

GRADE 7 · GEOMETRY · ANSWER KEY

VOLUME OF COMPOSITE SOLIDS

Today's goal: I can determine the volume of three-dimensional objects composed of cubes and right prisms in mathematical problems and determine the volume of three-dimensional objects composed of cubes and right prisms in real-world problems.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: During the lesson after the deck. Items 3–6 force $V = Bh$ by using a base no $l \times w \times h$ can handle. Item 8 teaches a check that needs no calculation. **Timing:** 15–25 min · pairs or independent · circulate on 6 and 9. Insist on square units for B and cubic for V ; the unit change marks where the multiplication happened.

- 1 36 cubic units 2 108 cubic units 3 22 square units 4 110 cubic units 5 136 cubic units
- 6 108 cubic units — the same as the stack 7 Because it is cut right through, so it spans the full depth
- 8 They added the hole instead of subtracting; 184 exceeds the whole box

1 **MILD** Box $6 \times 3 \times 2$?

36 cubic units

1. $6 \times 3 \times 2 = 36$.

Watch for: Added the edges.

2 **MEDIUM** Stacked total?

108 cubic units

1. $36 + 72 = 108$ cubic units.

Watch for: Multiplied the two volumes.

3 **MEDIUM** L-shaped base from 5×2 and 4×3

22 square units

1. First rectangle: $5 \times 2 = 10$ square units.

2. Second: $4 \times 3 = 12$ square units.

3. Base area $B = 10 + 12 = 22$ square units.

4. Note this is an AREA, so square units — it is not yet a volume.

Watch for: Multiplied the two rectangle areas. The base is made by joining them, so their areas add.

4 **MEDIUM** Prism with that base, height 5

110 cubic units

1. $V = Bh = 22 \times 5 = 110$ cubic units.

2. The base can be any shape at all; once you know its area, the formula is the same.

Watch for: Tried $l \times w \times h$ with the original four numbers. Once the base is irregular, only $V = Bh$ works.

5 **MEDIUM** $8 \times 5 \times 4$ box with a $3 \times 2 \times 4$ hole

136 cubic units

1. Whole box: $8 \times 5 \times 4 = 160$ cubic units.

2. Hole: $3 \times 2 \times 4 = 24$ cubic units.

3. Remaining: $160 - 24 = 136$ cubic units.

Watch for: Added the hole, giving 184.

6 **SPICY** Single box $6 \times 3 \times 6$ — notice?

108 cubic units — the same as the stack

1. $6 \times 3 \times 6 = 108$ cubic units.
2. It matches question 2 exactly.
3. That is because the two stacked boxes share the same 6 by 3 footprint, and their heights 2 and 4 add to 6.
4. Stacking prisms with identical footprints is the same as making one taller prism.
5. This gives a useful check whenever the footprints match — but it does NOT work when they differ.

Watch for: Called it a coincidence. The footprints match and the heights add, so it must come out the same.

7 **SPICY** Why must the hole's third dimension be 4?

Because it is cut right through, so it spans the full depth

1. The box is 4 units deep in that direction.
2. A hole cut 'right through' goes all the way from one face to the opposite one, so it must be 4 units long.
3. If it were shorter it would be a recess, not a hole, and some material would remain behind it.
4. This is why the hole's dimensions are 3, 2 and 4 — two are the cross-section of the hole and the third is fixed by the box.

Watch for: Said the 4 was chosen arbitrarily. It is determined by the box, not by the hole.

8 **EXTRA SPICY** Student answered 184 — error, and why impossible?

They added the hole instead of subtracting; 184 exceeds the whole box

1. Their calculation: $160 + 24 = 184$.
2. The correct operation is subtraction, because material was REMOVED.
3. Why it is impossible without any arithmetic: the solid box is the largest the object could possibly be. Drilling a hole can only make it smaller.
4. So any answer greater than the volume of the whole box is wrong before you check a single multiplication.
5. That check costs nothing and catches the commonest error in the topic.

Watch for: Recomputed carefully and found the same 136 without explaining the impossibility. The question asks for the reasoning, which is the transferable part.

Homework

7 problems + Mr DJ's worked example

Use: Parallels the practice sheet. Item 6 asks for a range check before the arithmetic, which is the same estimation habit built in the "Multi-step problems with rational numbers" lesson applied to geometry. **Timing:** ~15 min at home · independent · review item 7 as the next-day warm-up; the doubled answer is a recognisable fingerprint.

1 42 cubic units

2 112 cubic units

3 96 cubic units

4 112 cubic units

5 $B = 10$; $V = 60$ cubic units

6 The answer must be less than 108 and not much less

7 They used a rectangular base; B is 10, not 20

1 **MILD** Box $7 \times 2 \times 3$?

42 cubic units

1. $7 \times 2 \times 3 = 42$.

Watch for: Added the edges.

2 **MEDIUM** Stacked total?

112 cubic units

1. $42 + 70 = 112$ cubic units.

Watch for: Multiplied the volumes.

3 **MEDIUM** $9 \times 4 \times 3$ box with a $2 \times 2 \times 3$ hole

96 cubic units

1. Whole box: $9 \times 4 \times 3 = 108$ cubic units.
2. Hole: $2 \times 2 \times 3 = 12$ cubic units.
3. Remaining: $108 - 12 = 96$ cubic units.

Watch for: Added the hole, giving 120.

4  **SPICY** Single box $7 \times 2 \times 8$ — why does it match?

112 cubic units

1. $7 \times 2 \times 8 = 112$ cubic units, matching question 2.
2. The two stacked boxes share the same 7 by 2 footprint.
3. Their heights, 3 and 5, add to 8.
4. So stacking them produces exactly one prism 7 by 2 by 8 — the same solid described two ways.
5. This only works because the footprints are identical; with different footprints the pieces must be added separately.

Watch for: Said the numbers happened to work out. The footprints match and the heights add, which forces the result.

5  **SPICY** Triangular prism: base 5, height 4, length 6

B = 10; V = 60 cubic units

1. Base area: the base is a triangle, so $B = (5 \times 4) \div 2 = 10$ square units.
2. $V = Bh = 10 \times 6 = 60$ cubic units.
3. The halving is what distinguishes a triangular prism from a box with the same measurements.

Watch for: Answered 120 by multiplying $5 \times 4 \times 6$ without halving.

6  **SPICY** How to tell it was reasonable early?

The answer must be less than 108 and not much less

1. The whole box is $9 \times 4 \times 3 = 108$ cubic units, which is the maximum possible.
2. Removing material can only reduce it, so the answer must be below 108.
3. The hole is small compared with the box, so the answer should be a little below 108, not dramatically so.
4. 96 fits both expectations.
5. An answer above 108, or one like 20, would be wrong on sight — a range check catches errors an arithmetic check might not.

Watch for: Said you cannot tell until you finish. Bounding the answer before computing is exactly what makes a slip visible.

7  **EXTRA SPICY** Student answered 120 for the triangular prism

They used a rectangular base; B is 10, not 20

1. They computed $5 \times 4 \times 6 = 120$, treating the base as a 5 by 4 rectangle.
2. But the base is a TRIANGLE with base 5 and height 4, so $B = (5 \times 4) \div 2 = 10$ square units.
3. Correct volume: $10 \times 6 = 60$ cubic units.
4. Their answer is exactly double, which is the fingerprint of a missed halving.
5. The lesson: identify the SHAPE of the base before computing its area. $V = Bh$ is the same formula for every prism, but B changes with the base.

Watch for: Said they multiplied in the wrong order. Order does not matter in multiplication; the error is the shape of the base.

Exit Ticket

4 problems

Use: End of the volume lesson. 5–8 min, independent. Item 4 is the $V = Bh$ idea; item 3 rewards checking by a second route.

Timing: 5–8 min · independent. An answer of 120 on item 4 means the base was not treated as a triangle.

1 60 cubic units

2 100 cubic units

3 100 cubic units

4 $B = 6$; $V = 60$ cubic units1 **MILD** Box $5 \times 4 \times 3$?**60 cubic units**

1. $V = l \times w \times h = 5 \times 4 \times 3 = 60$ cubic units.
2. Or $V = Bh$ with $B = 5 \times 4 = 20$ and $h = 3$: $20 \times 3 = 60$.

Watch for: Added the edges, or used square units. Volume is always cubic.2 **MEDIUM** $6 \times 4 \times 5$ box with a $2 \times 2 \times 5$ hole**100 cubic units**

1. Whole box: $6 \times 4 \times 5 = 120$ cubic units.
2. Hole: $2 \times 2 \times 5 = 20$ cubic units.
3. Remaining: $120 - 20 = 100$ cubic units.
4. The hole runs the full depth of 5, which is why its third dimension matches the box.
5. Sanity check: the answer must be less than 120.

Watch for: Added the hole, giving 140, which is more than the solid box. Anything removed must reduce the volume.3 **SPICY** Stack a $5 \times 4 \times 2$ on top**100 cubic units**

1. Second box: $5 \times 4 \times 2 = 40$ cubic units.
2. Total: $60 + 40 = 100$ cubic units.
3. Check another way: the two boxes have the same 5 by 4 footprint, so together they form a single box 5 by 4 by 5.
4. $5 \times 4 \times 5 = 100$. Same answer.
5. The second method works only because the footprints match — with different footprints you must add the pieces.

Watch for: Added the heights AND the volumes, double counting. Pick one method: add the volumes, or combine into one solid.4 **SPICY** Triangular prism, base 4, height 3, length 10 **$B = 6$; $V = 60$ cubic units**

1. First the base area: it is a triangle, so $B = (4 \times 3) \div 2 = 6$ square units.
2. Then $V = Bh = 6 \times 10 = 60$ cubic units.
3. Not $4 \times 3 \times 10 = 120$ — that treats the base as a rectangle and forgets the halving.
4. B is an AREA, so it always carries square units, and multiplying by a length gives cubic units.

Watch for: Answered 120 by multiplying all three numbers. The triangular base must be halved before it is used.**Lesson slides**

teaching notes & timing

Slide	Time	What to do
1	0:00	Frame it: one formula for every prism. The work is finding the base area.
2	1:00	Hook — poll 60 vs 120. Do NOT resolve. 120 wins, because three numbers invite three-way multiplication.
3	2:30	WE1 — base area FIRST, as a separate written step. Square units for B , cubic for V .
4	5:00	Name the fingerprint: exactly double means a halving was missed.
5	7:00	Composite solids. The same-footprint shortcut is useful but say clearly when it does NOT apply.
6	11:00	Holes. Teach the bound: the whole solid is the maximum, so anything larger is wrong on sight.
7		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
8		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.

9	9:00	CFU — all-student response. 110 cubic units. The L-base is chosen so $l \times w \times h$ is unusable. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #6 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	13:30	Launch the in-class practice sheet. Launch the practice sheet. Exit ticket in the last 5–8 minutes.