

GRADE 8 · FUNCTIONS · ANSWER KEY

LINEAR AND NON-LINEAR

Today's goal: I can interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line and give examples of functions that are not linear.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 3 is the exit ticket's trap with different numbers, deliberately, so you can see whether the correction stuck rather than whether it was heard. Item 8 asks for the general test, which is the transferable thing: divide every y-change by its own x-change. **Timing:** 15–20 min · pairs · items 7 and 8 on the board.

- 1 yes 2 no 3 yes — rate 2 throughout 4 not linear, but it IS a function
- 5 any curve rising throughout with a changing steepness — $y = x^2$ for $x > 0$, or $y = 2_x$ 6 $y = 2_x$, $y = x^3$, $y = 3/x$ or similar
- 7 an explanation that the differences only measure the rate when the x-steps are equal, so an unevenly sampled line looks non-linear
- 8 divide each change in y by its own change in x and check every rate is the same

1 **MILD** y: 5,8,11,14

yes

1. x-steps all 1, differences all 3, so rate 3.

Watch for: Fine for most; the check on x-steps is the habit being built.

2 **MEDIUM** y: 1,2,4,8

no

1. x-steps are all 1, so the difference test is fair.
2. Differences 1, 2, 4 — growing, not constant.
3. It is $y = 2_x$, which doubles rather than adds.

Watch for: Said yes because the pattern is obvious. Doubling is a pattern and is not a constant rate.

3 **MEDIUM** x: 2,4,8 → y: 7,11,19

yes — rate 2 throughout

1. x-steps: 2 then 4. Uneven, so do NOT use raw differences.
2. Rates: $\frac{4}{2} = 2$ and $\frac{8}{4} = 2$.
3. Constant rate, so linear. It is $y = 2x + 3$.

Watch for: Said no because the y-differences were 4 then 8. Same trap as the exit ticket, with different numbers, and it is here to check the correction stuck.

4 **MEDIUM** $y = 3/x$
not linear, but it IS a function

1. Each x (except 0) gives exactly one value, so it is a function.
2. Rates: from $x = 1$ to 2 the value drops 3 to 1.5, then 1.5 to 1 from 2 to 3. Not constant.
3. So it is a function whose graph is a curve.

Watch for: Said not a function because it is not a line, which conflates the two properties this unit keeps separating.

5 **MEDIUM** Increasing but not linear

any curve rising throughout with a changing steepness — $y = x^2$ for $x > 0$, or $y = 2_x$

1. It must go up the whole way and bend.
2. Doubling curves work well.

Watch for: Drew a straight line, or a curve that goes up then down. Increasing and linear are different, and this item is where that separates.

6 **MEDIUM** A non-linear function that is not $y = x^2$

$y = 2_x$, $y = x^3$, $y = 3/x$ or similar

1. Anything with a changing rate.

Watch for: Gave $y = 2x + 1$. Being told to avoid the obvious example is what forces the category to be understood rather than recalled.

7 **SPICY** Why equal differences is not enough

an explanation that the differences only measure the rate when the x-steps are equal, so an unevenly sampled line looks non-linear

1. A rate is a change in y PER change in x.
2. If the x-steps differ, the same rate produces different differences.
3. $x = 1, 2, 4$ on $y = 2x + 1$ gives 3, 5, 9 — differences 2 and 4, one rate.
4. The shortcut works whenever the x-steps are equal, which is why it usually looks reliable.

Watch for: Restated the rule without the condition. The condition IS the answer.

8 **SPICY** The test for uneven x-steps

divide each change in y by its own change in x and check every rate is the same

1. Work out change in y over change in x for each consecutive pair.
2. If every one of those is identical, it is linear.
3. This works whether the x-steps are even or not, so it is the test worth keeping.

Watch for: Described adding a column of differences without dividing. The dividing is the entire fix.

Homework

7 problems + Mr DJ's worked example

Use: Homework after the practice sheet. Item 7 is the best item in the skill: repeated ADDING is linear, repeated MULTIPLYING is not, and both feel “steady”. A child who calls both linear has the right instinct about regularity and the wrong test. **Timing:** 20–25 min · independent · items 3, 5 and 7 first when marking.

1 yes — rate 4 2 no 3 yes — rate 3 throughout 4 a function, but not linear

5 an explanation that a bigger x-step produces a bigger y-jump at the same rate, so unequal differences do not mean an unequal rate

6 for example $y = x^2$ and $y = 2_x$ (or $y = 3/x$) 7 the phone is linear; the bacteria are not

1 **MILD** y: 2,6,10,14

yes — rate 4

1. Even x-steps, constant difference 4.

Watch for: —

2 **MEDIUM** y: 1,3,9,27

no

1. Even x-steps, so differences are fair: 2, 6, 18.
2. Growing, so not linear. It triples — $y = 3_x$.

Watch for: Said yes because tripling is a pattern.

3 **MEDIUM** x: 1,3,7 → y: 4,10,22

yes — rate 3 throughout

1. x-steps: 2 then 4.
2. Rates: $\frac{6}{2} = 3$ and $\frac{12}{4} = 3$.
3. Constant, so linear. It is $y = 3x + 1$.

Watch for: Said no from the differences 6 and 12. Third appearance of this trap across the three sheets, on purpose.

4 **MEDIUM** $y = x^2$: function? linear?

a function, but not linear

1. Each x gives exactly one x^2 , so it is a function.
2. Its rate changes — differences 1, 3, 5 — so it is not linear.
3. Two independent properties.

Watch for: Answered only one half, or said it fails both.

5 **SPICY** Why an uneven sample misleads

an explanation that a bigger x-step produces a bigger y-jump at the same rate, so unequal differences do not mean an unequal rate

1. The rate is per one unit of x.
2. Skip two units and you get twice the jump — same rate, bigger difference.
3. So the differences only line up when the steps do.

Watch for: Repeated that you should check the rate, without saying WHY the differences mislead. The why is the item.

6 **SPICY** Two counterexamples for Ben

for example $y = x^2$ and $y = 2_x$ (or $y = 3/x$)

1. Both contain x and neither has a constant rate.
2. Containing x makes it an equation, not a line.

Watch for: Gave one, or gave two linear equations. Two are asked for because one can be dismissed as a special case.

7 **EXTRA SPICY** Phone against bacteria

the phone is linear; the bacteria are not

1. The phone loses the SAME 6 percentage points each hour — a constant rate, so linear.
2. The bacteria DOUBLE each hour: 100, 200, 400. The increase itself grows, so the rate is not constant.
3. Adding the same amount each step is linear; multiplying by the same amount each step is not.
4. That distinction — repeated adding against repeated multiplying — is the whole difference.

Watch for: Called both linear because both change steadily. “Steadily” is the trap: doubling is steady and is not a constant rate.

Exit Ticket

4 problems

Use: End of the linear/non-linear lesson. 5–8 min, independent. Item 3 is the diagnostic and the marking note matters: a child who agrees with Nia has learned a shortcut that is right on nearly every table they have ever seen. Tell them the rule is usually correct, then show the condition — that lands better than calling it wrong. **Timing:** 5–8 min · independent.

1 yes 2 no — it is not linear 3 no — it IS linear

4 for example $y = x^2$: each input still has exactly one output, so it is a function, but its rate changes

1 **MILD** Is $y = 5x - 2$ linear?

yes

1. It is in the form $y = mx + b$, with $m = 5$ and $b = -2$.
2. A constant rate means a straight line.

Watch for: Said no because of the minus, or because no table was given.

2 **MEDIUM** $x: 0,1,2,3 \rightarrow y: 0,1,4,9$

no — it is not linear

1. The x -steps are all 1, so the difference test is fair here.
2. Differences: 1, 3, 5 — not constant.
3. So the rate changes and it is not linear. (It is $y = x^2$.)

Watch for: Said yes because there is a clear pattern. A pattern in the differences is not a constant difference — growing by 2 each time is still growing.

3 **SPICY** Is Nia right?

no — it IS linear

1. Her differences are right: $3 \rightarrow 5 \rightarrow 9$ goes up 2 then 4.
2. But look at the x -values: $1 \rightarrow 2 \rightarrow 4$. Those step by 1 then 2, not evenly.
3. The test is the RATE: $\frac{2}{1} = 2$ and $\frac{4}{2} = 2$. Constant, so linear.
4. In fact it is $y = 2x + 1$, just sampled at uneven x -values.
5. Her rule is right whenever the x -steps are equal — which is nearly every table she has ever seen, and why nobody warned her.

Watch for: Agreed with her. This is the misconception this whole skill is built around, and it is worth saying that her rule is USUALLY right rather than simply wrong — she has learned a reliable shortcut and met the case it does not cover.

4 **SPICY** A non-linear function

for example $y = x^2$: each input still has exactly one output, so it is a function, but its rate changes

1. $y = x^2$ is a function: put in 3, get 9, always.
2. It is not linear: the differences grow 1, 3, 5 for equal steps.
3. Non-linear and not-a-function are different things.
4. $y = 2_x$ and $y = 3/x$ also work.

Watch for: Gave a relation that is not a function at all — a circle, say — which answers a different question. Or gave a line. The item needs both halves: not linear, still a function.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Say the second half of the standard out loud: give examples that are NOT linear. It is the half that gets skipped, and the “What a function is” lesson showed why — the class lives in linear functions, so linear starts to mean function.
2	1:00	$y = mx + b$ as a constant rate. Quick.
3	2:00	The familiar test, on a table that steps by one. Let them be confident.
4	3:00	THE slide. Same function, sampled unevenly. Ask for a verdict BEFORE revealing — most of the class will say non-linear, using a rule that is usually right.
5	4:30	Fix the rule properly: constant RATE, not constant difference. Equal differences are a shortcut that only holds for equal steps, and it is worth saying that the shortcut works on nearly every textbook table, which is why nobody notices.
6	6:00	Now genuinely non-linear, sampled evenly so the test is fair. Squares and doubling. Point at the growing climbs.
7	7:30	Non-linear is still a function. This closes the loop from the definition lesson: the two properties are independent.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.

10	8:30	Three to sort, then the exit ticket. The middle one, $y = 3/x$, is the one to talk about — it is a function, and it is nowhere near a line. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #7 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.