

GRADE 8 · GEOMETRY · ANSWER KEY

SIMILARITY AND DILATIONS

Today's goal: I can understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations and given two similar two-dimensional figures, describe a sequence that exhibits the similarity.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: In-class practice after the deck. Item 2 is important in a way that is easy to miss: here the figures ARE similar and the differences are UNEQUAL, so together with item 4 the sheet shows the difference test failing in both directions — it is not a weaker test, it is the wrong test. Item 5 wants the algebra $L(k-1)$, which is what proves the pattern rather than illustrating it. Item 8 is the best discussion item: same name, different shape. **Timing:** 15–20 min · pairs · items 5 and 8 on the board.

1 similar means some sequence of translations, reflections, rotations and dilations carries one figure onto the other

2 similar — every ratio is 3

3 similar — every ratio is $3/2$

4 not similar — ratios 2, $5/3$ and $3/2$, though the differences are all 2

5 because enlarging multiplies each side by the scale factor, so a long side gains more than a short one

6 both — they are congruent, and congruent figures are similar

7 $3 : 1$ — the sides are in the ratio 3

8 every square is similar to every other square; rectangles are not

9 because congruence is similarity with scale factor 1

1 **MILD** The definition
similar means some sequence of translations, reflections, rotations and dilations carries one figure onto the other

1. It is the congruence definition with dilation added.
2. So the sequence may move the figure, turn it, flip it and resize it.

Watch for: Wrote “same shape, different size”. That is the consequence, and it is also slightly wrong — the size may be the SAME, in which case the figures are congruent and still similar.

2 **MEDIUM** 3-4-5 and 9-12-15

similar — every ratio is 3

1. $9 \div 3 = 3$, $12 \div 4 = 3$, $15 \div 5 = 3$.

Watch for: Compared differences (6, 8, 10), which are unequal here — and the figures ARE similar, so the difference test fails in this direction too.

3 **MEDIUM** 6-8-10 and 9-12-15

similar — every ratio is $3/2$

1. $9 \div 6 = 1.5$, $12 \div 8 = 1.5$, $15 \div 10 = 1.5$.
2. Both are enlargements of 3-4-5, so they must be similar to each other.

Watch for: Said not similar because the ratio is not a whole number. A scale factor may be any positive number.

4 **MEDIUM** 2-3-4 and 4-5-6

not similar — ratios 2, 5/3 and 3/2, though the differences are all 2

1. Ratios: $4 \div 2 = 2$, $5 \div 3 \approx 1.67$, $6 \div 4 = 1.5$. Not equal.
2. Differences: 2, 2, 2. Equal.
3. So this is the same trap as the exit ticket with different numbers.

Watch for: Answered similar on the strength of the differences. Two examples of the same trap in one lesson is deliberate — the pattern is what has to be unlearned.

5 **SPICY** Why enlargements give unequal differences

because enlarging multiplies each side by the scale factor, so a long side gains more than a short one

1. A side of length L becomes kL , so it gains $kL - L = L(k - 1)$.
2. That gain depends on L , so a longer side gains more.
3. Doubling 3-4-5: the gains are 3, 4 and 5 — different for each side.
4. Equal gains would need every side to be the same length to begin with, or $k = 1$, in which case nothing grew at all.

Watch for: Gave an example without the reason. The algebra $L(k-1)$ is what shows the gain **MUST** depend on the length, so the pattern cannot be a coincidence of one example.

6 **SPICY** 3 by 5 and 5 by 3

both — they are congruent, and congruent figures are similar

1. It is the same rectangle rotated 90° , so a rigid motion carries one onto the other.
2. That makes them congruent.
3. Congruent figures are similar with scale factor 1, so they are similar too.

Watch for: Said similar but not congruent, treating the swap as a change. Turning a rectangle does not resize it.

7 **SPICY** Areas 4 and 36

3 : 1 — the sides are in the ratio 3

1. Area ratio is the square of the side ratio.
2. $36 \div 4 = 9$, and the square root of 9 is 3.
3. So the sides are in the ratio 3, and a check confirms it: tripling lengths multiplies area by 9.

Watch for: Answered 9, giving the area ratio rather than the side ratio. The question asks for sides, so the square root is the last step.

8 **SPICY** Squares and rectangles

every square is similar to every other square; rectangles are not

1. A square is fixed in shape: all angles 90° and all sides equal, so only the size can differ, which is exactly a dilation.
2. A rectangle has a free ratio of length to width, so a 1 by 2 and a 1 by 5 are different shapes.
3. Check: 1 by 2 against 1 by 5 gives ratios 1 and 2.5, which disagree.
4. So “same name” does not mean “same shape”.

Watch for: Said both are always similar. Testing 1 by 2 against 1 by 5 settles the rectangle case in one line.

9 **EXTRA SPICY** Why congruent implies similar

because congruence is similarity with scale factor 1

1. The similar definition allows dilations; it does not require one.
2. A dilation of scale factor 1 leaves every point where it is.
3. So any sequence of rigid motions is already an allowed similarity sequence.
4. The reverse is not true: similar figures need not be congruent.

Watch for: Said they are different things. The definitions are nested, and noticing which way the implication runs is the useful half.

Use: Homework after the practice sheet. Item 6 is the one to mark first and it has a precise target: the difference test does not sometimes fail, it tests a different property. The algebra $L(k-1)$ is what makes that a proof rather than an observation, and a child who writes it will not reach for differences again. Item 5 is the everyday version — unequal ratios are what a stretched photograph looks like. Item 8 returns to the “Congruence through transformations” lesson’s rule that a description must carry its parameters. **Timing:** 20–25 min · independent · items 6, 5 and 8 first when marking.

1 similar — every ratio is 2

2 not similar — ratios $3/2$, $4/3$ and $5/4$, with differences all 2

3 10, 15 and 20

4 75

5 yes — both ratios are 2.5

6 because similarity is multiplicative and a difference is additive — a side of length L becomes kL and gains $L(k-1)$, which depends on L , so equal gains and equal ratios cannot both hold unless every side is already the same length

7 yes

8 dilate by scale factor 3 about a chosen centre, rotate 180° , then translate to position — any sequence with a dilation of 3 and a half turn will do

1 MEDIUM 5-12-13 and 10-24-26

similar — every ratio is 2

1. $10 \div 5 = 2$, $24 \div 12 = 2$, $26 \div 13 = 2$.

Watch for: Checked one pair only.

2 MEDIUM 4-6-8 and 6-8-10

not similar — ratios $3/2$, $4/3$ and $5/4$, with differences all 2

1. Ratios: $6 \div 4 = 1.5$, $8 \div 6 \approx 1.33$, $10 \div 8 = 1.25$. Not equal.

2. Differences: 2, 2, 2. Equal.

3. The same trap once more, and the ratios again decrease as the sides grow.

Watch for: Answered similar from the differences. The ratios shrinking as the sides grow is the tell — it happens whenever a constant has been ADDED.

3 MEDIUM Enlarge by 2.5

10, 15 and 20

1. $4 \times 2.5 = 10$, $6 \times 2.5 = 15$, $8 \times 2.5 = 20$.

2. A scale factor need not be a whole number.

Watch for: Added 2.5, or rounded the factor to 3 first.

4 MEDIUM Ratio 5, smaller area 3

75

1. Area is multiplied by $5^2 = 25$.

2. $3 \times 25 = 75$.

Watch for: Answered 15, multiplying by 5 rather than 25.

5 MEDIUM 4 by 6 to 10 by 15

yes — both ratios are 2.5

1. $10 \div 4 = 2.5$ and $15 \div 6 = 2.5$.

2. Equal ratios, so the enlargement is similar and the photograph is not distorted.

Watch for: Checked the differences (6 and 9) and said no. Photographs are the everyday case where getting this wrong is visible: unequal ratios are what stretching looks like.

6 **SPICY** Why differences can never test similarity

because similarity is multiplicative and a difference is additive — a side of length L becomes kL and gains $L(k - 1)$, which depends on L , so equal gains and equal ratios cannot both hold unless every side is already the same length

1. Enlarging multiplies each length by the same k . The GAIN is then $L(k - 1)$, which is bigger for a bigger side.
2. So a genuine enlargement has unequal differences, and equal differences means something was ADDED — which is a translation, not an enlargement.
3. The two tests can even disagree in both directions: 3-4-5 to 6-7-8 has equal differences and is not similar, while 3-4-5 to 9-12-15 has unequal differences and IS similar.
4. So it is not a weaker test. It is a test of something else.

Watch for: Said the difference test “sometimes fails”. It does not sometimes fail — it tests a different property entirely, and saying so is what stops a child reaching for it again.

7 **SPICY** All circles similar?

yes

1. A circle's shape is completely determined by nothing but its radius — there is no second measurement that could differ.
2. So any circle can be carried onto any other by a translation to match centres and a dilation to match radii.
3. The scale factor is the ratio of the radii.
4. This is why the squares argument works too, and why rectangles fail: a rectangle has two independent measurements, a circle has one.

Watch for: Said no because they are different sizes. Different size is exactly what similarity allows.

8 **EXTRA SPICY** 3-4-5 onto an upside-down 9-12-15

dilate by scale factor 3 about a chosen centre, rotate 180° , then translate to position — any sequence with a dilation of 3 and a half turn will do

1. The ratio $9:3 = 12:4 = 15:5 = 3$, so the scale factor is 3.
2. “Upside down” is a rotation of 180° , which is a rigid motion and does not affect the ratios.
3. Then a translation to put it in the right place.
4. The order is flexible, and the answer must NAME the parameters — scale factor 3, half turn — or nobody could carry it out.

Watch for: Said “enlarge it and flip it”. Similar to the “Congruence through transformations” lesson: a description without a scale factor and an angle is not a description. Also worth noting a 180° rotation is not a reflection, so the orientation is unchanged.

Exit Ticket

4 problems

Use: End of the similarity lesson. Items 2 and 4 are one diagnostic in two parts, and the sheet deliberately asks for the DIFFERENCES first: a child who computes $+3$, $+3$, $+3$ and stops has done real work and reached a wrong conclusion, which is exactly the error being tested. The strongest answers to item 4 notice that a genuine enlargement has UNEQUAL differences — doubling 3-4-5 gains 3, 4 and 5 — so equal differences is a translation's fingerprint. This is the same additive-versus-multiplicative error as the previous skill's dilation-by-adding, and naming the connection aloud is worth the thirty seconds.

Timing: 5–8 min · independent · calculator not needed.

1 similar — every ratio is 2

2 differences 3, 3, 3; ratios 2, 7/4 and 8/5

3 24

4 the ratios are 2, 1.75 and 1.6 — not equal — so C is not similar to A, even though the differences are all 3

1 **MEDIUM** A and B

similar — every ratio is 2

1. $6 \div 3 = 2$, $8 \div 4 = 2$, $10 \div 5 = 2$.
2. All three ratios agree, so a dilation of scale factor 2 (after any needed motions) carries A onto B.
3. The sides were paired smallest to smallest, which is what makes the ratios comparable.

Watch for: Checked only one pair. One matching ratio proves nothing — all of them have to agree.

2 **MEDIUM** C's differences and ratios

differences 3, 3, 3; ratios 2, 7/4 and 8/5

1. Differences: $6 - 3 = 3$, $7 - 4 = 3$, $8 - 5 = 3$.
2. Ratios: $6 \div 3 = 2$, $7 \div 4 = 1.75$, $8 \div 5 = 1.6$.
3. The differences are all equal. The ratios are not.

Watch for: Worked out only one of the two. Both are needed, because the whole point of item 4 is the contrast between them.

3 **MEDIUM** B's area

24

1. Lengths are multiplied by 2, so the area is multiplied by $2^2 = 4$.
2. $6 \times 4 = 24$.
3. Check directly: B is a 6-8-10 triangle, and half of 6×8 is 24.

Watch for: Answered 12, multiplying the area by 2. Both dimensions grew, so the area factor is the square of the length factor — the same result as the dilation work in the previous skill.

4 **SPICY** Correct Ben

the ratios are 2, 1.75 and 1.6 — not equal — so C is not similar to A, even though the differences are all 3

1. Similarity is about ratios, because enlarging MULTIPLIES every length by the same number.
2. Multiplying means a long side gains more than a short one, so a genuine enlargement has UNEQUAL differences: doubling 3-4-5 gains 3, then 4, then 5.
3. So equal differences is the fingerprint of a translation, not of an enlargement.
4. On this pair the difference test does not just fail to help — it actively points the wrong way, which is why it is worth testing with ratios every time.

Watch for: Said “you have to multiply, not add” without using the ratios. That is the right instinct and it is not yet the argument — the answer needs the three unequal ratios, and ideally the observation that a real enlargement's differences are unequal.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Open by connecting to congruence rather than starting fresh: same definition, one more move allowed. Children meet congruent and similar as two unrelated words, and they are the same idea with and without dilation.
2	0:45	Say the scale-factor-1 sentence explicitly: congruent figures ARE similar. It is usually a surprise and it makes the relationship between the two clear.
3	1:30	The ratio test. Emphasise ALL ratios — one matching pair proves nothing.
4	2:15	The clean case. Fast.
5	3:00	THE slide. Do the differences FIRST and let the class enjoy the neat +3, +3, +3. Then do the ratios. The point lands hardest if they have committed to the wrong test before it fails.
6	4:30	Why differences cannot work, and this is the reason to spend time on: enlarging MULTIPLIES, so a long side gains more than a short one. Doubling 3-4-5 gains 3, 4 and 5 — the differences are deliberately unequal. Equal differences is what a TRANSLATION looks like, which is exactly what the previous skill showed.
7	6:15	Pairing. Order both lists first. $10 \div 3$ is a real division of two real numbers that answers no question.
9	7:30	Both pairs, both tests, then the exit ticket. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #5 on the practice sheet.

10	Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11	Launch the practice sheet. Exit ticket in the last 5–8 minutes.
