



Slope as a unit rate

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the slope-as-rate lesson. 5–8 min, independent. Item 2 is the diagnostic and item 4 is the one that matters most for later work: comparing raw totals across different denominators is a habit that breaks every rate question in Grade 8 and beyond.

Timing: 5–8 min · independent. Graph paper available.

1. Slope through (0,0) and (3,12) **FLUENCY**

Answer: **4**

1. Change in y is 12, change in x is 3.
2. $\frac{12}{3} = 4$. So 4 units of y per 1 of x.

If wrong: Answered $\frac{1}{3} = 0.25$ by inverting.

2. Is the slope $\frac{1}{4}$? **REASONING**

Answer: **no — it is $\frac{10}{4} = 2.5$**

1. Slope is change in y over change in x, so y goes on top: $\frac{10}{4}$.
2. $\frac{10}{4} = 2.5$ and $\frac{1}{4} = 0.4$ — not close.
3. As a rate it has to be 2.5: for every 1 across, the line goes up 2.5. Going up 0.4 for every 1 across would be a much flatter line than the points describe.
4. Sketching the two would settle it instantly.

If wrong: Agreed. Inverting is the commonest error in the topic and it is hard to spot because the wrong answer still looks like a slope. Checking against the rate in words is the reliable defence.

3. Slope with a scaled axis APPLICATION

Answer: **4**

1. The scale matters: 6 squares up at 2 units each is a rise of 12.
2. The run is 3 squares at 1 unit each, so 3.
3. Slope = $\frac{12}{3} = 4$.
4. Counting squares would give $\frac{6}{3} = 2$ — exactly half, because the y-scale was doubled.

If wrong: Answered 2 by counting squares rather than units. Same skipped step as reading a bar graph without its scale, and it is invisible on the page — the counting was correct.

4. Car A or Car B? REASONING

Answer: **B — 20 mph against 15 mph**

1. Reduce both to a rate per one hour.
2. A: $45 \div 3 = 15$ mph.
3. B: $40 \div 2 = 20$ mph.
4. So B is faster, even though 45 is a bigger number than 40.
5. 45 and 40 are not comparable — they cover different amounts of time. Only the rates are.

If wrong: Chose A because $45 > 40$. The distances are over different times, so comparing them directly answers a question nobody asked. Reducing to a unit rate is the whole point of the standard.

Quick answers for fast checking:

#	Level	Answer
1	fluency	4
2	reasoning	no — $1\frac{0}{4} = 2.5$
3	application	4
4	reasoning	B, at 20 mph against 15

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 2 is the one to check carefully: inverting gives 2, which is a completely plausible slope, so the error hides. Items 6 and 8 are the raw-total trap in two settings, and item 6 is the crueller one because the bigger total is the more efficient recipe.

Timing: 15–20 min · pairs · graph paper · items 4 and 9 on the board.

1. (0,0) to (5,15) FLUENCY

Answer: **3**

1. $15/5 = 3$.

If wrong: Answered $5/15$.

2. (0,0) to (8,4) FLUENCY

Answer: **$\frac{1}{2}$**

1. $4/8 = \frac{1}{2}$.

2. Less than 1, so the line is shallow — half a unit up for every one across.

If wrong: Answered 2 by inverting. This item is where inverting is hardest to spot, because 2 is a perfectly plausible slope.

3. (0,8) to (4,0) APPLICATION

Answer: **-2 — the line falls**

1. Change in y is $0 - 8 = -8$. Change in x is 4.

2. $-8/4 = -2$.

3. The negative means the line goes DOWN as x increases: two units down for every one across.

If wrong: Gave 2 by ignoring the direction, or $-1/2$ by inverting. The sign carries meaning and should be interpreted, not just carried.

4. Why y over x REASONING

Answer: **an explanation that slope answers how much y changes per ONE unit of x, so x must be the divisor**

1. A rate is always ‘how much of this per one of that’.

2. Slope answers: for every 1 across, how far up?

3. Dividing by the run is what makes the run equal 1.

4. Inverting it answers a different question — how far across per one up — which is a real quantity and not the slope.

If wrong: Said “because that is the formula”. The per-one-of-x framing is the reason, and it is also what makes the inverting error self-correcting.

5. Scaled y-axis, up in 5s APPLICATION

Answer: **10**

1. 4 squares up at 5 units each is a rise of 20.
2. Run of 2 squares at 1 unit each is 2.
3. $2\frac{0}{2} = 10$.
4. Counting squares would give $\frac{1}{2} = 2$ — five times too small.

If wrong: Answered 2. The factor of error equals the scale, which is a useful thing to point out.

6. Recipe A or B? APPLICATION

Answer: **A — 3 cups per loaf against 2.4**

1. A: $6 \div 2 = 3$ cups per loaf.
2. B: $12 \div 5 = 2.4$ cups per loaf.
3. A uses more per loaf, even though B uses more flour in total.
4. 'More flour' and 'more flour per loaf' are different questions.

If wrong: Chose B because $12 > 6$. The raw-totals comparison again, in a setting where the bigger total is the more efficient recipe.

7. Printer through (0,0) and (4,10) APPLICATION

Answer: **15 pages**

1. Unit rate: $1\frac{0}{4} = 2.5$ pages per minute.
2. $6 \times 2.5 = 15$ pages.

If wrong: Multiplied by the inverted rate to get 2.4, or added on rather than scaling.

8. $y = 3x$ against a table REASONING

Answer: **$y = 3x$ — a rate of 3 against 2.5**

1. From $y = 3x$ the unit rate is 3.
2. From the table: $10 \div 4 = 2.5$.
3. $3 > 2.5$, so the EQUATION has the greater unit rate.
4. Both had to be reduced to a per-one rate before they could be compared at all.

If wrong: Compared 10 with 3 and chose the table. The equation's 3 is already a rate; the table's 10 is not, so they are not comparable as written.

9. Why through the origin REASONING

Answer: **an explanation that a proportional relationship means a constant rate, so zero of one quantity must give zero of the other**

1. Proportional means y is always the same multiple of x .
2. Put $x = 0$: y must be 0 too.
3. Concretely: zero hours of driving covers zero miles. A line with a starting amount is linear but not proportional.

If wrong: Said “because the graphs always do”. The zero-of-one-means-zero-of-the-other argument is the reason and it is what distinguishes $y = mx$ from $y = mx + b$ in the next lesson.

Quick answers for fast checking:

#	Level	Answer
1	fluency	3
2	fluency	$\frac{1}{2}$
3	application	-2, the line falls
4	reasoning	per one unit of x
5	application	10
6	application	A, 3 cups per loaf
7	application	15 pages
8	reasoning	$y = 3x$, rate 3 against 2.5
9	reasoning	zero of one gives zero of the other

Homework — answer key

Use: Homework after the practice sheet. Item 5 is deliberately one where comparing raw totals gives the RIGHT answer, so the working is the only way to tell whether the method was sound. Item 8 sets up the distinction between $y = mx$ and $y = mx + b$ for the next lesson.

Timing: 20–25 min · independent · items 5, 7 and 8 first when marking.

1. (0,0) to (6,18) FLUENCY

Answer: **3**

1. $1\frac{8}{6} = 3$.

If wrong: Inverted to $\frac{1}{3}$.

2. (0,0) to (10,4) FLUENCY

Answer: $\frac{2}{5}$ or 0.4

1. $\frac{4}{10} = 0.4$.

If wrong: Answered 2.5 by inverting — and 2.5 looks more like a ‘proper’ slope than 0.4, which is why this direction of the error is the more common one.

3. (0,6) to (3,0) APPLICATION

Answer: **-2 — the line falls two units for every one across**

1. $-\frac{6}{3} = -2$.

2. Negative means decreasing.

If wrong: Gave 2, dropping the sign.

4. y-axis in 4s APPLICATION

Answer: **6**

1. 3 squares at 4 units each is a rise of 12, over a run of 2.

2. $1\frac{1}{2} = 6$.

If wrong: Answered 1.5 by counting squares.

5. Tap A or B? APPLICATION

Answer: **A — 8 litres per minute against 7.5**

1. A: $24 \div 3 = 8$ l/min.

2. B: $15 \div 2 = 7.5$ l/min.

3. A is faster, and this time the bigger total is also the faster tap — so getting it right by comparing totals proves nothing.

If wrong: Chose A for the wrong reason. Read the working: this item is deliberately one where the lazy comparison happens to give the right answer, which is why the rates must be shown.

6. $y = 2.5x$ at $x = 8$ APPLICATION

Answer: **20, with a unit rate of 2.5**

- $2.5 \times 8 = 20$.
- The coefficient of x IS the unit rate.

If wrong: Did not recognise that the coefficient is the rate, and recomputed it from the point.

7. Correct Zoe REASONING

Answer: **the slope is $\frac{7}{2} = 3.5$**

- Slope is change in y over change in x , so 7 goes on top.
- $\frac{7}{2} = 3.5$, not $\frac{2}{7} \approx 0.29$.
- She has inverted it. Checking as a rate catches it: from $(0,0)$ to $(2,7)$ the line climbs 7 while going across only 2, so it must be steep — a slope below 1 would be nearly flat.

If wrong: Corrected the fraction without saying why. The rate check is the transferable part, because it works without remembering which way up the formula goes.

8. Not through the origin REASONING

Answer: **no — proportional requires zero to give zero**

- Proportional means a constant multiple: $y = mx$.
- At $x = 0$ that forces $y = 0$, so the graph must pass through the origin.
- A cost graph starting at £3 has a fixed charge, so it is linear but NOT proportional.
- The unit rate still exists as a slope; it just no longer equals $y \div x$ at every point.

If wrong: Said yes because the graph is a straight line. Straight and proportional are not the same thing, and this is the distinction the next lesson depends on.

Quick answers for fast checking:

#	Level	Answer
1	fluency	3
2	fluency	0.4
3	application	-2
4	application	6
5	application	A, 8 l/min against 7.5
6	application	20, rate 2.5
7	reasoning	$\frac{7}{2} = 3.5$

8	reasoning	no — zero must give zero
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Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Slope as a RATE from the first minute. Children who meet it as a formula about grids struggle to use it about anything else.
2/8	1:00	Rise over run, and say out loud that the order is not arbitrary. Write both $\frac{10}{4}$ and $\frac{1}{4}^{10}$ and evaluate them — 2.5 against 0.4 is a big visible gap.
3/8	2:30	The bridge to unit rate, with units attached. This is what makes slope mean something rather than being a number extracted from a picture.
4/8	4:00	THE slide they skip. Draw a grid whose y-axis goes up in 2s and ask for the slope. Half the class will count squares. Same demand as reading a bar graph's scale.
5/8	5:30	Steeper means a bigger rate. Quick.
6/8	6:30	The comparison across representations. Do NOT let them answer before reducing both to a per-hour rate — the temptation to compare 45 with 40 is strong.
7/8	8:00	Name why the raw numbers are incomparable: different denominators. This is Grade 6 ratio reasoning being used, not new content.
8/8	9:30	Two to try, then the exit ticket. Item 3 there is the scale one.