

GRADE 7 · RATIOS & PROPORTIONAL RELATIONSHIPS · ANSWER KEY

RECOGNIZING PROPORTIONAL RELATIONSHIPS

Today's goal: I can represent proportional relationships between quantities and recognize proportional relationships between quantities.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Items 4 and 7 are the reason the ratio test is taught as primary: $y = x^2$ passes the origin test and fails proportionality. **Timing:** 15–25 min · pairs or independent · circulate on 6 and 9. Require every ratio written out; a conclusion from one pair is the commonest error.

- 1 3 2 3 3 30 4 No 5 $\frac{1}{2}$ 6 10 7 The ratios are 1, 2, 3 — they grow, so there is no single k
8 The second student; the constant-ratio test settles it

1 **MILD** $6 \div 2$?

3

1. $6 \div 2 = 3$.

Watch for: Computed $2 \div 6 = \frac{1}{3}$.

2 **MILD** $9 \div 3$?

3

1. $9 \div 3 = 3$.

Watch for: Reversed the division.

3 **MEDIUM** y when $x = 10$, $k = 3$?

30

1. $y = kx = 3 \times 10 = 30$.

2. Check the ratio: $30 \div 10 = 3$, matching the others.

Watch for: Added 3 to get 13. The rule multiplies by k , it does not add k .

4 **MEDIUM** (1,1), (2,4), (3,9) — proportional?

No

1. $1 \div 1 = 1$, $4 \div 2 = 2$, $9 \div 3 = 3$.

2. The ratios are 1, 2, 3 — not constant, so not proportional.

3. This is $y = x^2$, which passes through the origin but still is not proportional.

Watch for: Answered yes because it passes through (0, 0). Passing through the origin is NECESSARY but not sufficient — the ratio must also be constant.

5 **MEDIUM** k for (4,2), (10,5), (14,7)?

$\frac{1}{2}$

1. $2 \div 4 = \frac{1}{2}$, $5 \div 10 = \frac{1}{2}$, $7 \div 14 = \frac{1}{2}$. All equal, so $k = \frac{1}{2}$.

Watch for: Answered 2 by computing $x \div y$.

6 **MEDIUM** y when $x = 20$, $k = \frac{1}{2}$?

10

1. $y = \frac{1}{2} \times 20 = 10$.

2. Check: $10 \div 20 = \frac{1}{2}$.

Watch for: Answered 40 by multiplying by 2 instead of $\frac{1}{2}$.

7 **SPICY** The pattern in those ratios

The ratios are 1, 2, 3 — they grow, so there is no single k

1. Ratios in order: 1, 2, 3.
2. They increase steadily, which means y grows FASTER than x rather than in step with it.
3. Proportionality needs one number k that works for every pair; here no single number does.
4. Note this table passes through the origin ($0 \div 0$ aside, $y = x^2$ gives $y = 0$ at $x = 0$), so the origin test alone would have wrongly passed it. The constant-ratio test is the reliable one.

Watch for: Said the growing ratios prove it is proportional because there is a pattern. A pattern in the ratios is exactly what rules it out — proportionality needs NO pattern, just repetition.

8 **SPICY** Who is right about $y = 2x + 3$?

The second student; the constant-ratio test settles it

1. At $x = 0$, $y = 3$, so the graph misses the origin and cannot be proportional.
2. The first student noticed a constant DIFFERENCE of 2, which makes the relationship linear, not proportional.
3. The settling test is the ratio: $5 \div 1 = 5$ but $7 \div 2 = 3\frac{1}{2}$, so no single k exists.
4. Both the origin test and the ratio test agree here. Where they disagree — as in question 4 — trust the ratio test, since the origin test can pass a non-proportional relationship.
5. Every proportional relationship is linear; not every linear relationship is proportional.

Watch for: Sided with the first student because the numbers change predictably. Predictable is not the criterion; constant $y \div x$ is.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Items 6 and 7 introduce the doubling test as a fast screen, derived rather than announced.

Timing: ~15 min at home · independent · review items 6 and 7 together as the next-day warm-up; the contrast is the lesson.

1 3 **2** 4 **3** 4 **4** 9 **5** No — the ratios are 3 and 4, so no single k fits both

6 y goes from 6 to 12, which is double **7** y goes from 7 to 11 — not double, so it is not proportional

1 **MILD** $12 \div 4$?

3

1. $12 \div 4 = 3$.

Watch for: Reversed the division.

2 **MILD** $20 \div 5$?

4

1. $20 \div 5 = 4$.

Watch for: Reversed the division.

3 **MEDIUM** k for (5,20), (7,28), (9,36)?

4

1. $20 \div 5 = 4$, $28 \div 7 = 4$, $36 \div 9 = 4$. All equal, so $k = 4$.

Watch for: Checked only the first pair.

4 **MEDIUM** x when $y = 36$, $k = 4$?

9

1. $y = kx$, so $x = y \div k = 36 \div 4 = 9$.
2. Check: $4 \times 9 = 36$.

Watch for: Multiplied to get 144. Finding x from y reverses the operation.

5 **SPICY** Same relationship?

No — the ratios are 3 and 4, so no single k fits both

1. (4, 12) gives 3. (5, 20) gives 4.
2. A proportional relationship has ONE constant k, and $3 \neq 4$.
3. So these two pairs cannot both lie on the same proportional relationship.
4. They could both lie on some other relationship, just not a proportional one.

Watch for: Answered yes because both ratios are whole numbers. Being tidy is not being equal.

6 **SPICY** $k = 3$, x from 2 to 4 — show y doubles

y goes from 6 to 12, which is double

1. At $x = 2$: $y = 3 \times 2 = 6$.
2. At $x = 4$: $y = 3 \times 4 = 12$.
3. 12 is exactly double 6, matching the doubling of x .
4. This always happens when a relationship is proportional: multiply x by any number and y is multiplied by the same number, because the constant k is untouched.

Watch for: Showed the values but did not connect the doubling of x to the doubling of y , which is the point of the question.

7 **EXTRA SPICY** Same doubling on $y = 2x + 3$

y goes from 7 to 11 — not double, so it is not proportional

1. At $x = 2$: $y = 2(2) + 3 = 7$.
2. At $x = 4$: $y = 2(4) + 3 = 11$.
3. Double of 7 would be 14, but y is 11, so doubling x did NOT double y .
4. The $+3$ does not double along with everything else, which is exactly what breaks the proportion.
5. This gives a third test, and often the fastest one: in a proportional relationship, doubling x must double y . If it does not, stop — it is not proportional.

Watch for: Concluded y nearly doubled so it is roughly proportional. Proportionality is exact; 'nearly' means no.

Exit Ticket

4 problems

Use: End of the proportionality lesson. 5–8 min, independent. Items 1 and 2 are a matched pair; a student answering yes to both has the misconception squarely. **Timing:** 5–8 min · independent. Item 2 is the diagnostic — the table grows by a tidy 2 each step and is designed to look proportional.

1 Yes, $k = 3$, rule $y = 3x$

2 No — the ratios differ

3 At $x = 0$, $y = 3$, not 0 — so it misses the origin

4 Yes, $k = \frac{1}{2}$, rule $y = \frac{1}{2}x$

1 **MILD** (2,6), (3,9), (5,15), (8,24) — proportional?

Yes, $k = 3$, rule $y = 3x$

1. $6 \div 2 = 3$, $9 \div 3 = 3$, $15 \div 5 = 3$, $24 \div 8 = 3$.
2. Every ratio is 3, so the relationship is proportional with $k = 3$.
3. Rule: $y = 3x$.

Watch for: Checked only the first pair. One matching ratio proves nothing — you need them all.

2 **MEDIUM** (1,5), (2,7), (3,9), (4,11) — proportional?

No — the ratios differ

1. $5 \div 1 = 5$, but $7 \div 2 = 3\frac{1}{2}$.
2. The ratios are already different at the second pair, so it is not proportional.
3. It does grow by a steady 2 each time, which is why it looks proportional at a glance — but steady growth is not the test.

Watch for: Answered yes because y increases by 2 every time. A constant DIFFERENCE is not a constant RATIO.

3 **SPICY** Use $x = 0$ on $y = 2x + 3$

At $x = 0$, $y = 3$, not 0 — so it misses the origin

1. Substituting: $y = 2(0) + 3 = 3$.
2. A proportional relationship must give $y = 0$ when $x = 0$, because $y = kx$ and $k \times 0 = 0$.
3. This one gives 3, so its graph crosses the y -axis at 3 rather than passing through the origin.
4. In context that +3 is a starting amount — a flat fee, a head start — and any fixed starting amount destroys proportionality.

Watch for: Said it is proportional because the graph is a straight line. Straight is necessary but not sufficient; it must also pass through (0, 0).

4 **SPICY** (4,2), (10,5), (14,7) — proportional?

Yes, $k = \frac{1}{2}$, rule $y = \frac{1}{2}x$

1. $2 \div 4 = \frac{1}{2}$, $5 \div 10 = \frac{1}{2}$, $7 \div 14 = \frac{1}{2}$.
2. All three ratios are $\frac{1}{2}$, so it is proportional with $k = \frac{1}{2}$.
3. Rule: $y = \frac{1}{2}x$, or equivalently $x = 2y$.
4. Here y is SMALLER than x , so k is less than 1. Proportional does not mean growing — it means growing at a constant rate, which can be a shrinking one.

Watch for: Answered no because y is smaller than x , or computed $x \div y = 2$ and reported $k = 2$. The constant is $y \div x$; the order matters.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: Grade 6 asked what the ratio IS. Grade 7 asks whether there is one at all.
2	1:00	Hook — poll both tables. Do NOT resolve. Table B is built to look proportional: tidy numbers, steady growth.
3	3:00	Run the ratios on both, out loud, every pair. Resist stopping after one — students copy what you model.
4	5:30	WE1 — name the difference-versus-ratio confusion directly. It is the whole misconception.
5	7:30	WE2 — the origin test, WITH its limitation. Teaching it as sufficient sets up the $y = x^2$ error on the practice sheet.
6	11:30	The doubling screen. Derive it from $y = kx$ rather than announcing it.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	9:30	CFU — all-student response. Yes, $k = \frac{1}{2}$. Watch for $k = 2$, which is $x \div y$. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #8 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:30	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.