

GRADE 7 · GEOMETRY · ANSWER KEY

SURFACE AREA OF PRISMS

Today's goal: I can determine the surface area of three-dimensional prisms in mathematical problems and determine the surface area of three-dimensional prisms in real-world problems.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Item 5 is unusual and worth the time: two different errors produce the same number, so the answer alone cannot diagnose. Item 8 derives the general formula. **Timing:** 15–25 min · pairs or independent · circulate on 5 and 9. Unrolling a paper prism makes item 8 obvious in a way explanation cannot.

- 1 72 square units 2 36 cubic units 3 96 square units 4 240 square units
5 36 is the VOLUME here — a coincidence worth noticing 6 Five: two triangles and three rectangles
7 72 square units — matches 8 Because every prism is two bases plus one band wrapped around

1 **MILD** Box $6 \times 3 \times 2$ SA?

72 square units

- Pairs: $6 \times 3 = 18$, $6 \times 2 = 12$, $3 \times 2 = 6$.
- $2(18) + 2(12) + 2(6) = 36 + 24 + 12 = 72$ square units.

Watch for: Answered 36 by counting one of each pair.

2 **MILD** Volume?

36 cubic units

- $6 \times 3 \times 2 = 36$ cubic units.

Watch for: Used square units.

3 **MEDIUM** Cube edge 4 SA?

96 square units

- Each face: $4 \times 4 = 16$. Six faces: $6 \times 16 = 96$ square units.

Watch for: Answered 64, the volume.

4 **MEDIUM** Triangular prism SA

240 square units

- Two triangular ends: each $(8 \times 3) \div 2 = 12$, so 24 square units.
- Three rectangles, each 12 long: $12 \times 5 = 60$, $12 \times 5 = 60$, $12 \times 8 = 96$. Total 216.
- Surface area: $24 + 216 = 240$ square units.

Watch for: Used a six-face formula, or forgot to halve the ends.

5 **SPICY** A student says SA is 36

36 is the VOLUME here — a coincidence worth noticing

- The surface area is 72 square units; the volume is 36 cubic units.
- So 36 is the volume, not the surface area.
- But there is a second possibility: 36 is also exactly half of 72, which is what you get by counting three faces instead of six.
- Both errors land on 36 for this particular box, so the number alone cannot tell you which they made.
- Asking for their working, or for the units, distinguishes them — cubic units means volume, square units means the halving error.

Watch for: Assumed one explanation without noticing the other. This box makes two different errors coincide, which is exactly when you must look at the method rather than the answer.

6 **SPICY** How many faces? List them

Five: two triangles and three rectangles

1. Two triangular ends.
2. Three rectangular sides, one for each edge of the triangle.
3. Total five faces.
4. The number of faces on a prism is always the number of sides of the base, plus 2 for the two ends.
5. A triangular base has 3 sides, so $3 + 2 = 5$. A rectangular base has 4, so $4 + 2 = 6$. A hexagonal prism would have 8.

Watch for: Answered six out of habit. Count the base's sides and add two.

7 **SPICY** $SA = 2B + Ph$ on the 6 by 3 by 2 box

72 square units — matches

1. Base is the 6 by 3 face: $B = 18$, so $2B = 36$.
2. Perimeter of the base: $P = 2(6 + 3) = 18$.
3. Height $h = 2$, so $Ph = 18 \times 2 = 36$.
4. Total: $36 + 36 = 72$ square units. Matches question 1.

Watch for: Used the perimeter of the wrong face, or multiplied P by the wrong dimension. h is the distance BETWEEN the two bases.

8 **EXTRA SPICY** Why does $2B + Ph$ work for both?

Because every prism is two bases plus one band wrapped around

1. Every prism has two identical bases, contributing $2B$ whatever their shape.
2. The remaining faces form a continuous band around the outside, joining one base to the other.
3. Unrolled flat, that band is a single rectangle: its length is the PERIMETER of the base, and its height is the distance between the bases.
4. So the lateral area is always $P \times h$, regardless of how many sides the base has.
5. The six-face method only works when there are exactly six faces, which is true for rectangular prisms and false for every other kind.
6. $2B + Ph$ does not care how many faces the band is divided into, which is why it generalises.

Watch for: Said it works because both are prisms, without explaining the band. The unrolling is the reason, and it is what makes the formula memorable rather than arbitrary.

Homework

7 problems + Mr DJ's worked example

Use: Closes G7 Geometry. Item 7 gives students a plain-language test — covering or filling — that decides between surface area and volume without recalling either formula. **Timing:** ~15 min at home · independent · review item 7 as the next-day warm-up; it is the most portable idea in the batch.

1 118 square units

2 70 cubic units

3 150 square units

4 136 square units

5 Eight — six sides plus two ends

6 118 square units — matches

7 Wrapping needs surface area in square units; sand needs volume in cubic units

1 **MILD** Box $7 \times 5 \times 2$ SA?

118 square units

1. Pairs: $7 \times 5 = 35$, $7 \times 2 = 14$, $5 \times 2 = 10$.
2. $2(35) + 2(14) + 2(10) = 70 + 28 + 20 = 118$ square units.

Watch for: Answered 59 by counting three faces.

2 **MILD** Volume?

70 cubic units

1. $7 \times 5 \times 2 = 70$ cubic units.

Watch for: Used square units.

3 **MEDIUM** Cube edge 5 SA?

150 square units

1. $6 \times (5 \times 5) = 150$ square units.

Watch for: Answered 125, the volume.

4 **MEDIUM** Triangular prism SA

136 square units

1. Two ends: each $(6 \times 4) \div 2 = 12$, so 24 square units.
2. Three rectangles, each 7 long: $7 \times 5 = 35$, $7 \times 5 = 35$, $7 \times 6 = 42$. Total 112.
3. Surface area: $24 + 112 = 136$ square units.

Watch for: Forgot the two triangular ends, giving 112.

5 **SPICY** Faces on a hexagonal prism?

Eight — six sides plus two ends

1. A hexagon has 6 sides, so the band around the prism is divided into 6 rectangles.
2. Add the two hexagonal ends: $6 + 2 = 8$ faces.
3. The rule: faces = (number of sides of the base) + 2.
4. Check it: triangular prism $3 + 2 = 5$. Rectangular prism $4 + 2 = 6$. Both correct.
5. This is why 'a prism has six faces' is wrong as a general statement — it is true only for a rectangular base.

Watch for: Answered six. The rule depends on the base, and stating it as a rule is what the question asks for.

6 **SPICY** $SA = 2B + Ph$ on the 7 by 5 by 2 box

118 square units — matches

1. Base is the 7 by 5 face: $B = 35$, so $2B = 70$.
2. Perimeter: $P = 2(7 + 5) = 24$. Height $h = 2$, so $Ph = 48$.
3. Total: $70 + 48 = 118$ square units. Matches question 1.

Watch for: Used 2 as the perimeter or 24 as the base area. B is an area; P is a distance.

7 **EXTRA SPICY** Gift-wrapping versus filling with sand

Wrapping needs surface area in square units; sand needs volume in cubic units

1. Gift-wrapping covers the OUTSIDE, so it needs the surface area: 118 square units of paper.
2. Filling with sand fills the INSIDE, so it needs the volume: 70 cubic units of sand.
3. The units follow from what is being measured — paper covers a surface, sand occupies space.
4. Both use the same three dimensions, which is why they are easy to confuse; the question being asked is what decides.
5. A good habit: before calculating, ask whether the situation is about covering or filling.

Watch for: Chose one calculation for both. They are genuinely different quantities, and the covering/filling test settles which is needed.

Exit Ticket

4 problems

Use: End of the surface area lesson, closing G7 Geometry. 5–8 min, independent. Item 4 derives the general formula from the face-by-face answer rather than presenting it cold. **Timing:** 5–8 min · independent. An answer of 47 on item 1 is exactly half and means three faces were counted.

1 94 square units

2 60 cubic units

3 184 square units; five faces

4 94 square units — it matches

1 **MILD** Box $5 \times 4 \times 3$ surface area?**94 square units**

1. Faces come in three matching pairs.
2. Front and back: $5 \times 4 = 20$ each. Sides: $4 \times 3 = 12$ each. Top and bottom: $5 \times 3 = 15$ each.
3. Total: $2(20) + 2(12) + 2(15) = 40 + 24 + 30 = 94$ square units.

Watch for: Added only $20 + 12 + 15 = 47$, counting three faces instead of six. The answer is exactly half, which is the fingerprint.

2 **SPICY** Volume, and how units distinguish them**60 cubic units**

1. $V = 5 \times 4 \times 3 = 60$ cubic units.
2. Surface area was 94 SQUARE units; volume is 60 CUBIC units.
3. Surface area multiplies two lengths at a time and adds up the faces, so the unit is squared.
4. Volume multiplies three lengths at once, so the unit is cubed.
5. The numbers 94 and 60 are not comparable — they measure different things. The unit is the fastest way to check which you computed.

Watch for: Compared 94 and 60 as though one should be larger. They answer different questions and cannot be ranked.

3 **SPICY** Triangular prism surface area**184 square units; five faces**

1. Count the faces first: two triangular ends and three rectangles — FIVE, not six.
2. Two ends: each $(6 \times 4) \div 2 = 12$, so $2 \times 12 = 24$ square units.
3. Three rectangles, each 10 long: $10 \times 5 = 50$, $10 \times 5 = 50$, $10 \times 6 = 60$. Total 160 square units.
4. Surface area: $24 + 160 = 184$ square units.
5. Counting the faces before computing is what prevents the automatic ‘six faces’ assumption.

Watch for: Applied the six-face rectangular formula, or forgot to halve the triangular ends.

4 **EXTRA SPICY** $SA = 2B + Ph$ for the same box**94 square units — it matches**

1. Take the 5 by 4 face as the base: $B = 20$ square units, so $2B = 40$.
2. Perimeter of that base: $P = 2(5 + 4) = 18$ units.
3. Height $h = 3$, so the lateral area is $Ph = 18 \times 3 = 54$ square units.
4. Total: $40 + 54 = 94$ square units. Matches question 1.
5. Why it works: the lateral faces form one band wrapped around the prism. Unrolled, that band is a rectangle of length P and height h .
6. This version works for ANY prism, including the triangular one, which is why it is worth knowing.

Watch for: Used the wrong perimeter, or treated P as the base area. P is a distance around the base; B is the area of it.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: surface area is a covering question, so unfolding turns it into areas you already know.
2	1:00	Hook — poll the number of faces. Do NOT resolve. Six is the reflex answer after the “Nets and surface area” sheet (Grade 6).
3	2:30	WE1 — count them together, then give the sides-plus-two rule and test it on a box.
4	5:00	Total it face by face. Insist the two ends are halved.
5	7:00	WE2 — unroll an actual paper prism if you have one. The band becoming a rectangle is the whole derivation.

6	11:30	Covering versus filling, with the halved-answer fingerprint. Both are portable beyond this lesson.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	9:30	CFU — all-student response. 94 square units both ways. Two methods agreeing is the strongest check available. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #6 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:30	Launch the in-class practice sheet. This closes G7 Geometry. Launch the practice sheet. Exit ticket in the last 5–8 minutes.