

GRADE 8 · GEOMETRY · ANSWER KEY

ANGLES IN TRIANGLES AND PARALLEL LINES

Today's goal: I can use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Items 4 and 5 are the standard's actual demand — establish, not state — and both are marked on the argument. For item 4, a torn paper triangle is good evidence and is not an argument, and saying so kindly is worth doing. Item 7 is the sharpest item on the sheet: the perpendicular case is where the co-interior error gives the RIGHT answer, so a class that practises only on right angles never meets the rule that differs. **Timing:** 15–20 min · pairs · items 4, 5 and 7 on the board.

- 1 60°
- 2 alternate 40°, corresponding 40°, co-interior 140°
- 3 130°, 120°, 110° — and they add to 360°
- 4 draw a line through the apex parallel to the base; the two outer angles at the apex equal the base angles by alternate angles, and all three angles at the apex lie on a straight line
- 5 the exterior angle and its neighbour make 180, and the three interior angles make 180, so subtracting the neighbour from each leaves the exterior angle equal to the other two interior angles
- 6 because the third angle is forced — the three must add to 180
- 7 all three are 90°, and it is misleading because the co-interior pair is equal here by coincidence
- 8 no — they are similar, and could be any size

1 **MILD** Third angle

60°

1. $180 - 35 - 85 = 60^\circ$.

Watch for: Added the two given angles and stopped at 120.

2 **MEDIUM** A 40° transversal

alternate 40°, corresponding 40°, co-interior 140°

1. $180 - 40 = 140$ for the co-interior angle.

Watch for: Gave 40° three times.

3 **MEDIUM** Three exterior angles

130°, 120°, 110° — and they add to 360°

1. At the 50° vertex: $180 - 50 = 130^\circ$, and the opposite interiors are $60 + 70 = 130^\circ$.

2. At the 60° vertex: 120° , and $50 + 70 = 120^\circ$.

3. At the 70° vertex: 110° , and $50 + 60 = 110^\circ$.

4. Total: $130 + 120 + 110 = 360^\circ$. Each exterior angle uses two interior angles, so the total counts every interior angle twice:
 $3 \times 180 - 180 = 360^\circ$.

Watch for: Gave the interior angles again, or found one exterior angle and assumed the others matched. The 360° total is worth noticing — it is the same for every triangle.

4 **SPICY** Why the angles make 180°

draw a line through the apex parallel to the base; the two outer angles at the apex equal the base angles by alternate angles, and all three angles at the apex lie on a straight line

1. Draw the parallel line through the top vertex.
2. Each base angle has an alternate angle at the apex, so those two angles are copies of the base angles.
3. The three angles at the apex sit on a straight line, so they add to 180° .
4. Those three angles are the triangle's three angles, so the triangle's angles add to 180° .

Watch for: Said “because they always do”, or tore up a paper triangle. Tearing is good evidence and it is not an argument — the standard asks you to ESTABLISH the fact, and the parallel line is what does that.

5 **SPICY** The exterior angle, derived

the exterior angle and its neighbour make 180, and the three interior angles make 180, so subtracting the neighbour from each leaves the exterior angle equal to the other two interior angles

1. Exterior + neighbouring interior = 180 (straight line).
2. The three interior angles = 180 (angle sum).
3. So exterior + neighbour = neighbour + other two.
4. Take the neighbour off both sides: exterior = the other two interior angles.
5. So it is the angle sum and a straight line, not a new fact.

Watch for: Verified it with numbers only. A numerical check is worth doing and it does not establish the general fact — the two-180s argument does, and it also explains why the rule says OPPOSITE angles.

6 **SPICY** Why two angles suffice

because the third angle is forced — the three must add to 180

1. If two angles match, the third is 180 minus their sum in both triangles, so it matches too.
2. All three angles equal means the same shape.
3. So checking a third would be checking something already known.
4. Note what this does NOT give you: the size. Hence similar rather than congruent.

Watch for: Said the third is “probably” the same. It is forced, not probable, and the angle sum is what forces it.

7 **SPICY** A perpendicular transversal

all three are 90° , and it is misleading because the co-interior pair is equal here by coincidence

1. Alternate: 90° . Corresponding: 90° . Co-interior: $180 - 90 = 90^\circ$.
2. So in this one case the co-interior angles ARE equal.
3. That happens only when both are 90° , since it needs a number equal to 180 minus itself.
4. A class that practises only on perpendicular transversals would never see the co-interior rule differ from the other two — which is part of why the error survives.

Watch for: Said the case is just easy. It is easy, and it is also the one case where the common error gives the right answer, which is a different and more useful observation.

8 **EXTRA SPICY** Three 60° angles

no — they are similar, and could be any size

1. Both are equilateral, so both have the same shape.
2. An equilateral triangle of side 2 and one of side 200 both have three 60° angles.
3. Angles never determine size.

Watch for: Answered yes because “all equilateral triangles are the same”. They are all the same SHAPE, which is similarity.

Use: Homework after the practice sheet. Item 3 runs the exterior angle fact backwards, which separates children who know the relationship from those who have memorised one direction of it — and the angle-sum check confirms all three answers. Item 6 is worth a comment when marking: a child who computes -10° has done the arithmetic correctly and drawn the wrong conclusion, and the useful move is reading a negative angle as proof of impossibility. Item 8 asks for the standard's own reasoning. **Timing:** 20–25 min · independent · items 3, 6 and 8 first when marking.

1 alternate 72° , co-interior 108°

2 third angle 60° , exterior angle at 95° is 85°

3 the other opposite interior angle is 75° , and the third is 60°

4 because a co-interior angle lies on a straight line with the alternate angle, and two angles on a straight line add to 180 — so they are only equal if both are 90

5 similar — the second triangle's third angle must be 80°

6 no — that is already 190° , more than the whole angle sum

7 45° , 45° and 90° — and no, the size is not fixed

8 because an argument can be rebuilt when the fact is forgotten, and it tells you which cases the fact applies to

1 **MEDIUM** A 72° transversal

alternate 72° , co-interior 108°

1. $180 - 72 = 108^\circ$.

Watch for: Gave 72° for both.

2 **MEDIUM** Third and exterior angle

third angle 60° , exterior angle at 95° is 85°

1. $180 - 25 - 95 = 60^\circ$.

2. Exterior at the 95° vertex: $180 - 95 = 85^\circ$.

3. Check: the opposite interior angles are $25 + 60 = 85^\circ$.
Agreed.

Watch for: Gave the exterior angle as 95° . The check against $25 + 60$ catches it.

3 **MEDIUM** Working backwards

the other opposite interior angle is 75° , and the third is 60°

1. The exterior angle equals the two opposite interior angles: $120 = 45 + \text{the other}$, so the other is 75° .

2. The third interior angle is the one adjacent to the exterior angle: $180 - 120 = 60^\circ$.

3. Check the angle sum: $45 + 75 + 60 = 180^\circ$.

Watch for: Subtracted 45 from 180, or gave only one angle. Two of the three angles come from the exterior angle fact and the third from the straight line, and the angle-sum check confirms all three.

4 **SPICY** Why co-interior are not equal

because a co-interior angle lies on a straight line with the alternate angle, and two angles on a straight line add to 180 — so they are only equal if both are 90

1. The alternate angle equals the original angle.

2. The co-interior angle and that alternate angle sit on a straight line, so they add to 180° .

3. Equal would mean each is 90° , since a number equal to 180 minus itself must be 90.

4. So for any other transversal angle the two differ, and calling them equal makes the straight line add to less or more than 180.

Watch for: Restated the rule. The question asks why, and the straight-line argument is both the reason and the recovery route when the rule is forgotten.

5 **SPICY** 40-60-80 and 40-60

similar — the second triangle's third angle must be 80°

1. $180 - 40 - 60 = 80^\circ$, so the second triangle has the same three angles.
2. Two matching angles force the third, which is the angle-angle criterion.
3. Equal angles give the same shape, so similar.

Watch for: Said there is not enough information. The angle sum supplies the missing angle, which is exactly why two angles are enough.

6 **SPICY** Two angles of 95° ?

no — that is already 190° , more than the whole angle sum

1. $95 + 95 = 190^\circ$.
2. The three angles must total 180° , and 190 already exceeds it before the third angle is counted.
3. So no such triangle exists.

Watch for: Tried to find the third angle and gave -10° . The arithmetic is right and the conclusion is that the triangle is impossible, not that an angle is negative.

7 **SPICY** Two equal and a right angle

45° , 45° and 90° — and no, the size is not fixed

1. The two equal angles share $180 - 90 = 90^\circ$, so each is 45° .
2. Every triangle with these angles has the same shape.
3. Any size is possible: legs of 1 and 1, or 100 and 100.
4. So the angles fix the shape and leave the size entirely open — similar, not congruent.

Watch for: Gave the angles and said the triangle is fixed. The second half of the question is the point.

8 **EXTRA SPICY** Why establish rather than learn

because an argument can be rebuilt when the fact is forgotten, and it tells you which cases the fact applies to

1. Five separate rules is five things to forget, and there is no way to check a half-remembered one.
2. One fact plus straight lines lets you rebuild the other four in a step each.
3. It also protects against over-generalising: someone who has ARGUED the co-interior case knows it comes from a straight line, so they never reach for “equal”.
4. And it is how the facts were found in the first place — a rule handed over with no reason is a rule you cannot extend to an unfamiliar diagram.

Watch for: Said “so you understand it”. True and vague. The concrete answers are recoverability and knowing the limits of the rule — the co-interior error is the exact thing an argument prevents.

Exit Ticket

4 problems

Use: End of the angle-facts lesson. Item 1 is the diagnostic and item 3 is what makes it useful: a child who answers 65° three times has applied a pattern that genuinely fits two of the three named pairs, so this is over-generalisation rather than carelessness. Mark item 3 on the ARGUMENT — “because co-interior angles add to 180” restates the fact the question is asking them to establish. The strongest answers notice that 65 and 65 make 130 where a straight line needs 180, so the wrong answer refutes itself on the page. Item 4 carries the similarity distinction forward. **Timing:** 5–8 min · independent · protractor not needed.

1 alternate 65° , corresponding 65° , co-interior 115°

2 third angle 70° , exterior angle 110°

3 the co-interior angle sits on a straight line with the alternate angle, so it is $180 - 65 = 115^\circ$

4 similar — the angles fix the shape but not the size

1 **MEDIUM** Three angles at a transversal

alternate 65° , corresponding 65° , co-interior 115°

1. Alternate angles are equal, so 65° .
2. Corresponding angles are equal, so 65° .
3. Co-interior angles add to 180° , so $180 - 65 = 115^\circ$.

Watch for: Gave 65° for all three. Two of the three named pairs ARE equal, so the pattern fits two cases out of three — which is exactly why the third is the one that gets missed.

2 **MEDIUM** Third angle and exterior angle

third angle 70° , exterior angle 110°

1. Angles sum to 180: $180 - 50 - 60 = 70^\circ$.
2. The exterior angle and the 70° angle lie on a straight line: $180 - 70 = 110^\circ$.
3. Check with the exterior angle fact: $50 + 60 = 110^\circ$. The two routes agree.

Watch for: Gave the exterior angle as 70° , taking it to be the interior angle at that vertex, or as 180° , adding all three. Checking against $50 + 60$ catches both instantly.

3 **SPICY** Correct Nadia

the co-interior angle sits on a straight line with the alternate angle, so it is $180 - 65 = 115^\circ$

1. The alternate angle is 65° .
2. The co-interior angle and that alternate angle together lie on a straight line.
3. Angles on a straight line add to 180° , so the co-interior angle is 115° .
4. Her answer also contradicts itself: 65 and 65 make 130, and those two angles are supposed to make a straight line, which is 180.

Watch for: Said “because co-interior angles add to 180” and stopped. That is the fact, not the argument, and the question asks for the argument — which is also what makes the fact recoverable when it is forgotten. The 130-instead-of-180 observation is the strongest form of the answer.

4 **SPICY** Congruent, similar, or unknown

similar — the angles fix the shape but not the size

1. Equal angles mean the triangles have the same shape.
2. Nothing in the angles says how big either triangle is: a 3-4-5 triangle and a 9-12-15 triangle have identical angles.
3. So they are similar, and they may or may not also be congruent — there is no way to tell from angles alone.
4. This is the angle-angle criterion, and it gives similarity rather than congruence for exactly this reason.

Watch for: Answered congruent. Equal angles is the definition of same shape, and size is a separate question — which is the whole distinction from the similarity skill.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Say the design out loud: one fact, four arguments. Five separate rules is five ways to forget; one rule plus straight lines is recoverable under pressure.
2	0:45	Alternate angles as the single starting point. Draw it large.
3	1:30	Build the angle sum. The parallel line through the apex is the move worth rehearsing — the standard says establish, and this is what establishing looks like. Ask what the argument would be without the parallel line.
4	3:00	Exterior angle, derived from two 180s rather than stated. The same subtraction gives it, so it is not a fifth fact to hold.
5	4:15	Name the three pairs, and name the trap immediately: two are equal, one is not. Children generalise from two cases out of three, which is reasonable and wrong.
6	5:15	THE slide. Rebuild co-interior from the alternate angle and a straight line. Then show what the error costs: 65 and 65 make 130, and a straight line is 180, so the answer contradicts itself on the page. Also mention the perpendicular case — when the transversal is at 90° both are 90 and the error is invisible, which is part of why it survives.

7	6:45	AA. Two angles are enough because the third follows. Then the important half: same angles means similar, not congruent. Use the 3-4-5 and 9-12-15 pair from the similarity work.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	8:00	The 65° transversal, then the exit ticket. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #4 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.