

GRADE 8 · THE NUMBER SYSTEM · ANSWER KEY

LOCATING IRRATIONAL NUMBERS

Today's goal: I can use rational approximations of irrational numbers to compare their size and locate them approximately on a number line diagram.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 2 is deliberately close — 4.472 against 4.5 — so guessing fails and squaring succeeds. Item 7 runs the bracket backwards and is the best test of whether the idea is understood rather than the procedure memorised. **Timing:** 15–20 min · pairs · no calculators. Put items 3 and 7 on the board.

- 1 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 2 4.5 is bigger 3 nearer 5 4 $\sqrt{2}$, π , $\sqrt{10}$ 5 between 12 and 13
6 an explanation that the square root asks what multiplies by itself to give n, while halving asks what adds to itself
7 any whole number from 37 to 48 8 $\sqrt{10}$ is bigger

1 **MILD** Perfect squares to 100

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

1. 1^2 through 10^2 .

Watch for: Missed 1 or stopped at 81. Knowing these fluently is most of this topic, so a gap here explains later errors.

2 **MEDIUM** $\sqrt{20}$ or 4.5?

4.5 is bigger

1. $4.5^2 = 20.25$ and $(\sqrt{20})^2 = 20$.

2. $20.25 > 20$, so $4.5 > \sqrt{20}$.

3. It is close — $\sqrt{20} = 4.472\ldots$ — which is why squaring beats guessing.

Watch for: Chose $\sqrt{20}$. The two are within 0.03 of each other, so any method other than squaring is a coin flip here, and that is why this pair was chosen.

3 **MEDIUM** Nearer 5 or 6?

nearer 5

1. Bracket: $25 < 30 < 36$, so between 5 and 6.

2. Test the midpoint: $5.5^2 = 30.25$, which is ABOVE 30.

3. So $\sqrt{30}$ is below 5.5, and therefore nearer 5.

Watch for: Answered nearer 6 because 30 looks closer to 36 than to 25. It is — but the midpoint test settles it and intuition does not.

4 **MEDIUM** Order $\sqrt{2}$, π , $\sqrt{10}$

$\sqrt{2}$, π , $\sqrt{10}$

1. $\sqrt{2} = 1.41\ldots$, clearly smallest.

2. $\pi \approx 3.14$ and $\sqrt{10} \approx 3.16$.

3. Square the close pair: $\pi^2 \approx 9.87 < 10$, so $\pi < \sqrt{10}$.

Watch for: Put π last, assuming the famous constant is the biggest. The close pair is the whole difficulty.

5 **MEDIUM** Bracket $\sqrt{150}$

between 12 and 13

1. $144 < 150 < 169$.

2. $12^2 = 144$ and $13^2 = 169$.

Watch for: Needed squares beyond 100, so a child who stopped their list at 10^2 stalls here. That is the link back to item 1.

6 SPICY Why halving is not rooting

an explanation that the square root asks what multiplies by itself to give n, while halving asks what adds to itself

1. $\sqrt{n}$ asks: what times itself gives n?
2. Halving asks: what plus itself gives n?
3. For 100: the root is 10, the half is 50. Very different questions.
4. They only agree at 4, where half of 4 and $\sqrt{4}$ are both 2 — which is probably where the habit comes from.

Watch for: Said “because it just does not work”. Naming the two questions is the answer, and the coincidence at 4 is worth mentioning if a child raises it.

7 SPICY $\sqrt{n}$ between 6 and 7

any whole number from 37 to 48

1. $6^2 = 36$ and $7^2 = 49$.
2. So n is above 36 and below 49.
3. That is 37 up to 48 inclusive — twelve possible values.

Watch for: Gave a single value such as 42, answering a narrower question than the one asked. Or included 36 and 49, whose roots are exactly 6 and 7 rather than between them.

8 EXTRA SPICY π or $\sqrt{10}$?

$\sqrt{10}$ is bigger

1. Square both: $\pi^2 \approx 9.87$, and $(\sqrt{10})^2 = 10$ exactly.
2. $9.87 < 10$, so $\pi < \sqrt{10}$.
3. Decimals confirm it: 3.14159... against 3.16227..., decided at the second place.
4. Squaring is easier here than remembering either decimal.

Watch for: Chose π . Squaring is the point: $(\sqrt{10})^2$ is exactly 10, which makes the comparison trivial once you take it.

Homework

7 problems + Mr DJ's worked example

Use: Homework after the practice sheet. Item 2 reverses the practice sheet's comparison on purpose: there the decimal was bigger (4.5 beat $\sqrt{20}$), here the root is ($\sqrt{13}$ beats 3.5). A child who took away ‘the decimal wins’ instead of the squaring technique gets it wrong, so read the working rather than the answer. **Timing:** 20–25 min · independent · no calculator. Items 2 and 7 first when marking.

- 1 between 8 and 9 2 $\sqrt{13}$ is bigger 3 nearer 8 4 any three of 17 to 24 5 $\sqrt{3}$, 2, $\sqrt{6}$
6 find the perfect squares either side of n 7 he halved — the answer is just under n

1 MILD Bracket $\sqrt{70}$

between 8 and 9

1. $64 < 70 < 81$.

Watch for: Halved to 35 and 36.

2 MEDIUM 3.5 or $\sqrt{13}$?

$\sqrt{13}$ is bigger

1. Square both: $3.5^2 = 12.25$, and $(\sqrt{13})^2 = 13$.
2. $12.25 < 13$, so $3.5 < \sqrt{13}$.
3. Check: $\sqrt{13} = 3.605...$, which is indeed above 3.5.

Watch for: Chose 3.5. This item reverses the practice sheet on purpose — there the decimal was bigger, here the root is — so a child who took away ‘the decimal wins’ as a rule rather than squaring gets it wrong. Read the working, not the answer.

3 MEDIUM Nearer 8 or 9?

nearer 8

1. Between 8 and 9, since $64 < 70 < 81$.
2. Midpoint test: $8.5^2 = 72.25$, which is above 70.
3. So $\sqrt{70}$ is below 8.5, and nearer 8. ($\sqrt{70} = 8.366...$)

Watch for: Answered nearer 9 by feel.

4 MEDIUM $\sqrt{n}$ between 4 and 5

any three of 17 to 24

1. $16 < n < 25$.
2. So 17 through 24.

Watch for: Included 16 or 25, whose roots are exactly 4 and 5.

5 MEDIUM Order $\sqrt{3}$, 2, $\sqrt{6}$

$\sqrt{3}$, 2, $\sqrt{6}$

1. Square all three: 3, 4, 6.
2. So the order is $\sqrt{3}$, 2, $\sqrt{6}$.
3. Squaring lets you compare a whole number with two roots in one step.

Watch for: Put 2 first, assuming a whole number must be smaller than any root.

7 SPICY Correct Sam

he halved — the answer is just under 10

1. Halving 99 gives 49.5, which is not a square root.
2. $81 < 99 < 100$, so $9 < \sqrt{99} < 10$.
3. In fact $\sqrt{99} = 9.949\dots$, only just under 10.
4. Sam's answer is five times too big.

Watch for: Agreed, or corrected the number without naming the halving. The named error is what stops it recurring.

6 SPICY First thing to do

find the perfect squares either side of n

1. Bracket first, always.
2. Everything else — refining, comparing, placing — depends on having the two whole numbers.

Watch for: Said “use a calculator”. Not available, and not the skill.

Exit Ticket

4 problems

Use: End of the locating lesson. 5–8 min, independent. Item 4 is the diagnostic and it is worth reading first: a child who agrees with Rosa has a method that will fail on every root, while looking systematic. **Timing:** 5–8 min · independent. No calculators — the whole point is that none is needed.

1 between 7 and 8

2 1.5 is bigger

3 between 2 and 3, nearer 3

4 between 9 and 10 — halving is not rooting

1 **MILD** Bracket $\sqrt{50}$

between 7 and 8

1. Perfect squares either side of 50: 49 and 64.
2. $7^2 = 49 < 50 < 64 = 8^2$.
3. So $7 < \sqrt{50} < 8$. ($\sqrt{50} = 7.07\dots$)

Watch for: Answered between 25 and 26 by halving, or gave 7 and 8 with no squares shown — the squares are what makes it a method rather than a recollection.

2 **MEDIUM** 1.5 or $\sqrt{2}$?

1.5 is bigger

1. Square both: $1.5^2 = 2.25$, and $(\sqrt{2})^2 = 2$.
2. $2.25 > 2$, so $1.5 > \sqrt{2}$.
3. Check against the decimal: $\sqrt{2} = 1.414\dots$, which is indeed below 1.5.

Watch for: Chose $\sqrt{2}$ because the root sign looks like it should be larger, or compared decimals from memory without squaring. Squaring is the reliable move and the one being assessed.

3 **MEDIUM** Place $\sqrt{8}$

between 2 and 3, nearer 3

1. $4 < 8 < 9$, so $2 < \sqrt{8} < 3$.
2. 8 is much closer to 9 than to 4, so $\sqrt{8}$ sits high in that gap.
3. Refine: $2.8^2 = 7.84$ and $2.9^2 = 8.41$, so $\sqrt{8}$ is between 2.8 and 2.9. Nearer 3.

Watch for: Answered nearer 2, splitting the gap evenly. The squares are not evenly spaced, so the midpoint of the bracket is not the midpoint of the numbers.

4 **SPICY** Correct Rosa

between 9 and 10 — halving is not rooting

1. Halving and square-rooting are different operations. $45 \times 45 = 2025$, not 90.
2. Bracket properly: $81 < 90 < 100$, so $9 < \sqrt{90} < 10$.
3. $\sqrt{90} = 9.486\dots$
4. Rosa's method would give a number about five times too big.

Watch for: Agreed, or fixed the answer without naming the error. Halving is the commonest wrong method in this topic and it needs saying out loud, because it feels like undoing something.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Name the day's habit before any mathematics: bracket first. It is one move and it makes every question on the topic tractable.
2	1:00	Offer the halving answer yourself and let the class kill it with one multiplication. Doing it this way round means nobody has to be wrong out loud.
3	2:30	Write the perfect squares along the top of the board and leave them there for the whole lesson. Most of the difficulty in this topic is not knowing them fluently.
4	4:00	Refining with 5.5. This is the step that turns a bracket into an estimate, and it is worth doing by hand so they see a calculator is not needed.
5	7:00	THE technique for comparing. Squaring both sides removes all the guesswork, and it is what the standard is actually asking for.
6	9:00	Pi against root 10. Ask which is bigger before showing the decimals — most will say pi, because pi is the famous one.
7	10:30	Place them all on one line, then the exit ticket. Watch whether they bracket before placing, or place by feel and check afterwards.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		

Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.

10	5:30	Three to try. Circulate for anyone still halving — they will put $\sqrt{8}$ between 3 and 4 or worse. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #6 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.