



Rational and irrational numbers

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the rational/irrational lesson, and the first Grade 8 exit ticket. 5–8 min, independent. Item 2 is the diagnostic: a child who agrees with Amara is using ‘goes on forever’ as the definition, and will get every repeating decimal wrong for the rest of the unit.

Timing: 5–8 min · independent. Read item 2 first when marking — it predicts items 1 and 3.

1. Is 0.125 rational? **FLUENCY**

Answer: **rational**

1. The decimal terminates — it stops after three digits.
2. And it can be written as a fraction: $0.125 = \frac{125}{1000} = \frac{1}{8}$.
3. Either reason is enough.

If wrong: Answered irrational because it is written as a decimal rather than a fraction. Notation is not a property of the number.

2. Is $\frac{2}{3}$ irrational? **REASONING**

Answer: **no — it is rational**

1. Amara is right that 0.666... goes on forever.
2. But that is not the test. The test is whether it terminates or REPEATS.
3. 0.666... repeats — the 6 goes on identically — so it is rational.
4. And the fraction is already in front of her: $\frac{2}{3}$ is two integers, one over the other.

If wrong: Agreed with her. This is THE misconception in the topic: ‘goes on forever’ is being used as the definition of irrational, and it makes every repeating decimal wrong.

3. Sort the four numbers APPLICATION

Answer: **rational: $\sqrt{16}$ and 0.181818...; irrational: $\sqrt{5}$ and π**

1. $\sqrt{16} = 4$, a whole number, so rational. Not every root is irrational.
2. $\sqrt{5} = 2.2360...$ and never repeats, so irrational.
3. $\pi = 3.14159...$ and never repeats, so irrational.
4. 0.181818... repeats, so rational (it is $\frac{2}{11}$).

If wrong: Put $\sqrt{16}$ with the irrationals, over-generalising from the radical sign. Or put 0.181818... with the irrationals, using the forever-test again.

4. Is $\pi = \frac{22}{7}$? REASONING

Answer: **no — $\frac{22}{7}$ is only an approximation**

1. $\frac{22}{7} = 3.142857142857...$ — it repeats, so $\frac{22}{7}$ IS rational.
2. $\pi = 3.14159265...$ and never repeats.
3. They already disagree at the third decimal place: 3.142 against 3.141.
4. So $\frac{22}{7}$ is a useful approximation of π , not equal to it. π cannot be written as any fraction.

If wrong: Accepted the equality. Worth showing both decimals to four places side by side — the disagreement is visible and settles it faster than any explanation.

Quick answers for fast checking:

#	Level	Answer
1	fluency	rational
2	reasoning	no — 0.666... repeats, so it is rational
3	application	rational: $\sqrt{16}$, 0.181818...; irrational: $\sqrt{5}$, π
4	reasoning	no — $\frac{22}{7}$ repeats and only approximates π

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 8 is the hardest and the most worthwhile: a decimal with an obvious pattern and no repeating block. It separates children who have the actual criterion from those who have learned ‘looks patterned’.

Timing: 15–20 min · pairs then share · items 3 and 7 on the board.

1. 0.6 and -0.125 as fractions FLUENCY

Answer: $\frac{3}{5}$ and $-\frac{1}{8}$

1. $0.6 = \frac{6}{10} = \frac{3}{5}$.

2. $-0.125 = -\frac{125}{1000} = -\frac{1}{8}$.

If wrong: Left them unsimplified, which is not wrong; or wrote $0.6 = \frac{6}{10}$, reading the digit as a denominator.

2. Which are rational? FLUENCY

Answer: **all four of them**

1. $7 = \frac{7}{1}$.

2. $-\frac{3}{4}$ is already a ratio.

3. $0.5 = \frac{1}{2}$.

4. $0.121212\dots$ repeats, so it is rational too.

If wrong: Excluded $0.121212\dots$ on the forever-test. Every item here is rational on purpose — a child expecting a mixed answer often invents one.

3. $\sqrt{36}$ and $\sqrt{37}$ APPLICATION

Answer: **$\sqrt{36}$ is rational, $\sqrt{37}$ is irrational**

1. 36 is a perfect square, so $\sqrt{36} = 6$ exactly. Rational.

2. 37 is not a perfect square, so $\sqrt{37} = 6.0827\dots$ and never repeats. Irrational.

3. The radical sign tells you nothing on its own — what matters is whether the number under it is a perfect square.

If wrong: Called both irrational because of the sign, which is the over-generalisation this pair exists to catch.

4. Why forever is not the test REASONING

Answer: **an explanation that a repeating decimal such as $\frac{1}{3} = 0.333\dots$ goes on forever and is still a ratio of two integers**

1. The test terminates OR repeats, not stops or does not stop.

2. $\frac{1}{3} = 0.333\dots$ runs forever and is one over three.

3. So forever tells you nothing by itself.

If wrong: Restated the rule without an example. The example is the answer — it is what makes the rule believable.

5. Is $\sqrt{7}$ rational? APPLICATION

Answer: **rational**

1. It is already written as one integer over another, so it is rational before any decimal work.
2. And the decimal repeats: the block 142857 comes round again.

If wrong: Called it irrational because the repeating block is six digits long. Length of the block is irrelevant; repeating at all is enough.

6. An irrational number that is not π APPLICATION

Answer: **$\sqrt{2}$ (accept any non-perfect-square root, or e)**

1. Any root of a number that is not a perfect square works: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.
2. $\sqrt{2} = 1.41421\dots$ and never repeats.

If wrong: Wrote a long decimal such as 0.123456789, which terminates and is therefore rational. Or wrote $22/7$.

7. $\sqrt{20}$ APPLICATION

Answer: **between 4 and 5; irrational**

1. $16 < 20 < 25$, so $4 < \sqrt{20} < 5$.
2. 20 is not a perfect square, so $\sqrt{20}$ is irrational.
3. $\sqrt{20} = 4.472\dots$

If wrong: Answered between 10 and 11, halving 20 rather than finding the nearest squares. This is the estimating error that NS-02 is about.

8. 0.101001000100001... REASONING

Answer: **irrational**

1. There is a pattern — one more zero each time — but no block of digits repeats.
2. A pattern is not the same as a repeating block, and only a repeating block makes it rational.

If wrong: Called it rational because they could see a pattern. A pattern you can describe and a repeating block are different things, and this item is the only place that distinction gets tested.

9. Every fraction of two integers is rational REASONING

Answer: **true (provided the bottom is not zero)**

1. That is the definition of rational: a ratio of two integers.
2. The one exclusion is a denominator of zero, which is not a number at all.

If wrong: Answered false, usually by thinking of $\pi = \frac{22}{7}$ — but $\frac{22}{7}$ is rational; it simply is not π .

Quick answers for fast checking:

#	Level	Answer
1	fluency	3^5 and $-\sqrt[4]{8}$
2	fluency	all four
3	application	$\sqrt{36}$ rational, $\sqrt{37}$ irrational
4	reasoning	a repeating example such as $\frac{1}{3}$
5	application	rational
6	application	$\sqrt{2}$ or similar
7	application	between 4 and 5; irrational
8	reasoning	irrational
9	reasoning	true

Homework — answer key

Use: Homework after the practice sheet. Item 8 is a deliberate stretch: recognising it as rational is the credit, and spotting that it equals a half is a bonus worth reading aloud.

Timing: 20–25 min · independent · items 4 and 6 are the ones to look at first.

1. 0.35 as a fraction FLUENCY

Answer: $\frac{7}{20}$

1. $0.35 = \frac{35}{100}$.
2. Divide top and bottom by 5: $\frac{7}{20}$.

If wrong: Left it as $\frac{35}{100}$, which is correct and unsimplified.

2. $\sqrt{49}$ and $\sqrt{50}$ FLUENCY

Answer: $\sqrt{49}$ rational, $\sqrt{50}$ irrational

1. 49 is a perfect square: $\sqrt{49} = 7$.
2. 50 is not: $\sqrt{50} = 7.071\dots$, never repeating.

If wrong: Called both irrational.

3. Is 2.5 rational? FLUENCY

Answer: **yes**

1. It terminates.
2. And $2.5 = \frac{5}{2}$.

If wrong: Said no because it is not a whole number.

4. Repeats vs goes on forever REASONING

Answer: **an explanation that repeating means a fixed block of digits comes round again, which forever alone does not guarantee**

1. $0.333\dots$ repeats: the same digit, endlessly.
2. $\sqrt{2} = 1.41421356\dots$ goes on forever with no block ever returning.
3. Both are forever. Only one repeats, and only that one is rational.

If wrong: Treated the two as the same thing, which is the misconception restated rather than resolved.

5. $\sqrt{30}$ APPLICATION

Answer: **between 5 and 6**

1. $25 < 30 < 36$, so $5 < \sqrt{30} < 6$.

If wrong: Answered between 15 and 16, halving again.

6. Correct Kai REASONING

Answer: $\sqrt{100} = 10$, which is rational

1. The root sign says nothing by itself.
2. 100 is a perfect square, so its root is the whole number 10.
3. A root is irrational only when the number underneath is not a perfect square.

If wrong: Agreed with him, or corrected the answer to 10 without saying why the reasoning was wrong. The reasoning is what this item marks.

7. One of each APPLICATION

Answer: for example $\sqrt{3}$ is irrational; $\frac{22}{7}$ or 0.121212... looks irrational and is not

1. Irrational: any non-perfect-square root.
2. Looks irrational: a long repeating decimal, or $\frac{22}{7}$.

If wrong: Gave two irrational numbers, or two rational ones. The pair is the point.

8. 0.4999... with the 9 repeating REASONING

Answer: rational — it equals $\frac{1}{2}$

1. It repeats, so it is rational before any conversion.
2. And $0.4999... = 0.5 = \frac{1}{2}$. Two decimals can name the same number.
3. If that is surprising: $0.999... = 1$ for the same reason.

If wrong: Answered irrational, or gave $\frac{5}{100}$ without noticing it simplifies to a half. The equality with 0.5 is a stretch item and is not required for credit — recognising it as rational is.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\neq 0$
2	fluency	$\sqrt{49}$ rational, $\sqrt{50}$ irrational
3	fluency	yes
4	reasoning	a repeating block returns; forever alone does not
5	application	between 5 and 6
6	reasoning	$\sqrt{100} = 10$, rational
7	application	one of each, correctly labelled
8	reasoning	rational — it is $\frac{1}{2}$

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Set up the lesson by naming the wrong test out loud, because most of the class arrives holding it: “irrational means it goes on forever”.
2/8	1:00	Rational as ratio. Point at the word. Integers, terminating decimals and mixed numbers are all easy cases and worth doing fast.
3/8	2:30	Quick practice writing them as fractions. The turn-and-talk question is the hook for slide 6 — let them believe yes for now.
4/8	4:00	THE slide. A third goes on forever and is rational. Once this lands, the forever-test is dead and the rest of the lesson is straightforward.
5/8	5:30	State the criterion cleanly: terminates or repeats. Both are rational; neither is irrational. Write it where it stays for the whole unit.
6/8	7:00	Root 2, and then immediately root 9. Children over-generalise from the radical sign, so the counterexample has to arrive in the same breath as the example.
7/8	9:00	Pi against $22/7$. Put both decimals on the board to four places and let them see the disagreement at the third. This is where “my calculator says” gets addressed.
8/8	10:30	Sorting task, then the exit ticket. Circulate for anyone still using the forever-test — they will classify $2/3$ as irrational.