

GRADE 8 · GEOMETRY · ANSWER KEY

CONGRUENCE THROUGH TRANSFORMATIONS

Today's goal: I can understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations and given two congruent figures, describe a sequence that exhibits the congruence.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: In-class practice after the deck. Items 2 and 3 build the exit ticket's sequence one step at a time, so a child who could not produce it whole can still get there. Items 6, 7 and 9 are the orientation argument in three forms: a rotation preserving it, an even number of reflections cancelling, and the two-reflection case that makes “even” concrete rather than a rule. Item 8 is worth discussing — the squared-length habit pays off again in Pythagoras. **Timing:** 15–20 min · pairs · squared paper, tracing paper · items 7, 8 and 9 on the board.

1 congruent means some sequence of translations, reflections and rotations carries one figure exactly onto the other

2 $(-1,1)$, $(-4,1)$, $(-1,3)$

3 $(5,3)$, $(2,3)$, $(5,5)$

4 $(-1,1)$, $(-1,4)$, $(-3,1)$

5 not congruent — every side is twice as long

6 congruent, and the orientation is unchanged

7 the sequence needs an EVEN number of reflections — zero is the usual answer

8 because they stay whole numbers, so the comparison is exact

9 the figure returns to exactly where it started — a translation of zero

1 **MILD** The definition

congruent means some sequence of translations, reflections and rotations carries one figure exactly onto the other

1. The sequence is what does the work — congruence is the existence of one.
2. “Looks the same” is not the definition and cannot be checked.

Watch for: Wrote “the same shape and size” only. True, and it is the consequence rather than the definition the standard asks for — and it gives you nothing to DO.

2 **MEDIUM** Reflect in the y-axis

$(-1,1)$, $(-4,1)$, $(-1,3)$

1. Across the y-axis, x changes sign and y stays.
2. The axis you reflect in is the coordinate that survives.

Watch for: Flipped y instead of x, which is the x-axis reflection.

3 **MEDIUM** Then translate

$(5,3)$, $(2,3)$, $(5,5)$

1. Add 6 to every x and 2 to every y.
2. This is PQR from the exit ticket, reached in two steps.

Watch for: Applied the translation to the ORIGINAL rather than to the reflected image. A sequence is applied in order.

4 **MEDIUM** Rotate 90° anticlockwise

$(-1,1)$, $(-1,4)$, $(-3,1)$

1. Anticlockwise about the origin sends (x, y) to $(-y, x)$.
2. $(1,1) \rightarrow (-1,1)$. $(4,1) \rightarrow (-1,4)$. $(1,3) \rightarrow (-3,1)$.

Watch for: Swapped the coordinates without the sign change, or turned it clockwise.

5 **MEDIUM** $(2,2)$, $(8,2)$, $(2,6)$

not congruent — every side is twice as long

1. Squared sides of the original: 4, 9, 13. Of this one: 16, 36, 52.
2. Each is four times as large, so each length is doubled, since 4 is 2 squared.
3. Same shape, different size: similar, not congruent.

Watch for: Said congruent because it is the same shape. Same shape is similar; congruent needs the size too.

6 **SPICY** Congruent, and orientation

congruent, and the orientation is unchanged

1. Squared sides are 4, 9 and 13 before and after, so congruent.
2. A rotation preserves orientation, so the corners still run anticlockwise.
3. So this pair could be matched with NO reflection at all — unlike the exit ticket's pair.

Watch for: Said the orientation reversed because the triangle looks different. It looks different because it turned; reading the corners round is what settles it.

7 **SPICY** Same direction

the sequence needs an EVEN number of reflections — zero is the usual answer

1. Slides and turns preserve orientation; each reflection reverses it.
2. Same direction means the reversals cancelled, so there is an even number of them.
3. Zero is even, so a sequence of slides and turns alone is normally enough.

Watch for: Said no reflections are allowed. Two reflections are fine — they cancel — and item 9 is that case.

8 **SPICY** Why squared lengths

because they stay whole numbers, so the comparison is exact

1. This triangle's third side is the square root of 13, which has no exact decimal.
2. Comparing 13 to 13 is certain; comparing 3.6055... to 3.61 is a rounding argument.
3. Rigid motions preserve length, so they preserve squared length — the test is just as valid and much cleaner.
4. This is the same reasoning that makes Pythagoras easier to check in squares than in lengths.

Watch for: Said it is quicker. It is, but the real reason is exactness — a rounded decimal can make two equal lengths look different.

9 **EXTRA SPICY** Reflecting twice in the same line

the figure returns to exactly where it started — a translation of zero

1. The first reflection reverses orientation; the second reverses it back.
2. Every point returns to its original position.
3. So the pair of reflections together is the identity, which is a translation by the zero vector.
4. Which is why an EVEN number of reflections behaves like no reflection at all, as far as orientation is concerned.

Watch for: Said the shape ends up mirrored. One reflection mirrors it; the second undoes that. Tracing paper settles it in a few seconds.

Homework

8 problems

Use: Homework after the practice sheet. Item 4 rewards checking whether ONE motion suffices before assembling a sequence — every corner shifts identically, so a translation does it alone. Item 7 is the one most likely to be answered wrongly

with confidence, and comparing two students' valid sequences settles it faster than any explanation; the parity is the part that IS fixed. Item 8 reaches forward to similarity in the “Similarity and dilations” lesson. **Timing:** 20–25 min · independent · squared paper · items 4, 7 and 8 first when marking.

- 1 (1,3), (4,3), (1,1) 2 yes — both have squared sides 4, 9 and 13 3 similar, not congruent
 4 a translation 3 right and 4 up 5 no — an even number of reflections, and zero is the usual case
 6 because a rotation needs a centre and an angle before anyone can carry it out 7 no — there are usually many
 8 because it gives a test you can run and a reason the figures match, rather than a list of measurements that happen to agree

1 **MEDIUM** Reflect then translate

(1,3), (4,3), (1,1)

1. Reflecting in the x-axis: $(2,1) \rightarrow (2,-1)$, $(5,1) \rightarrow (5,-1)$, $(2,3) \rightarrow (2,-3)$.
2. Then 1 left and 4 up: $(1,3)$, $(4,3)$, $(1,1)$.
3. Order matters — do the reflection first, as written.

Watch for: Translated first, or added 1 instead of subtracting. Left is negative.

2 **MEDIUM** Congruent?

yes — both have squared sides 4, 9 and 13

1. First: $(1,2)-(3,2)$ is 4; $(1,2)-(1,5)$ is 9; $(3,2)-(1,5)$ is $4+9 = 13$.
2. Second: $(4,6)-(6,6)$ is 4; $(4,6)-(4,9)$ is 9; $(6,6)-(4,9)$ is $4+9 = 13$.
3. The same three squared lengths, so congruent.

Watch for: Compared only one side, or measured with a ruler and rounded. All three lengths are needed, and squares keep it exact.

3 **MEDIUM** 5-12-13 and 10-24-26

similar, not congruent

1. Every side of the second is exactly twice the first, so the shape is the same.
2. The sizes differ, so no rigid motion maps one to the other.
3. Same shape only is similar.

Watch for: Said congruent because both are right triangles. Being the same shape is what similar means.

4 **SPICY** A single motion

a translation 3 right and 4 up

1. Each corner moves the same way: $(1,2) \rightarrow (4,6)$ is 3 right and 4 up, and so are the others.
2. Because every point moves by the same vector, one translation does it — no sequence needed.
3. The orientations match, which is consistent: a translation cannot reverse it.

Watch for: Gave a reflection and a translation. Check whether ONE motion suffices before building a sequence — here every corner shifts identically.

5 **SPICY** Clockwise to clockwise

no — an even number of reflections, and zero is the usual case

1. Orientation is unchanged, so any reflections present must cancel in pairs.
2. Zero reflections is the simplest answer, so slides and turns alone will normally do it.
3. But two reflections would also be allowed, so “must not contain one” would be too strong.

Watch for: Said no reflections are permitted. Two are, and saying so is the precise answer.

6 **SPICY** Why “rotate it” is incomplete

because a rotation needs a centre and an angle before anyone can carry it out

1. Rotating 90° about the origin and 90° about a corner give different images.
2. So “rotate it” does not identify one transformation — it names a family of them.
3. The test of a description is whether another person can execute it without asking a question.

Watch for: Said it is not detailed enough, without saying which details. Naming the centre and the angle is the answer, and the two-different-images argument is what shows they are needed.

7 **SPICY** Always exactly one?

no — there are usually many

1. The exit ticket's pair can be done as a reflection then a translation, or a translation then a reflection with a different mirror line.
2. Any sequence that lands correctly is valid; the standard asks for A sequence, not THE sequence.
3. What IS fixed is the parity: the number of reflections is always odd, or always even, for a given pair.
4. So different answers can both be right, while the orientation constraint holds for all of them.

Watch for: Said yes. Worth checking against a classmate's answer — two different valid sequences in one room settles it faster than any explanation.

8 **EXTRA SPICY** Why transformations

because it gives a test you can run and a reason the figures match, rather than a list of measurements that happen to agree

1. Measuring every side and angle can only report that the numbers agree; it never says why.
2. A sequence of motions shows the figures are the SAME figure in two positions.
3. It also extends: the same idea with dilation gives similarity, which is the next skill.
4. And measuring has a practical problem — a rounded measurement can make equal lengths look different, which is why squared lengths are used here.

Watch for: Said measuring is slower. Speed is not the reason; the transformation definition explains the congruence rather than merely detecting it.

Exit Ticket

4 problems

Use: End of the congruence lesson, following rigid motions. Item 3 is the diagnostic and it is the whole method: orientation decides whether a reflection is needed, before any motion is attempted. A child who does not have it will slide and turn a reversed triangle and then conclude the figures are not congruent — a wrong answer arrived at by careful work, which is why item 1 asks for the reasoning rather than the verdict. Item 2 is marked on whether another person could actually follow it.

Timing: 5–8 min · independent · squared paper, tracing paper.

- 1 yes — the side lengths match exactly
- 2 reflect in the y-axis, then translate 6 right and 2 up
- 3 that the sequence must contain an odd number of reflections
- 4 they are similar, not congruent — congruent needs the same size as well as the same shape

1 **MEDIUM** Congruent?

yes — the side lengths match exactly

1. Squared side lengths of ABC: 9, 4 and 13.
2. Squared side lengths of PQR: 9, 4 and 13.
3. The same three lengths, so one can be carried onto the other.
4. Comparing SQUARED lengths avoids the irrational 13 entirely — exact arithmetic instead of a decimal that has to be rounded.

Watch for: Said no because the triangles point different ways. Orientation differing does not stop them being congruent — it only tells you the sequence needs a reflection.

2 **SPICY** Describe a sequence

reflect in the y-axis, then translate 6 right and 2 up

1. Reflecting (1,1), (4,1), (1,3) in the y-axis gives (−1,1), (−4,1), (−1,3).
2. Translating each by 6 right and 2 up gives (5,3), (2,3), (5,5), which is PQR exactly.
3. Other sequences work too — any answer with an odd number of reflections that lands correctly is right.
4. The test of a description is whether another person could execute it without asking a question.

Watch for: Wrote “reflect it then slide it”. A reflection without its mirror line and a translation without its vector cannot be carried out, and the standard asks for a description, not a list of motion names.

3

SPICY

What the reversal tells you

that the sequence must contain an odd number of reflections

1. Translations and rotations both preserve orientation, every time.
2. So no sequence of only slides and turns can reverse it, however long.
3. A reversed image therefore requires a reflection — one, or three, or any odd number.
4. This is worth reading FIRST: it decides what your sequence must contain before you try anything.

Watch for: Said it means they are not congruent, or that it makes no difference. Both miss that orientation is a decision procedure — it narrows the search instead of ending it.

4

EXTRA SPICY

Correct Maya

they are similar, not congruent — congruent needs the same size as well as the same shape

1. 6, 8, 10 is exactly twice 3, 4, 5, so the shape is identical and the size is not.
2. Rigid motions never resize a figure, so no sequence of them can turn one into the other.
3. Same shape only is **similar**. Congruent means same shape AND same size.

Watch for: Agreed with her. “Congruent” sounding like “matching” is doing the damage — a scaled copy does match in shape, so the word invites exactly this error.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Open with the definition as a TEST rather than a property. Congruent is not a way two figures look, it is a claim that a sequence exists — and that is a claim you can settle.
2	1:00	Draw two congruent figures in different positions and orientations. Ask what would settle it. Steer away from “they look the same”.
3	2:00	Sequences. Most pairs need two motions. Order is the student's choice and different sequences can both be right — worth saying, because children expect one answer.
4	3:00	THE slide. Label corners, read them round, compare directions. This is the whole method and it takes one glance.
5	4:30	Why it works, and it needs the reason rather than the rule: slides and turns cannot reverse orientation, so if the image is reversed the reflection is not optional. A child without this slides and turns a reversed triangle for ten minutes and then says “not congruent” — a wrong answer from correct-looking work.
6	6:00	Parameters. Have one student describe a sequence and another EXECUTE it literally. “Reflect it” stalls immediately, and the class sees why the mirror line is part of the answer rather than a detail.
7	7:00	Congruent vs similar, briefly. Dilation arrives in the “Similarity and dilations” lesson with its own word; here it is enough that rigid motions do not resize, so a bigger copy is out.
9	8:00	The pair from the exit ticket. Orientation first, then a followable sequence. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11		Launch the practice sheet. Exit ticket in the last 5–8 minutes.