

GRADE 8 · STATISTICS & PROBABILITY · ANSWER KEY

SCATTER PLOTS

Today's goal: I can construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities and describe patterns such as clustering, outliers, positive or negative association, linear association and non-linear association.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: In-class practice after the deck. Item 9 is the sharpest item on the sheet and it is the one that separates understanding from recall: the slogan fits every association equally, so it cannot be evidence about any particular claim. Item 6 is worth the discussion time because the answer “that is silly” feels sufficient and is not — the same structure appears in claims that are not silly at all, which is where the reasoning earns its keep. Item 4 catches children who read “negative” as “weak”. **Timing:** 15–20 min · pairs · squared paper · items 6 and 9 on the board.

1 two measurements taken from the same individual

2 negative and roughly linear

3 a curve falling steeply at first and then levelling off, or falling gently and then steeply — the points must fall overall and clearly not lie on a straight line

4 a negative association is a real pattern in which one variable falls as the other rises; no association means no pattern at all

5 because direction says which way it goes and shape says whether the rate is constant — a plant growing quickly then flattening is positive and non-linear

6 no — age drives both: older children have bigger feet and read better

7 the plot may hold two different groups — for example two year groups of different ages, or two classes measured with different rulers

8 it hides real information, and it makes the pattern look stronger than the evidence supports

9 because it says the claim might be wrong without showing that it is — a third variable is what actually explains the pattern

1 MILD What a point represents

two measurements taken from the same individual

1. The horizontal position is one variable, the vertical position is the other.
2. Both belong to the same person, plant or object.

Watch for: Said “a number” or “a person”. It is a person AND both of their measurements, which is what makes a scatter plot different from a bar chart.

2 MEDIUM Hot chocolate

negative and roughly linear

1. As temperature rises, sales fall every time, so the association is negative.
2. The drops are fairly steady — about 5 or 6 per 3 degrees — so it is roughly linear.
3. Total fall of 34 across the range.

Watch for: Called it positive because the numbers along the bottom increase. The direction is about how the two variables move TOGETHER, not about the horizontal axis.

3 MEDIUM Sketch a negative non-linear pattern

a curve falling steeply at first and then levelling off, or falling gently and then steeply — the points must fall overall and clearly not lie on a straight line

1. Draw axes and label them with two plausible variables.
2. Plot points that fall overall, with the steepness clearly changing along the way.
3. Check both properties: falling means negative, changing steepness means non-linear.

Watch for: Drew a straight downward line, which is negative and LINEAR. The changing steepness is what makes it non-linear, so the curve has to bend visibly.

4 SPICY Negative versus none

a negative association is a real pattern in which one variable falls as the other rises; no association means no pattern at all

1. Negative: knowing one value tells you something useful about the other.
2. None: the dots are scattered with no trend, and knowing one value tells you nothing.
3. So negative is a strong finding, and “no association” is also a finding — it just is not a relationship.

Watch for: Said negative means “bad” or “weak”. Negative describes the direction, not the strength: a negative association can be extremely strong.

5 **SPICY** Two separate questions

because direction says which way it goes and shape says whether the rate is constant — a plant growing quickly then flattening is positive and non-linear

1. Direction: are the two rising together, or moving oppositely?
2. Shape: is the rate of change constant, or does it change?
3. The plant data rises throughout, so it is positive; the weekly gains shrink from 8 to 0, so it is curved.
4. Answering only “positive” would suggest a straight line is safe, which it is not.

Watch for: Gave the two words without an example. The example is what shows they are independent, and the practical consequence — a straight-line prediction failing at the ends — is what makes the distinction matter.

6 **SPICY** Shoe size and reading age

no — age drives both: older children have bigger feet and read better

1. The association is real for children aged 4 to 14.
2. Age is a third variable increasing both.
3. Within a single age group the association would largely disappear, which is a good way to test the idea.
4. So growing feet do not improve reading, and putting a child in bigger shoes would not help.

Watch for: Said no because “that is silly”. The absurdity is a clue, not the explanation — naming age is the answer, and the same reasoning applies to claims that are NOT obviously silly, which is where it actually matters.

7 **SPICY** Two clusters

the plot may hold two different groups — for example two year groups of different ages, or two classes measured with different rulers

1. Clustering usually means two populations have been plotted together.
2. Age is the most likely cause here, since both height and arm span grow with age.
3. A measurement difference is also possible: one group measured in inches and the other in centimetres would split the plot.
4. Either way the fix is to identify the groups and look at them separately.

Watch for: Said “the data is wrong”. Two clusters is a finding about structure in the data, and it is worth investigating rather than dismissing.

8 **SPICY** Removing an outlier

it hides real information, and it makes the pattern look stronger than the evidence supports

1. First: the outlier may be the most informative point — a fault, a mistake, or a genuinely unusual case worth knowing about.
2. Second: deleting points that disagree makes any pattern look better, so the reported result is no longer a fair description of the data.
3. The honest route is to investigate the point and report what was found, including if it was removed and why.
4. “It spoils the pattern” is a reason to look harder, not a reason to delete.

Watch for: Gave one reason twice. The two reasons are different in kind: one is about losing information, the other is about misrepresenting the strength of the evidence.

9 **EXTRA SPICY** Why the slogan is not enough

because it says the claim might be wrong without showing that it is — a third variable is what actually explains the pattern

1. The slogan is a general warning, and it applies to every association whether the claim is true or false.
2. So repeating it does not distinguish a misleading claim from a sound one.
3. Naming a plausible alternative — hot weather, age — shows there IS another explanation, which is what makes the objection concrete.
4. It also protects against the opposite error: sometimes an association DOES reflect a cause, and dismissing every claim with the slogan is no better than accepting them all.

Watch for: Said the slogan is “too short”. The real problem is that it does not discriminate — it fits every case equally, so it cannot be evidence about any particular one.

Use: Homework after the practice sheet. Item 8 is the most valuable and the answer has two halves: name the plausible third variables, and say that an intervention would settle it. Watch for the mirror-image error — a child who rejects the claim flatly has made the same mistake as one who accepts it, since the association is real and only the causal step is unproven. Item 2's correct answer is “no association”, which children resist because it feels like failing to find something. Item 7's shuffling argument is the neatest way to see where the information in a scatter plot lives. **Timing:** 20–25 min · independent · items 2, 5 and 8 first when marking.

1 negative, and probably non-linear

2 no association

3 population size explains both — bigger towns have more libraries and more crime

4 positive but non-linear — a straight line would over-predict improvement from extra practice at the top end

5 an outlier, and most likely a recording error — probably the arm span was measured or typed in the wrong unit

6 for example the amount of water given to a plant and its growth, or hours worked and pay at a fixed hourly rate

7 because the pattern lives in the PAIRING — which value goes with which — and two separate lists lose that

8 whether something else explains both — such as household income or a settled home routine — and ideally whether grades improve when breakfast is actually provided

1 **MEDIUM** Car age and value

negative, and probably non-linear

- Older cars are generally worth less, so the association is negative.
- The fall is steepest in the first few years and then slows, so it is non-linear.
- Both answers are asked for.

Watch for: Gave only “negative”. The shape matters here in practice: most of the loss happens early.

2 **SPICY** Height and birth day-of-month

no association

- There is no reason for the day of the month to relate to height.
- The points would scatter with no trend, so knowing one tells you nothing about the other.
- “No association” is a real finding, not a failure to find one.

Watch for: Invented a weak pattern. If there is no reason for a relationship, no association is the expected and correct answer.

3 **SPICY** Libraries and crime

population size explains both — bigger towns have more libraries and more crime

- The association is between TOTALS, and both totals grow with the number of people.
- So population is the third variable driving each of them.
- Comparing crime PER PERSON instead of in total would remove the effect, which is the test.
- Closing libraries would not reduce crime, since the libraries were never the cause.

Watch for: Said crime attracts libraries, or the reverse. Reversing the direction is still a causal claim; the useful move is the third variable, and per-person rates are how it would be checked.

4 **SPICY** Practice and free throws

positive but non-linear — a straight line would over-predict improvement from extra practice at the top end

- Success rises with practice, so positive.
- The rise slows sharply, so the shape is a curve, not a line.
- A straight line fitted to the early steep part would keep climbing at that rate, predicting success rates far above what actually happens — possibly above 100%.
- So the shape decides whether a prediction is trustworthy, especially beyond the data.

Watch for: Answered “positive” and stopped. The second half of the question is where the value is — the over-prediction is the practical cost of ignoring shape.

5 **SPICY** (150 cm, 15 cm)

an outlier, and most likely a recording error — probably the arm span was measured or typed in the wrong unit

1. Arm span is normally close to height, so 15 cm beside 150 cm is far outside the pattern.
2. The most likely cause is a mistake: a decimal point, a unit mix-up, or a mistyped entry.
3. So it should be investigated — go back to the original measurement and check.
4. Delete it only after establishing it is an error, and say so when reporting.

Watch for: Deleted it immediately. It is almost certainly an error and that still has to be CHECKED rather than assumed — and the check is cheap.

6 **SPICY** A genuine cause

for example the amount of water given to a plant and its growth, or hours worked and pay at a fixed hourly rate

1. A good example has a mechanism you can state: water is used by the plant to grow.
2. Better still, one you could TEST by changing the cause and watching the effect — give one plant more water and see.
3. That controlled change is what separates a cause from an association.
4. So the answer should name the mechanism, not just two things that move together.

Watch for: Gave an example with no mechanism, or one with a lurking variable. Being able to state HOW one produces the other is what makes it a causal claim rather than a hopeful one.

7 **SPICY** Why paired measurements

because the pattern lives in the PAIRING — which value goes with which — and two separate lists lose that

1. A scatter plot asks whether people who are high on one variable tend to be high on the other.
2. That question is about individuals, so each point must carry both of one individual's values.
3. Two unpaired lists would let you find the average of each and nothing about their relationship.
4. Shuffling one list would destroy the association while leaving both lists unchanged, which shows where the information actually is.

Watch for: Said you need both variables. True, and the sharper point is that the PAIRING carries the information — the shuffling argument makes that concrete.

8 **EXTRA SPICY** The breakfast claim

whether something else explains both — such as household income or a settled home routine — and ideally whether grades improve when breakfast is actually provided

1. The association may be real and still not causal: families able to provide breakfast may differ in other ways that affect grades.
2. Income, sleep, and a stable routine are all plausible third variables.
3. The strong test is an intervention: provide breakfast to some schools and compare, rather than observing who already eats it.
4. Note the recommendation might be worth following anyway on other grounds — but this evidence alone does not establish it.

Watch for: Accepted or rejected the claim outright. The useful answer names what would settle it, and the intervention is the strongest form. Rejecting it flatly is the mirror-image error of accepting it: the association is real, it is the causal step that is unproven.

Exit Ticket

4 problems

Use: End of the scatter plots lesson and the first Grade 8 Statistics exit ticket. Item 3 is the diagnostic and it is deliberately not answerable with a slogan: it asks for a THIRD THING, so “correlation is not causation” earns nothing. A child who names hot weather has the idea; a child with the phrase has a phrase. Item 2's misconception is the one worth naming aloud — deleting an outlier to tidy a pattern is how a real finding gets discarded, and the (9, 5) point is about 83 marks below the trend, further than any step in the data. Items 1 and 4 together test that direction and shape are two answers. **Timing:** 5–8 min · independent · squared paper for sketching.

1 positive and roughly linear

2 an outlier — it should be investigated, not deleted

3 association is not causation — hot weather explains both: it raises ice cream sales and it also means more people swim

4 positive but non-linear — it rises, then flattens off

1 **MEDIUM** Describe the revision data

positive and roughly linear

1. As hours rise, scores rise, so the association is positive.
2. The score goes up by between 4 and 9 marks per extra hour throughout, which is close to a constant rate, so the pattern is roughly linear.
3. Both answers are needed: direction and shape are separate questions.

Watch for: Gave only “positive”. Direction alone is half the description, and the shape is what decides whether a straight line may be used later.

2 **SPICY** The ninth student

an outlier — it should be investigated, not deleted

1. Every other student scored above 30; this one scored 5 after the most revision of all.
2. The trend would predict roughly 88 at 9 hours, so the point sits around 83 marks below the pattern — far more than any step in the data.
3. So it is an outlier, and the right response is to find out WHY: illness on the day, a recording error, the wrong row copied, or a genuine and important case.
4. Deleting it would tidy the plot and might throw away the most informative point on it.

Watch for: Said to remove it so the pattern is clearer. That is the misconception exactly — an outlier is data, and a pattern made tidy by deletion is no longer a description of what happened.

3 **SPICY** The newspaper's error

association is not causation — hot weather explains both: it raises ice cream sales and it also means more people swim

1. The two do rise together, so the association is real.
2. A real association does not say which way any influence runs, or whether there is any.
3. Hot weather is a third variable that raises both independently: people buy more ice cream AND more people go swimming, so more drownings occur.
4. Notice ice cream sales could be reduced to nothing and the drownings would continue, because the cause was never the ice cream.

Watch for: Wrote “correlation is not causation” and stopped. The phrase is correct and it demonstrates nothing — the question asks for a third thing, and naming hot weather is what shows the idea is understood rather than remembered.

4 **EXTRA SPICY** The plant data

positive but non-linear — it rises, then flattens off

1. Height never falls, so the association is positive.
2. The weekly increases are 7, 8, 6, 4, 2, 1 and 0, so they shrink towards nothing — the growth is slowing.
3. A shrinking rate of change means the pattern is a curve, not a line, so it is non-linear.
4. A straight line through the early points would predict about 53 cm by week 8 instead of 32, so the shape matters for any prediction.

Watch for: Answered “positive” and stopped, or called it linear because it goes up. Rising and rising-at-a-constant-rate are different claims, and the shrinking gaps are the evidence.

Lesson slides

teaching notes & timing

Slide	Time	What to do
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1	0:00	
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Open with the data, not the vocabulary. Collect two measurements from the class if there is time — height and arm span works and gives a strong positive pattern.

2	0:45	One point per person. Worth labouring: children often plot one variable and forget each dot carries two facts about one individual.
3	1:30	Direction. Quick.
4	2:15	Shape as a SEPARATE question, and say so explicitly. The plant-growth curve is positive and non-linear, and children who have only one word for a pattern will answer “positive” and think they are finished.
5	3:15	Outliers. Ask what could produce the (9, 5) point — illness, a mistake in recording, the wrong row. Then say plainly: investigate, never delete. Deleting an outlier to tidy a pattern is how a real finding gets thrown away.
6	4:30	Clustering, briefly. Two clusters usually means two populations on one plot.
7	5:15	THE slide, and give it real time. The ice cream example, then insist on the THIRD variable. Do not accept “correlation is not causation” as an answer — ask what else could explain it, every time. A child who can produce the lurking variable has the idea; a child with the phrase has a phrase.
9	7:30	The revision plot described fully, then the exit ticket. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #4 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11		Launch the practice sheet. Exit ticket in the last 5–8 minutes.