



Fitting and using a line

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the fitting-a-line lesson. Items 1 and 2 are the standard's actual demand and the most-skipped part of this topic: an interpretation carries two units, so “the slope is 8” earns nothing. Item 4 is the subtlest — the fit is CLOSE and it is also systematically high, so a child who checks only the size of the gaps calls it good. Both sides of the line, all the way along, is the test. Item 3 rewards judging a prediction rather than producing it; 345 out of 100 is the model announcing its own limits.

Timing: 5–8 min · independent · calculator allowed.

1. Interpret the slope **REASONING**

Answer: **for each extra hour of revision, the model predicts about 8 more marks**

1. The slope measures how much y changes for a one-unit change in x .
2. Here x is hours and y is marks, so the slope is 8 marks per hour.
3. Saying “the slope is 8” is not an interpretation — a rate needs both units to mean anything.
4. So three extra hours would predict 24 more marks.

If wrong: Wrote “the slope is 8” or “it goes up by 8”. Neither names what is going up or per what, and the units are what turn the number into something a student could act on.

2. Interpret the intercept REASONING

Answer: **25 marks — the score the model predicts for a student who did no revision at all. It is meaningful here**

1. The intercept is the value of y when $x = 0$.
2. $x = 0$ means zero hours of revision, which is a real possibility, so the intercept describes something.
3. 25 marks is between 0 and 100, so it is also plausible.
4. It is worth checking both: some intercepts are impossible or describe nothing real.

If wrong: Said “the line starts at 25” without saying what $x = 0$ means. The interpretation is about no revision, not about where the line meets an axis.

3. Two predictions REASONING

Answer: **5 hours gives 65 marks, which is trustworthy; 40 hours gives 345 marks, which is not — it is far outside the data and above the maximum possible score**

1. At 5 hours: $8 \times 5 + 25 = 65$ marks. This is inside the range the line was fitted to, so it is reasonable.
2. At 40 hours: $8 \times 40 + 25 = 345$ marks.
3. The test is marked out of 100, so 345 is impossible — the model has been used far beyond the data it came from.
4. Also, real revision does not keep paying at a constant rate: the pattern is unlikely to stay straight, so even 15 hours would be doubtful.

If wrong: Gave both numbers without judging them, or said 40 is fine because the arithmetic works. The arithmetic always works — the question is whether the model still describes anything, and 345 out of 100 answers it.

4. Every point on or below the line REASONING

Answer: **the line sits slightly too high; lowering it by about 3 marks would put points on both sides**

1. A good informal fit has points scattered on BOTH sides along its whole length.
2. All eight residuals here are zero or negative, so the line is consistently above the data.
3. The gaps are small — at most 7 marks — so the SLOPE looks about right; it is the height that is off.
4. Lowering the intercept from 25 to about 22 gives misses on both sides, which is a better fit.

If wrong: Said the fit is good because the points are close. Closeness is only half of it: a line can be close to every point and still be systematically above them all, which means it will over-predict everywhere.

Quick answers for fast checking:

#	Level	Answer
1	reasoning	8 marks per extra hour of revision
2	reasoning	25 marks predicted for zero revision; meaningful here
3	reasoning	65 trustworthy; 345 impossible and far outside the data
4	reasoning	the line is too high; lower the intercept by about 3

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 6 is the most valuable and the most counter-intuitive: a line can be a good model and have a meaningless intercept, because $x = 0$ sits nowhere near the data. Children want to conclude the line is wrong. Item 7 is the both-sides test in comparative form, and item 3 sets it up by asking for signed distances — the signs are the evidence. Item 8 quantifies what fitting a line to a curve costs: 21 cm, against a worst miss of 7 on data that really was linear.

Timing: 15–20 min · pairs · rulers · items 6 and 7 on the board.

1. Check first FLUENCY

Answer: **whether the pattern is roughly linear**

1. A straight line is only a sensible model for a straight-ish pattern.
2. If the data curves, a line will fit the middle and fail at both ends.

If wrong: Said to check for outliers. Worth doing too, and the shape is the question that decides whether a line is the right tool at all.

2. Two predictions FLUENCY

Answer: **3 hours: 49 marks. 7 hours: 81 marks**

1. $8 \times 3 + 25 = 49$.

2. $8 \times 7 + 25 = 81$.

If wrong: Forgot to add the intercept.

3. Distance from the predictions APPLICATION

Answer: **at 3 hours the actual is 1 mark below; at 7 hours it is 7 marks below**

1. $48 - 49 = -1$.

2. $74 - 81 = -7$.

3. Both actuals are BELOW the line, which is a hint that the line sits high.

If wrong: Gave the distances without signs. The signs are what reveal whether a line is systematically above or below the data.

4. Plant slope REASONING

Answer: **the plant grows about 3 cm per week**

1. The slope is the change in y per one-unit change in x.

2. x is weeks and y is cm, so the rate is 3 cm per week.

If wrong: Wrote “3” or “it grows by 3”. Per week is the missing half.

5. Car slope REASONING

Answer: **the car loses about \$1200 of value per year**

1. A negative slope means y falls as x rises.

2. x is years and y is dollars, so the rate is $-\$1200$ per year.

3. Stating it as a loss of \$1200 per year is the clearest form.

If wrong: Said “it goes down by 1200” without units, or read the minus sign as an error. A negative slope is the normal way to describe something decreasing.

6. An intercept of -15 REASONING

Answer: **no — it would be the shoe size of an adult of height 0 cm, which does not exist, and a negative shoe size is impossible anyway**

1. The intercept is the predicted y when $x = 0$.
2. Here $x = 0$ means a height of 0 cm, which describes no person.
3. And -15 is not a possible shoe size.
4. So the intercept is a feature of the LINE that keeps it in the right place over the real range of heights, and not a meaningful prediction. The line can be perfectly useful with a meaningless intercept.

If wrong: Said the line is wrong. The line may be a good model for adult heights of 150 to 200 cm and still have a meaningless intercept — because $x = 0$ is nowhere near the data.

7. Which fit is better REASONING

Answer: **the one with points on both sides — all points below means the line is systematically too high**

1. A fitted line should pass through the middle of the data, so misses should occur in both directions.
2. If every point is below, the line over-predicts everywhere, however small the gaps.
3. That is a fixable error: move the line down until the misses balance.
4. Closeness alone is not enough — a consistently biased line can still be close.

If wrong: Chose the one with smaller gaps without checking the sides. Small AND balanced is the test; small alone can hide a systematic bias.

8. The line on the curved data APPLICATION

Answer: **it predicts 53 cm at week 8, an over-prediction of 21 cm**

1. Starting at 4 cm and rising 7 cm per week: $4 + 7 \times 7 = 53$ cm at week 8.
2. The actual height was 32 cm.
3. $53 - 32 = 21$ cm too high.
4. That miss is three times the worst miss on the revision data, and it happens because the growth slowed while the line kept climbing.

If wrong: Predicted correctly and did not compare to the actual. The comparison is the point — 21 cm is what fitting a line to a curve costs.

9. Inside versus outside the data REASONING

Answer: **because inside the range the line has been checked against real points, and outside it there is no evidence that the pattern continues**

1. Every point used to fit the line lies inside the range, so the fit is supported there.
2. Beyond the data, nothing has been observed — the line is an assumption that the pattern keeps going.
3. Real patterns often change: growth flattens, a test has a maximum, a price cannot go below zero.
4. The revision line reaching 345 marks out of 100 is the clearest possible example.

If wrong: Said the line “stops” at the data. It can be extended arithmetically at any time; the point is that there is no evidence for it out there, not that it is impossible to compute.

Quick answers for fast checking:

#	Level	Answer
1	fluency	whether the pattern is roughly linear
2	fluency	49 and 81
3	application	–1 and –7
4	reasoning	about 3 cm per week
5	reasoning	about \$1200 lost per year
6	reasoning	no — $x = 0$ is nowhere near the data
7	reasoning	both sides — one-sided means systematically high
8	application	53 cm, out by 21 cm
9	reasoning	inside is supported by evidence; outside is an assumption

Homework — answer key

Use: Homework after the practice sheet. Item 4 pairs deliberately with practice item 6: here $x = 0$ is birth, a real and useful moment, so the intercept IS meaningful — and children who learned “intercepts are often nonsense” will over-apply it. The contrast between the two items is the lesson. Item 3 asks where the model fails rather than whether, and the answer is startling: one hour past the data. Item 5 wants the computed 153.5 kg, since a number is a stronger objection than a rule about which data a line came from.

Timing: 20–25 min · independent · calculator allowed · items 3, 4 and 8 first when marking.

1. Pay line REASONING

Answer: **slope: \$15 earned per hour worked. Intercept: \$20 paid before any hours — perhaps a fixed call-out fee**

1. Slope: y changes by 15 for each extra hour, and the units are dollars per hour.
2. Intercept: at $x = 0$ hours the pay is \$20, so there is a payment independent of hours worked.
3. That is meaningful here — a fixed fee, a travel allowance or a minimum payment.
4. So the model says: \$20 to turn up, then \$15 an hour.

If wrong: Gave 15 and 20 without units or meaning. The last sentence — \$20 to turn up, then \$15 an hour — is what an interpretation sounds like.

2. Pay for 6 hours FLUENCY

Answer: **\$110**

1. $15 \times 6 + 20 = 90 + 20 = 110$.

If wrong: Forgot the \$20.

3. At 9 and 10 hours REASONING

Answer: **97 and 105 marks — the second is impossible on a test out of 100, so the line has already left the range where it can mean anything**

1. $8 \times 9 + 25 = 97$. $8 \times 10 + 25 = 105$.
2. The data went up to 8 hours, so both are extrapolations.
3. 97 is at least possible; 105 is not, because the test has a maximum of 100.
4. So the model breaks down between 9 and 10 hours — only one hour past the data, which is sooner than most people expect.

If wrong: Reported both numbers without noticing 105 is impossible. Finding WHERE a model fails is more useful than knowing that it eventually does, and here it fails almost immediately.

4. Baby weight line REASONING

Answer: **slope: the baby gains about 0.5 kg per month. Intercept: 3.5 kg at 0 months, which is birth weight — so it is meaningful**

1. Slope: 0.5 kg per month of age.
2. Intercept: the predicted weight at age 0 months, which is the moment of birth.
3. 3.5 kg is a plausible birth weight, so the intercept is both meaningful and checkable.
4. This is a case where $x = 0$ is a real, interesting value — unlike the shoe-size line.

If wrong: Said the intercept is meaningless because you cannot be 0 months old. Age 0 months is birth, which is a real moment and a useful prediction — worth contrasting with an intercept that genuinely describes nothing.

5. A 25-year-old REASONING

Answer: **no — 25 years is 300 months, far outside the data, and the line would predict about 153.5 kg**

1. $0.5 \times 300 + 3.5 = 153.5$ kg.
2. That is far heavier than any adult, so the prediction is clearly wrong.
3. Babies do not keep gaining 0.5 kg per month for 25 years: growth slows and then stops, so the pattern is not linear over that range.
4. The line is a good model over the first months and says nothing about adults.

If wrong: Said no because “the line is for babies”. True, and the reason is that the RATE does not continue — and computing 153.5 kg makes the objection concrete rather than a rule about labels.

6. All points above REASONING

Answer: **the line sits too low, so it under-predicts everywhere; move it up until points fall on both sides**

1. A good fit has misses in both directions.
2. All points above means the line is consistently below the data.
3. Raising the intercept moves the line up without changing the slope, which is what a one-sided pattern calls for.
4. If the gaps also grow along the line, the SLOPE needs changing too, not just the height.

If wrong: Said the line is fine because the gaps are small. Systematically low is a fixable error, and it will under-predict every single time it is used.

7. Why “12” is not an interpretation REASONING

Answer: **because it does not say 12 of what, per what — a rate needs both units to mean anything**

1. A slope is a rate: a change in y per one-unit change in x.
2. 12 could be dollars per hour, cm per week, or marks per hour of revision.
3. Without the units it cannot be used to answer any question about the situation.
4. So an interpretation always has the shape “*this much* of y for each *one* of x”.

If wrong: Said it needs more detail. The specific missing thing is the two units, and naming that is what makes the answer usable next time.

8. Why interpret in context REASONING

Answer: **because the point of a model is to answer a question about the real situation, and a number without units answers nothing — the interpretation is also what reveals when a model has stopped making sense**

1. A slope of 8 is a fact about a line; 8 marks per hour is a fact about revision, and only the second can inform a decision.
2. The context also tells you the limits: knowing the test is out of 100 is what makes 345 obviously wrong.
3. And it tells you whether the intercept means anything — birth weight does, a person of height zero does not.
4. So context is not decoration on the calculation; it is what turns the calculation into information.

If wrong: Said context makes it “more real”. The two concrete answers are that units make the number usable, and that the context is what exposes a model's limits.

Quick answers for fast checking:

#	Level	Answer
1	reasoning	\$15 per hour; \$20 fixed before any hours
2	fluency	\$110
3	reasoning	97 and 105; 105 is impossible, so it fails by 10 hours
4	reasoning	0.5 kg per month; 3.5 kg is birth weight, so meaningful
5	reasoning	no — 153.5 kg, and the rate does not continue
6	reasoning	too low; raise it until misses balance
7	reasoning	a rate needs both units
8	reasoning	units make it usable; context exposes the limits

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Frame the lesson in three parts: fit, judge, interpret. The interpreting is the part that is usually skipped and the part the standard is about.
2/8	0:45	Shape first, using SP-01's plant data as the counterexample. Fitting a line to a curve is a mistake made before any arithmetic happens.
3/8	1:30	Fit by eye with a ruler. The instruction that helps most: aim for the middle of the cloud, not through the most points.
4/8	2:30	Judging the fit. Measure a few vertical gaps. The key idea is BOTH SIDES — if every point is below the line, the line is too high, however small the gaps are. The revision line at $y = 8x + 25$ is a good example: every residual is zero or negative, so it is a decent fit that could be improved by lowering it.
5/8	4:00	THE slide. Refuse “the slope is 8” and ask “8 what?” until someone says marks per hour. Two units is the test of an interpretation.
6/8	5:15	The intercept, and the honest complication: sometimes $x = 0$ is meaningless. Ask for an example where it would be — a line relating adult height to shoe size has an intercept describing a person with no feet.
7/8	6:30	Extrapolation. Predict at 40 hours, get 345, and let the class notice the test is out of 100. The model does not know that; they do.
8/8	7:45	Both interpretations and both predictions, then the exit ticket.