



Modelling with a linear function

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the modelling lesson. 5–8 min, independent. Item 4 is the diagnostic and item 2 is the one most often under-marked: the standard requires the rate and initial value to be INTERPRETED in the situation, so “£5” alone is half an answer however correct the number is.

Timing: 5–8 min · independent.

1. Rate per mile **APPLICATION**

Answer: **£2 a mile**

1. The cost rose $\pounds 19 - \pounds 11 = \pounds 8$.
2. Over $7 - 3 = 4$ miles.
3. $\pounds 8 \div 4 \text{ miles} = \pounds 2 \text{ a mile}$.

If wrong: Answered £8 — the difference rather than the rate. A difference and a rate are different quantities, and only one of them is per mile.

2. Initial value and its meaning **REASONING**

Answer: **£5 — the fixed charge before any distance**

1. At 3 miles the fare was £11.
2. Three miles cost $3 \times \pounds 2 = \pounds 6$.
3. So before travelling at all: $\pounds 11 - \pounds 6 = \pounds 5$.
4. In this situation that is the flag fall — what you pay just for getting in.
5. The interpretation is required, not a bonus: “ $b = 5$ ” does not answer “what does it mean”.

If wrong: Gave £5 with no interpretation, or gave £11. The standard names interpretation explicitly, so a bare number is half an answer.

3. The model, checked APPLICATION

Answer: $y = 2x + 5$

1. Rate 2, initial value 5.
2. Check at 3 miles: $2(3) + 5 = 11$ ✓.
3. Check at 7 miles: $2(7) + 5 = 19$ ✓.
4. Both readings, because the one you built it from will always fit.

If wrong: Checked only one reading. Same trap as checking a system in one equation — the reading you used to build the model cannot test it.

4. Correct Ali REASONING

Answer: **£11 is the value at 3 miles, not at 0 miles**

1. The initial value means the value when $x = 0$ — before any distance at all.
2. £11 already includes three miles of travelling.
3. Strip those out: $3 \times £2 = £6$, so the fixed part is £5.
4. Nothing in the question said “starting”, which is exactly why this has to become a habit: the first number given is almost never the initial value.

If wrong: Agreed, or corrected to £5 without saying WHY £11 is not it. The reason — that £11 is a reading at $x = 3$ — is the transferable part.

Quick answers for fast checking:

#	Level	Answer
1	application	£2 a mile
2	reasoning	£5 — the fixed charge
3	application	$y = 2x + 5$, both readings checked
4	reasoning	£11 is the value at 3 miles

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 4 is chosen so the rate and the initial value are BOTH 4 — a child who writes “ $m = 4$ and $b = 4$ ” has said nothing, and one who writes “grows 4 cm a week, was 4 cm tall to begin with” has the whole idea. Item 7 is the case where the initial value IS given, so working back does not become a ritual.

Timing: 15–20 min · pairs · items 4 and 6 on the board.

1. Plant growth rate APPLICATION

Answer: **4 cm a week**

1. Grew $24 - 12 = 12$ cm.
2. Over $5 - 2 = 3$ weeks.
3. $12 \div 3 = 4$ cm a week.

If wrong: Answered 12 cm — the difference, not the rate.

2. Height at week 0 APPLICATION

Answer: **4 cm**

1. At week 2 it was 12 cm.
2. Two weeks of growth is $2 \times 4 = 8$ cm.
3. $12 - 8 = 4$ cm at week 0.

If wrong: Answered 12 cm, taking the first reading as the start.

3. The model, checked APPLICATION

Answer: **$y = 4x + 4$**

1. Check at week 2: $4(2) + 4 = 12$ ✓.
2. Check at week 5: $4(5) + 4 = 24$ ✓.

If wrong: Wrote $y = 4x + 12$, using a reading as the intercept — which fits week 0 and fails both readings.

4. Interpret both REASONING

Answer: **the plant grows 4 cm each week, and it was 4 cm tall when the measuring started**

1. $m = 4$: growth per week, in centimetres.
2. $b = 4$: the height before any of the weeks counted.
3. Both numbers happen to be 4 here, and they mean completely different things — one is a speed, one is a length.

If wrong: Wrote “ $m = 4$ and $b = 4$ ”. That restates the model rather than interpreting it, and this item is deliberately one where the two numbers coincide so the difference in MEANING has to carry the answer.

5. Phone APPLICATION

Answer: **–6% an hour, starting at 80%**

1. Fell $74 - 56 = 18$ percentage points over 3 hours, so –6 an hour.
2. At 1 hour it was 74, so at 0 it was $74 + 6 = 80\%$.

If wrong: Gave the rate as 18, or the start as 74. Note the start is the LARGEST value here, which surprises children expecting to work upwards.

6. Why the start is usually not given REASONING

Answer: **because the readings are taken at times after the start, so each one already includes some change**

1. A reading at $x = 2$ has already had two units of change in it.
2. The initial value is at $x = 0$, which is usually before anyone measured.
3. So it has to be reconstructed by removing the change that had already happened.
4. It IS one of the given numbers when a reading happens to be at $x = 0$ — like the pool in item 7.

If wrong: Said “because it is a trick”. Naming that a reading carries accumulated change is what makes the working-back step obvious rather than arbitrary.

7. When is the pool empty? APPLICATION

Answer: **after 50 minutes**

1. At 0 minutes: 500 litres, so $b = 500$ — here the reading IS at $x = 0$.
2. Lost 100 litres over 10 minutes, so –10 a minute.
3. Model: $y = -10x + 500$.
4. Empty means $y = 0$: $10x = 500$, so $x = 50$ minutes.

If wrong: Answered 10 or 100. This item also shows the case where the initial value IS given, which stops working-back becoming a ritual applied blindly.

8. Why check both readings REASONING

Answer: **because the model was built from one of them, so that one always fits and tests nothing**

1. If you used the reading at $x = 2$ to find b , then $x = 2$ will fit by construction.
2. Only the OTHER reading is evidence.
3. Same reason a solution to a system must satisfy both equations.

If wrong: Said “to be careful”. The reason is specific: one reading is not independent of the model.

9. Invent a situation for $y = 3x + 12$ REASONING

Answer: **any situation with a starting amount of 12 and a rate of 3 per unit, with both interpreted**

1. Pick something that starts at 12 and gains 3 each step.
2. Say what the 3 is per, and what the 12 was before anything happened.

If wrong: Wrote a situation whose numbers are swapped — starting at 3 and gaining 12. Writing one is much harder than reading one, and the swap is the commonest slip.

Quick answers for fast checking:

#	Level	Answer
1	application	4 cm a week
2	application	4 cm
3	application	$y = 4x + 4$
4	reasoning	grows 4 cm a week; was 4 cm at the start
5	application	−6% an hour, from 80%
6	reasoning	a reading already includes accumulated change
7	application	50 minutes
8	reasoning	the model was built from one reading
9	reasoning	a situation with both numbers interpreted

Homework — answer key

Use: Homework after the practice sheet. Item 2 is the sharpest: working back on a FALLING quantity means adding, and children who have only ever worked back on rising situations subtract out of habit. Item 8 is the third appearance in Grade 8 of “a check that cannot fail is not a check” — alongside systems and estimating before rather than after.

Timing: 20–25 min · independent · items 2, 4 and 8 first when marking.

1. Candle rate APPLICATION

Answer: **−2 cm an hour**

1. Fell $18 - 10 = 8$ cm over 4 hours.
2. $8 \div 4 = 2$, and it is shrinking, so −2 cm an hour.

If wrong: Answered 8, or 2 without the sign. A burning candle cannot have a positive rate.

2. Height when lit APPLICATION

Answer: **22 cm**

1. At 2 hours it was 18 cm.
2. Two hours of burning is $2 \times 2 = 4$ cm.
3. So at 0 hours: $18 + 4 = 22$ cm.
4. Note we ADD, because it was taller earlier.

If wrong: Answered 18. Working back ADDS when the quantity is falling, which trips children who have only worked back on rising situations.

3. The model, checked APPLICATION

Answer: **$y = -2x + 22$**

1. Check at 2: $-2(2) + 22 = 18$ ✓.
2. Check at 6: $-2(6) + 22 = 10$ ✓.

If wrong: Wrote $y = -2x + 18$.

4. Interpret both REASONING

Answer: **the candle burns down 2 cm every hour, and it was 22 cm tall when it was lit**

1. $m = -2$: centimetres lost per hour.
2. $b = 22$: the height before burning began.

If wrong: Restated the letters.

5. When does it burn out? APPLICATION

Answer: **after 11 hours**

1. Burnt out means $y = 0$.
2. $2x = 22$, so $x = 11$ hours.

If wrong: Answered 22, giving the height rather than the time.

6. Gym fees APPLICATION

Answer: **£12 a month, £22 to join**

1. Rose £36 over 3 months, so £12 a month.
2. At 2 months: $£46 - 2(£12) = £22$ to join.
3. Check at 5 months: $12(5) + 22 = £82$ ✓.

If wrong: Answered £36 a month, or £46 to join. This one has two named quantities in the question, which makes the interpretation half unavoidable.

7. Why divide REASONING

Answer: **because a rate is a change PER one unit, so the difference has to be shared over the units that passed**

1. A difference tells you how much changed. It does not say over how long.
2. Gaining 60 litres in 6 minutes and in 60 minutes are very different rates.
3. Dividing by the elapsed amount is what makes it per-one.

If wrong: Said “because that is the formula”. The 6-minutes-against-60-minutes comparison is what makes it obvious.

8. Is one check enough? REASONING

Answer: **no — the model was built from that reading**

1. She used that reading to find the rate or the initial value, so it fits by construction.
2. It cannot disagree, so it is not evidence.
3. The second reading is the only real test.
4. Same reason a solution to a system has to satisfy BOTH equations.

If wrong: Said yes because it fitted. A check that cannot fail is not a check, and this is the third place in Grade 8 where that idea appears — alongside systems and the estimate-first habit in two-step problems.

Quick answers for fast checking:

#	Level	Answer
1	application	-2 cm an hour
2	application	22 cm
3	application	$y = -2x + 22$
4	reasoning	burns 2 cm an hour; 22 cm when lit
5	application	11 hours

6	application	£12 a month, £22 to join
7	reasoning	a rate is per ONE unit
8	reasoning	no — that reading fits by construction

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Two jobs, said up front: build the model, then interpret it. The second is the part children treat as optional and the standard treats as half the content.
2/8	1:00	Give the two readings and ask which is the starting amount. Most will say 40. Let that stand for a moment — it is the misconception this lesson exists for.
3/8	2:00	Rate as a difference PER unit. Write 60 and 10 side by side and name which is which; a difference and a rate are different quantities.
4/8	3:30	THE slide. Work BACK two minutes to reach $x = 0$. This is the step nothing in the wording prompts, so it has to become a habit rather than a response to a cue.
5/8	5:00	Write the model and test it on BOTH readings. One reading always fits — you built it from that one — so checking one proves nothing, exactly as with systems of equations.
6/8	6:00	Interpretation. Insist on units and a noun: “gains 10 litres every minute”, not “ $m = 10$ ”. Say that this is marked.
7/8	7:30	A falling situation, so the rate is negative and the start is the LARGEST value. Children expect the start to be the smallest.
8/8	8:30	The taxi, with interpretation, then the exit ticket.