

GRADE 7 · RATIOS & PROPORTIONAL RELATIONSHIPS · ANSWER KEY

UNIT RATES WITH FRACTIONS

Today's goal: I can given a rate relationship between two quantities in a mathematical problem, calculate the unit rate, compute unit rates associated with ratios of fractions and given a rate relationship between two quantities in a real-world problem, calculate the unit rate.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Items 5–8 deliberately break the ‘the answer is always bigger’ pattern set by 1–5. Item 8 rewards reasoning over computing. **Timing:** 15–25 min · pairs or independent · circulate on 7 and 9. Insist on the sense check before the arithmetic on every item.

- 1 3 mph 2 4 cups per serving 3 2½ litres per minute 4 2 mph 5 ⅓ kg per box 6 ½ mph
- 7 Because ⅔ is greater than... no — because you divided by a fraction close to 1 while the amount was small
- 8 The first — 3 mph against ½ mph

1 **MILD** ⅓ mi in ⅑ h?

3 mph

$$1. \frac{1}{3} \div \frac{1}{9} = \frac{1}{3} \times 9 = 3.$$

Watch for: Answered ⅑₂₇ by multiplying.

2 **MILD** ½ cup per ⅛ serving?

4 cups per serving

$$1. \frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times 8 = 4.$$

Watch for: Answered ⅑₁₆.

3 **MEDIUM** ⅝ L in ⅓ min?

2½ litres per minute

$$1. \frac{5}{6} \div \frac{1}{3} = \frac{5}{6} \times 3 = \frac{15}{6} = \frac{5}{2} = 2\frac{1}{2}.$$

Watch for: Left the answer as ⅑₆ without simplifying.

4 **MEDIUM** ⅞ mi in ⅞ h?

2 mph

$$1. \frac{7}{8} \div \frac{7}{16} = \frac{7}{8} \times \frac{16}{7} = \frac{112}{56} = 2.$$

2. The 7s cancel, which is the quickest route.

Watch for: Multiplied to get ⅑₁₂₈.

5 **MEDIUM** ⅔ kg per ⅔ box?

⅓ kg per box

$$1. \frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1.$$

Watch for: Answered ⅑₂ by flipping the wrong fraction.

6 **MEDIUM** ¼ mi in ½ h?

½ mph

$$1. \frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2} \text{ mph.}$$

2. Half an hour to go a quarter mile is a slow walk.

Watch for: Answered 2 by dividing the wrong way round.

7 **SPICY** Why is question 5 less than 1?

Because $\frac{2}{3}$ is greater than... no — because you divided by a fraction close to 1 while the amount was small

1. In questions 1–5 the amount was divided by a SMALL fraction ($\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{10}$), and dividing by something small makes the answer big.
2. In question 5 the amount $\frac{2}{9}$ is itself small and $\frac{2}{3}$ is comparatively large, so the answer stays under 1.
3. The rule: the unit rate is bigger than the amount when you divide by a fraction less than 1, and the SIZE of that fraction decides how much bigger.
4. Dividing $\frac{2}{9}$ by $\frac{2}{3}$ still increases it — $\frac{1}{3}$ is bigger than $\frac{2}{9}$ — just not past 1.

Watch for: Concluded that dividing made the number smaller. It did not: $\frac{1}{3} > \frac{2}{9}$. What changed is the starting amount, not the direction.

8 **SPICY** Which is faster, $\frac{1}{3}$ mi in $\frac{1}{9}$ h or $\frac{1}{4}$ mi in $\frac{1}{2}$ h?

The first — 3 mph against $\frac{1}{2}$ mph

1. Question 1: 3 mph. Question 6: $\frac{1}{2}$ mph.
2. You could tell before dividing: the first covers MORE distance ($\frac{1}{3} > \frac{1}{4}$) in LESS time ($\frac{1}{9} < \frac{1}{2}$).
3. More distance in less time is faster on both counts, so no calculation is needed to rank them.
4. Comparing only becomes arithmetic when one quantity goes up and the other goes up too.

Watch for: Compared only the distances, or only the times. Either alone can mislead; here both point the same way, which is what makes the shortcut valid.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Items 6 and 7 both reward noticing structure instead of computing — the habit G7 proportional reasoning depends on. **Timing:** ~15 min at home · independent · review item 7 as the next-day warm-up; the reciprocal observation sets up the “Recognizing proportional relationships” lesson.

1 3 mph

2 $2\frac{2}{3}$ cups

3 $\frac{5}{6}$ litre per minute

4 3 mph

5 $\frac{1}{3}$ kg per bag

6 1 mph, because the two quantities are equal

7 The first — 2 gal/h against $\frac{1}{2}$ gal/h, four times faster

1 **MILD** $\frac{1}{2}$ mi in $\frac{1}{6}$ h?

3 mph

1. $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times 6 = 3$.

Watch for: Answered $\frac{1}{12}$.

2 **MILD** $\frac{2}{3}$ cup per $\frac{1}{4}$ batch?

$2\frac{2}{3}$ cups

1. $\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$.

Watch for: Left it as $\frac{8}{3}$ without converting, or answered $\frac{1}{6}$.

3 **MEDIUM** $\frac{7}{8}$ L in $\frac{3}{4}$ min?

$\frac{5}{6}$ litre per minute

1. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3} = \frac{28}{24} = \frac{7}{6}$.
2. Less than 1, because $\frac{3}{4}$ minute is close to a whole minute.

Watch for: Did not simplify $\frac{28}{24}$.

4 **MEDIUM** $\frac{9}{10}$ mi in $\frac{3}{10}$ h?

3 mph

1. $\frac{9}{10} \div \frac{3}{10} = \frac{9}{10} \times \frac{10}{3} = 3$. The tenths cancel.
2. With the same denominator you can just divide the numerators: $9 \div 3 = 3$.

Watch for: Multiplied to get $\frac{27}{100}$.

5 **MEDIUM** $\frac{1}{6}$ kg per $\frac{1}{2}$ bag?

$\frac{1}{3}$ kg per bag

1. $\frac{1}{6} \div \frac{1}{2} = \frac{1}{6} \times 2 = \frac{2}{6} = \frac{1}{3}$.

Watch for: Answered $\frac{1}{12}$ by multiplying.

6  **SPICY** $\frac{5}{8}$ mi in $\frac{5}{8}$ h — why?

1 mph, because the two quantities are equal

1. $\frac{5}{8} \div \frac{5}{8} = 1$.
2. Any non-zero number divided by itself is 1, so the fractions never need computing.
3. In context: covering five-eighths of a mile in five-eighths of an hour means one mile every hour.
4. Whenever the amount and the number of units are the same, the unit rate is exactly 1 — the fractions are a distraction.

Watch for: Computed $\frac{25}{64}$ by multiplying, missing that the two are identical.

7  **SPICY** Which tap is faster: $\frac{2}{3}$ gal in $\frac{1}{3}$ h, or $\frac{1}{3}$ gal in $\frac{2}{3}$ h?

The first — 2 gal/h against $\frac{1}{2}$ gal/h, four times faster

1. First tap: $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times 3 = 2$ gallons per hour.
2. Second tap: $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$ gallon per hour.
3. $2 \div \frac{1}{2} = 4$, so the first is four times faster.
4. Notice the two rates are reciprocals, because the fractions were swapped. Swapping the amount and the time always gives the reciprocal rate.

Watch for: Said they were the same because the same two fractions appear. Order decides which is divided by which, and that changes everything.

Exit Ticket

4 problems

Use: End of the opening G7 lesson. 5–8 min, independent. Item 4 introduces reciprocal rates, which the “Multi-step ratio problems” lesson leans on. **Timing:** 5–8 min · independent. Any answer SMALLER than the starting amount on items 1–3 is the wrong-way-round error.

1 2 miles per hour 2 $1\frac{1}{2}$ cups per batch 3 4 miles per hour

4 2 gallons per hour; $\frac{1}{2}$ hour per gallon — they are reciprocals

1 **MILD** $\frac{1}{2}$ mile in $\frac{1}{4}$ hour?

2 miles per hour

- Unit rate = amount $\div$ units = $\frac{1}{2} \div \frac{1}{4}$.
- Keep $\cdot$ Change $\cdot$ Flip: $\frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$.
- Sense check: a quarter hour is less than an hour, so the hourly distance must be MORE than $\frac{1}{2}$ mile. $2 > \frac{1}{2}$.

Watch for: Answered $\frac{1}{8}$ by multiplying instead of dividing. The sense check catches it instantly — $\frac{1}{8}$ is smaller than the half mile already run.

2 **MEDIUM** $\frac{3}{4}$ cup for $\frac{1}{2}$ batch?

$1\frac{1}{2}$ cups per batch

- $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$ cups.
- A whole batch is twice a half batch, so the answer must be double $\frac{3}{4}$.

Watch for: Answered $\frac{1}{8}$ by multiplying. A whole batch needs more flour than half a batch, not less.

3 **MEDIUM** $\frac{2}{3}$ mile in $\frac{1}{6}$ hour?

4 miles per hour

- $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$ miles per hour.
- $\frac{1}{6}$ hour is 10 minutes, so this is a brisk pace — 4 mph is reasonable.

Watch for: Answered $\frac{1}{9}$ by multiplying the fractions.

4 **SPICY** $\frac{4}{5}$ gallon in $\frac{2}{5}$ hour — both rates, and how are they related?

2 gallons per hour; $\frac{1}{2}$ hour per gallon — they are reciprocals

- Gallons per hour: $\frac{4}{5} \div \frac{2}{5} = \frac{4}{5} \times \frac{5}{2} = \frac{20}{10} = 2$.
- Hours per gallon: $\frac{2}{5} \div \frac{4}{5} = \frac{2}{5} \times \frac{5}{4} = \frac{10}{20} = \frac{1}{2}$.
- The two multiply to 1: $2 \times \frac{1}{2} = 1$, so each is the reciprocal of the other.
- That makes sense — ‘2 gallons each hour’ and ‘half an hour for each gallon’ describe the same tap.
- Which one you want depends on the question: filling time needs hours per gallon.

Watch for: Gave the same answer twice. The two rates measure the same situation from opposite sides, so they are different numbers unless the rate happens to be 1.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: same unit-rate idea as Grade 6, now with fractions on both sides.
2	1:00	Hook — poll bigger or smaller than $\frac{1}{2}$. Do NOT resolve. Many will say smaller, because dividing ‘should’ shrink.
3	3:00	Set up the division. Recall Keep $\cdot$ Change $\cdot$ Flip explicitly — it was the “Adding & subtracting rational numbers” lesson and it is load-bearing here.
4	5:00	WE1 — resolve the hook. Say the sense check aloud: less than an hour, so further in a full hour.
5	7:00	WE2 — generalise the sense check. This is the single most transferable idea in the lesson.
6	11:00	Reciprocal rates. Ask which one a plumber would want, and which a runner would. Sets up the “Recognizing proportional relationships” lesson.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
9	9:00	CFU — all-student response. Answer 4 mph. Watch for $\frac{1}{9}$, which is multiplication. Whiteboards: everyone tries Mild, then climbs. These are #1, #3, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.

