



Comparing functions

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the comparing lesson. 5–8 min, independent. Items 1 and 2 have DIFFERENT answers deliberately: a child who gives the same function for both has answered “which is bigger” rather than the question asked. Item 3 is where the misconception is named rather than just corrected.

Timing: 5–8 min · independent.

1. Greater rate: A or B? APPLICATION

Answer: **A — a rate of 3 against 2**

1. A: the coefficient of x is the rate, so 3.
2. B: the rate is the STEP between outputs. $5 \rightarrow 7 \rightarrow 9 \rightarrow 11$, so 2 each time.
3. $3 > 2$, so A rises faster.

If wrong: Compared 3 with 11, or with 5. Those are an OUTPUT and a STARTING VALUE; neither is a rate. The table never printed its rate, which is why it has to be worked out before anything can be compared.

2. Greater starting value? FLUENCY

Answer: **B — 5 against 2**

1. A: the constant is 2, so it starts at 2.
2. B: the output at $x = 0$ is 5.
3. $5 > 2$.

If wrong: Answered A, carrying over the item 1 result. The two questions have DIFFERENT answers on purpose — that is the point of asking both.

3. Correct Sam REASONING

Answer: **he compared an output with a rate**

1. B's 11 is an OUTPUT — the value when $x = 3$. A's 3 is a RATE — how much it climbs per step.
2. Those are different kinds of quantity, so the comparison means nothing.
3. A at $x = 3$ is $3(3) + 2 = 11$. Identical to B. They are equal there, and A is ahead after it: at $x = 5$, A gives 17 and B gives 15.
4. “Bigger function” is not a property. Bigger rate, bigger start, or bigger at a particular x — those are three different questions.

If wrong: Agreed, or corrected the answer without naming that two DIFFERENT KINDS of quantity were compared. That naming is the skill; the arithmetic is not the difficulty here.

4. Translate the sentence APPLICATION

Answer: **$y = -4x + 10$, rate -4**

1. “Starts at 10” is the value at $x = 0$, so the constant is 10.
2. “Falls 4 each hour” is the rate, and falling makes it negative: -4 .
3. So $y = -4x + 10$.
4. Check: after 2 hours, $-4(2) + 10 = 2$.

If wrong: Wrote $y = 4x + 10$, dropping the sign, or $y = 10x - 4$, swapping the roles. Verbal is the form children translate worst, because nothing in the sentence is labelled.

Quick answers for fast checking:

#	Level	Answer
1	application	A — rate 3 against 2
2	fluency	B — 5 against 2
3	reasoning	he compared an output with a rate
4	application	$y = -4x + 10$, rate -4

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 7 is the one to discuss: a greater rate wins EVENTUALLY, not everywhere, and a child who answers Q for both has replaced the question with the coefficient. Item 4 has both answers agreeing on purpose, so the exit ticket's disagreement lands.

Timing: 15–20 min · pairs · items 5 and 7 on the board.

1. $y = 5x - 3$ **FLUENCY**

Answer: **rate 5, starts at -3**

1. Coefficient of x is the rate; the constant is the start.

If wrong: Swapped them, or dropped the minus.

2. Table 4, 10, 16 **FLUENCY**

Answer: **rate 6, starts at 4**

1. The step is $10 - 4 = 6$, and again $16 - 10 = 6$.
2. At $x = 0$ the output is 4.

If wrong: Gave the rate as 4 or 10 — reading a printed output as the rate, which is the whole error of this skill.

3. Graph through (0,1) and (4,21) **APPLICATION**

Answer: **rate 5, starts at 1**

1. Rate = change in y over change in $x = \frac{21-1}{4-0} = 5$.
2. At $x = 0$, $y = 1$.

If wrong: Inverted the ratio to $\frac{4}{20}$, or read 21 as the rate.

4. Compare all three **APPLICATION**

Answer: **greatest rate: the table (6); starts highest: the table (4)**

1. Rates: 5, 6, 5.
2. Starts: -3, 4, 1.
3. The table wins both here — which happens, and does not make “bigger” a property.

If wrong: Picked one function as generally biggest without separating the two questions. Here they agree; on the exit ticket they do not, and that contrast is the lesson.

5. Why printed numbers do not compare REASONING

Answer: **an explanation that each form shows different quantities, so the visible numbers may be a rate in one and an output in the other**

1. An equation prints its rate. A table prints outputs and hides the rate.
2. So a number from one and a number from the other may not be the same kind of thing.
3. Comparing a rate with an output is like comparing a speed with a distance.
4. Translate both to rate and start, and then the numbers mean the same thing.

If wrong: Said “because they look different”. The speed-versus-distance comparison is what makes it land.

6. Tank draining APPLICATION

Answer: **$y = -5x + 60$**

1. Starts at 60, so the constant is 60.
2. Drains 5 a minute, so the rate is -5 .

If wrong: Wrote $y = 5x + 60$, which fills the tank instead of emptying it. Check by asking what happens after 12 minutes: 0 is right, 120 is not.

7. P and Q at $x = 1$ and $x = 4$ REASONING

Answer: **P at $x = 1$ (11 against 7); Q at $x = 4$ (25 against 17)**

1. P: $2(1) + 9 = 11$ and Q: $6(1) + 1 = 7$, so P leads at $x = 1$.
2. P: $2(4) + 9 = 17$ and Q: $6(4) + 1 = 25$, so Q leads at $x = 4$.
3. Q has the greater rate, so it overtakes — but not immediately.
4. “Which is bigger” depends on WHERE, which is the answer this item exists for.

If wrong: Answered Q for both, reasoning from the bigger coefficient alone. A greater rate wins eventually, not everywhere.

8. Two graphs, different scales REASONING

Answer: **the one whose axis goes up in 10s**

1. Equally steep on the page means equal numbers of SQUARES per step.
2. One square is 1 unit on the first and 10 on the second.
3. So the second climbs ten units where the first climbs one.
4. Read the axes before comparing the pictures.

If wrong: Said they are equal, comparing the pictures. Fifth appearance of the axis-scale demand, and the first where two diagrams are compared rather than one read.

9. A table and a sentence for the same function APPLICATION

Answer: **any matching pair, for example $x: 0,1,2 \rightarrow y: 3,8,13$ with “starts at 3 and rises 5 each step”**

1. Pick a rate and a start.
2. Build the table from them, then say the same thing in words.

If wrong: Wrote a table and a sentence that disagree — usually the sentence's starting value does not match the output at $x = 0$. Building both is much harder than reading either, and the mismatch is the useful part.

Quick answers for fast checking:

#	Level	Answer
1	fluency	rate 5, start -3
2	fluency	rate 6, start 4
3	application	rate 5, start 1
4	application	the table on both counts
5	reasoning	each form shows different quantities
6	application	$y = -5x + 60$
7	reasoning	P at $x = 1$, Q at $x = 4$
8	reasoning	the axis in 10s
9	application	a matching table and sentence

Homework — answer key

Use: Homework after the practice sheet. Item 8 is the skill in one question: over the range shown the graph's outputs ARE larger, and its rate is smaller. A child who answers yes has compared the wrong quantity while looking at the right picture.

Timing: 20–25 min · independent · items 6, 7 and 8 first when marking.

1. $y = -2x + 7$ **FLUENCY**

Answer: **rate -2 , starts at 7**

1. Coefficient, then constant.

If wrong: Dropped the minus.

2. Table 9, 12, 15 **FLUENCY**

Answer: **rate 3 , starts at 9**

1. Step of 3 ; output 9 at $x = 0$.

If wrong: Gave 9 as the rate.

3. Greater rate **APPLICATION**

Answer: **$y = 4x + 1$ — 4 against 3**

1. $4 > 3$.

If wrong: Compared 4 with 15 .

4. Starts higher **APPLICATION**

Answer: **the table — 9 against 1**

1. $9 > 1$.

If wrong: Carried over item 3's answer. The two differ on purpose.

5. Phone battery **APPLICATION**

Answer: **$y = -6x + 80$**

1. Starts at 80 , loses 6 an hour.

2. Check: after 10 hours, $-6(10) + 80 = 20\%$.

If wrong: Wrote $y = 6x + 80$, charging the phone rather than draining it.

6. M and N at $x = 2$ and $x = 10$ REASONING

Answer: **M at $x = 2$ (22 against 10); N at $x = 10$ (50 against 30)**

1. M: $2 + 20 = 22$, N: $5(2) = 10$, so M leads early.
2. M: $10 + 20 = 30$, N: $5(10) = 50$, so N leads later.
3. N's rate is five times M's, so it overtakes — at $x = 5$, where both give 25.

If wrong: Answered N for both. The crossing point is worth finding: before it the head start wins, after it the rate does.

7. Why the question is incomplete REASONING

Answer: **an explanation that a function has several properties — rate, starting value, and value at a particular x — which can rank differently**

1. One function can have the greater rate and the smaller starting value at once.
2. And which is bigger at a given x depends on the x .
3. So the question has to say WHICH property, or WHERE.

If wrong: Said “because you need more information” without naming the properties. Naming them is the answer.

8. Graph against $y = 3x - 1$ APPLICATION

Answer: **no — its rate is 2 against 3**

1. Graph: from $(0,4)$ to $(3,10)$, rise 6 over run 3, so rate 2.
2. $y = 3x - 1$ has rate 3.
3. $2 < 3$, so the graph rises more slowly.
4. Its starting value is higher (4 against -1), which is a different question.

If wrong: Answered yes because the graph's outputs are larger over that range. Larger outputs and a faster rate are different claims, which is this whole skill in one item.

Quick answers for fast checking:

#	Level	Answer
1	fluency	rate -2 , start 7
2	fluency	rate 3, start 9
3	application	$y = 4x + 1$
4	application	the table
5	application	$y = -6x + 80$
6	reasoning	M at $x = 2$; N at $x = 10$

7	reasoning	rate, start and value at an x can rank differently
8	application	no — rate 2 against 3

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Set the routine in one sentence: translate both, then compare. Almost every error on this topic is a comparison made before the translation.
2/8	1:00	Name what each form hides. This is the insight that makes the routine feel necessary rather than fussy.
3/8	2:00	The equation. Rate and start both visible, which is why it feels easy.
4/8	3:00	The table. Get the rate from the STEP, and be explicit that it was never printed — that is exactly why comparing printed numbers fails.
5/8	4:00	THE slide. A wins on rate, B wins on start. Do not resolve it — the point is that the question was underspecified, and children need to feel that.
6/8	5:30	Where they cross. A greater rate wins EVENTUALLY. Worth doing because it stops “bigger rate” becoming “bigger everywhere”.
7/8	7:00	Translating a sentence. Falls means negative; the starting amount is the intercept. Slow down here — verbal is the form children translate worst.
8/8	8:30	Different scales. Fifth time these children have had to read an axis before counting, and the first time two diagrams must be compared. Launch the exit ticket.