.5 — the same as for 7, which should look wrong.

9. '11 totals, so $\frac{1}{11}$ each' REASONING

Answer: **Wrong — the totals are not equally likely; the 36 PAIRS are**

1. From questions 1 to 5: total 3 has 2 ways, total 6 has 5, total 12 has 1. They are plainly not equal.
2. If each total had probability $\frac{1}{11}$, then 7 and 12 would be equally common — but 7 arises six ways and 12 only one.
3. Expected counts in 72 rolls make it concrete: 12 sevens against 2 twelves. The student's model predicts about 6.5 of each.
4. What they should have counted: the 36 equally likely PAIRS of dice, not the 11 totals.
5. The general lesson: being able to LIST outcomes does not make them equally likely. You must find the level at which the outcomes really are equally likely, and build from there.

If wrong: Said the student just needs to divide differently. The problem is not the arithmetic but the choice of what to treat as equally likely.

Quick answers for fast checking:

#	Level	Answer
1	fluency	2
2	fluency	5
3	fluency	1
4	application	$\frac{5}{36}$
5	application	$\frac{1}{36}$
6	reasoning	36
7	application	12
8	application	2
9	reasoning	count the 36 pairs

Homework — answer key

Use: Parallels the practice sheet. Item 8 offers two routes to a better model — measuring angles or estimating from data — which links this skill directly back to SP-06.

Timing: ~15 min at home · independent · review item 8 as the next-day warm-up before compound events.

1. Ways to total 5? FLUENCY

Answer: 4

1. 1+4, 2+3, 3+2, 4+1.

If wrong: Answered 2, ignoring order.

2. Ways to total 9? FLUENCY

Answer: 4

1. 3+6, 4+5, 5+4, 6+3.

If wrong: Answered 3.

3. Are 5 and 9 equally likely? REASONING

Answer: **Yes — both have 4 ways**

1. Total 5 arises 4 ways; total 9 arises 4 ways.
2. Both therefore have probability $\frac{4}{36} = \frac{1}{9}$.
3. They are equally likely, but not because they are both 'possible totals' — because they happen to have the same number of ways.
4. The distribution is symmetric about 7, and 5 and 9 sit the same distance either side, which is why they match.

If wrong: Said no because 9 is a bigger number. Size of the total has nothing to do with its likelihood.

4. P(total 5)? APPLICATION

Answer: $\frac{1}{9}$

1. $\frac{4}{36} = \frac{1}{9}$.

If wrong: Answered $\frac{1}{11}$.

5. Expected 5s in 90 rolls APPLICATION

Answer: 10

1. $\frac{1}{9}$ of 90 = 10.

If wrong: Used $\frac{1}{11}$.

6. Expected 7s in 90 rolls APPLICATION

Answer: **15**

1. $P(7) = \frac{1}{6}$, and $\frac{1}{6}$ of 90 = 15.

If wrong: Gave 10, the same as for 5.

7. 12 fives and 17 sevens in 90 rolls REASONING

Answer: **No — both are close to prediction**

1. Predicted: 10 fives and 15 sevens.
2. Observed: 12 fives and 17 sevens.
3. Deviations of 2 in each case, over 90 rolls. Small.
4. Both observations sit slightly above prediction, which is unremarkable — random variation does not alternate neatly above and below.
5. So the data is consistent with the model. Nothing here counts against it.
6. Compare a result like 40 sevens, which would be far beyond what chance explains and would call the dice into question.

If wrong: Concluded the dice are biased because neither count matched exactly. A model predicts a centre, and small deviations are expected rather than surprising.

8. Spinner with an oversized A sector REASONING

Answer: **Equal probabilities assume equal sectors, which is false here**

1. A model giving each letter $\frac{1}{4}$ assumes the four outcomes are equally likely.
2. They are not: the A sector is much larger, so the pointer is more likely to stop there.
3. The listing of four outcomes is correct; the assumption of equal likelihood is what fails.
4. How to build a better model, option one — measure the ANGLES. If A takes 180° and the others 60° each, then $P(A) = \frac{180}{360} = \frac{1}{2}$ and each other letter is $\frac{60}{360} = \frac{1}{6}$.
5. Option two — SPIN IT many times and use the relative frequencies as estimates, which is SP-06's method and needs no measuring.
6. The angle method gives exact probabilities if you can measure; the experimental method works even when you cannot.
7. Either way, the model must come from the situation rather than from the number of labels.

If wrong: Said the model just needs more spins. More spins would help you MEASURE the probabilities, but the specific error is assuming equal sectors when they are visibly unequal.

Quick answers for fast checking:

#	Level	Answer
1	fluency	4
2	fluency	4
3	reasoning	yes — 4 ways each
4	application	$\frac{1}{9}$
5	application	10
6	application	15
7	reasoning	no — deviations of 2
8	reasoning	sectors are unequal

Lesson slides — teaching notes & timing

Mini-lesson deck: ~12–15 min, then the practice sheet. Draw the 6-by-6 grid of dice pairs on the board and leave it up — every question on the practice sheet is readable from it.

Slide	~Time	Note
1/8	0:00	Frame it: a model is a complete list of outcomes with a probability attached to each.
2/8	1:00	Hook — poll $\frac{1}{11}$ or not. Do NOT resolve. Eleven outcomes makes $\frac{1}{11}$ look inevitable.
3/8	2:30	The key move: find the level at which outcomes ARE equally likely. Draw the 6-by-6 grid here.
4/8	5:00	WE1 — count the ways for 7 and 2 on the grid. The six diagonal cells against one corner cell is the picture.
5/8	7:30	State the general lesson. Listing outcomes and assuming they are equal is the commonest modelling error there is.
6/8	9:00	CFU — all-student response. 5 ways, $\frac{5}{36}$. Watch for $\frac{1}{11}$.
7/8	10:30	Model versus data. Both examples together; the contrast between deviations of 3 and 30 is the whole criterion.
8/8	13:30	Launch the in-class practice sheet.