



Square and cube roots

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the roots lesson. 5–8 min, independent. Items 1 and 2 are one diagnostic in two halves: the same answer to both means the equation/symbol distinction has not landed, whichever answer it is.

Timing: 5–8 min · independent. Mark 1 and 2 as a pair.

1. Solve $x^2 = 49$ **FLUENCY**

Answer: $x = 7$ or $x = -7$

1. $7 \times 7 = 49$.
2. And $(-7) \times (-7) = 49$, because a negative times a negative is positive.
3. So $x = \pm 7$ — both satisfy the equation.

If wrong: Gave only $x = 7$. This is the commonest error in the topic and it costs a root in every equation from here to quadratics.

2. Evaluate $\sqrt{49}$ **REASONING**

Answer: **7 — the symbol means the principal root only**

1. $\sqrt{49} = 7$, not ± 7 .
2. The radical symbol denotes the PRINCIPAL root, which is the non-negative one. That is a convention, and it exists so that $\sqrt{49}$ names a single number.
3. Item 1 was an EQUATION: two different numbers satisfy it.
4. Item 2 is a SYMBOL: it evaluates to one number.
5. Which is why $x^2 = 49$ is written $x = \pm\sqrt{49}$ — the $\pm$ comes from the equation.

If wrong: Answered ± 7 , carrying the equation's two solutions onto the symbol. Read items 1 and 2 together: getting both right is the skill, and getting the same answer to both means the distinction has not landed.

3. $\sqrt[3]{27}$ and $\sqrt[3]{125}$ FLUENCY

Answer: **3 and 5**

1. $3 \times 3 \times 3 = 27$, so $\sqrt[3]{27} = 3$.
2. $5 \times 5 \times 5 = 125$, so $\sqrt[3]{125} = 5$.

If wrong: Answered 9 and about 41 by dividing by 3. Check by cubing: $9^3 = 729$, nowhere near 27.

4. Is Noor right? REASONING

Answer: **no — $x = -2$**

1. She is right about SQUARE roots: no real number squares to a negative, because a square is never negative.
2. But cubes keep their sign. $(-2) \times (-2) \times (-2) = -8$.
3. So $\sqrt[3]{-8} = -2$, and $x = -2$ is the only real solution.
4. The rule she used is real; she extended it one operation too far.

If wrong: Agreed with her. Worth crediting the part she has right — the square-root rule — before correcting the over-extension, because the reasoning is sound and only its scope is wrong.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$x = \pm 7$
2	reasoning	7
3	fluency	3 and 5
4	reasoning	no — $x = -2$

Practice Worksheet — answer key

Use: In-class practice after the deck. Items 2, 3 and 5 form a set worth marking together: two solutions for a square equation, one value for a symbol, one solution for a cube equation. A child who gives $\pm$ to all three, or to none, has a rule rather than the distinction.

Timing: 15–20 min · pairs · items 4 and 6 on the board. Keep the squares and cubes visible.

1. Squares and cubes FLUENCY

Answer: **1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 — and 1, 8, 27, 64, 125**

1. Squares: 1^2 to 12^2 .
2. Cubes: 1^3 to 5^3 .
3. 64 appears in both lists, which is worth noticing.

If wrong: Stopped the squares at 100. The 121 and 144 matter because bracketing questions reach past 100.

2. $x^2 = 81$ and $x^2 = 1$ FLUENCY

Answer: **$x = \pm 9$ and $x = \pm 1$**

1. $9 \times 9 = 81$ and $(-9) \times (-9) = 81$.
2. $1 \times 1 = 1$ and $(-1) \times (-1) = 1$.

If wrong: Gave only the positive root. $x^2 = 1$ catches this especially often, because $x = 1$ feels complete.

3. $\sqrt{81}$, $\sqrt{144}$, $\sqrt[3]{64}$ FLUENCY

Answer: **9, 12 and 4**

1. $\sqrt{81} = 9$ and $\sqrt{144} = 12$ — principal roots, one value each.
2. $\sqrt[3]{64} = 4$, because $4 \times 4 \times 4 = 64$.

If wrong: Wrote ± 9 and ± 12 , carrying the equation habit onto symbols. Or answered $\sqrt[3]{64} = 8$, confusing it with $\sqrt{64}$.

4. Why two, and why one REASONING

Answer: **an explanation that the equation asks which numbers satisfy it (two do) while the symbol denotes the principal root by convention (one value)**

1. $x^2 = 36$ asks: which numbers, squared, give 36? Both 6 and -6 do.
2. $\sqrt{36}$ is a symbol for a specific number, and the convention is that it means the non-negative one.
3. So the equation has two solutions and the symbol has one value.
4. This is why we WRITE $x = \pm\sqrt{36}$ — the $\pm$ supplies the second solution the symbol will not.

If wrong: Said “because the symbol only means positive” without addressing the equation, which is half the answer. Both halves are the skill.

5. $x^3 = 64$ and $x^3 = -27$ APPLICATION

Answer: $x = 4$ and $x = -3$

1. $4^3 = 64$.
2. $(-3)^3 = -27$, since three negatives multiply to a negative.
3. One solution each — cube equations do not gain a second the way square equations do.

If wrong: Gave ± 4 , over-applying the two-solutions rule from squares. Or said $x^3 = -27$ has no solution.

6. Squares versus cubes and negatives REASONING

Answer: **an explanation that a square is never negative because the signs cancel in pairs, while an odd number of negative factors leaves the result negative**

1. Squaring multiplies two numbers of the SAME sign, so the answer is always positive. Nothing squares to -9 .
2. Cubing multiplies THREE. If they are all negative, two cancel and one is left over, so the answer is negative.
3. Odd and even is the whole difference.

If wrong: Said “you just can’t” for squares. The pairing-up argument is what makes both halves make sense at once, and it is worth insisting on.

7. $x^2 = 2$ APPLICATION

Answer: $x = \pm\sqrt{2}$, which is irrational

1. No whole number squares to 2: $1^2 = 1$ and $2^2 = 4$.
2. So $x = \pm\sqrt{2} \approx \pm 1.414$.
3. $\sqrt{2}$ is irrational — its decimal never repeats and no fraction gives it exactly.
4. Two solutions, and neither is rational.

If wrong: Gave a decimal such as 1.41 as if exact. An approximation is not the solution; $\pm\sqrt{2}$ is. Or forgot the negative root, which this item shares with item 2.

8. Cube of volume 125 APPLICATION

Answer: **5 cm**

1. Volume of a cube is edge $\times$ edge $\times$ edge.
2. So the edge is $\sqrt[3]{125} = 5$ cm.
3. Only the positive root makes sense — a length cannot be negative.

If wrong: Answered 41.7 by dividing by 3. Or gave ± 5 : mathematically the cube root is unique anyway, and a negative length would be meaningless regardless.

9. Correct Ivan REASONING

Answer: **$\sqrt[3]{64} = 4$ — rooting is not dividing**

1. Dividing by 3 and taking a cube root are different operations.
2. Check his answer: $21\frac{1}{3}$ cubed is over 9000, not 64.
3. $\sqrt[3]{64}$ asks what number times itself times itself gives 64. That is 4, since $4 \times 4 \times 4 = 64$.
4. Same error as halving to find a square root — one operation further out.

If wrong: Corrected the number without naming the error, or agreed with him. Cubing his answer is the fastest way to show it cannot be right, and it is a check they can always run themselves.

Quick answers for fast checking:

#	Level	Answer
1	fluency	squares to 144, cubes to 125
2	fluency	± 9 and ± 1
3	fluency	9, 12, 4
4	reasoning	equation has two solutions, symbol has one value
5	application	4 and -3
6	reasoning	signs cancel in pairs; odd leaves one over
7	application	$\pm\sqrt{2}$, irrational
8	application	5 cm
9	reasoning	4 — rooting is not dividing

Homework — answer key

Use: Homework after the practice sheet. Item 8 is the whole lesson in one question, and item 6 is its mirror: one asks a child to take a $\pm$ away from a symbol, the other asks them not to add one where nothing calls for it.

Timing: 20–25 min · independent · items 6 and 8 first when marking.

1. $x^2 = 100$ **FLUENCY**

Answer: $x = \pm 10$

1. $10^2 = 100$ and $(-10)^2 = 100$.

If wrong: Gave only 10.

2. $\sqrt{100}$ and $\sqrt[3]{8}$ **FLUENCY**

Answer: **10 and 2**

1. $\sqrt{100} = 10$, the principal root.
2. $\sqrt[3]{8} = 2$, since $2 \times 2 \times 2 = 8$.

If wrong: Wrote ± 10 , or answered $\sqrt[3]{8} = 4$ by confusing it with $\sqrt{}$.

3. $x^3 = -125$ **APPLICATION**

Answer: $x = -5$

1. $(-5)^3 = -125$.
2. One solution only, and it is negative — cubes keep their sign.

If wrong: Said no solution, or gave ± 5 .

4. $\sqrt{121}$ and $\sqrt{144}$ **FLUENCY**

Answer: **11 and 12**

1. $11^2 = 121$ and $12^2 = 144$.

If wrong: Left blank — these are the two squares most often missing from a memorised list, and they are exactly the ones bracketing questions need.

5. Two for a square, one for a cube **REASONING**

Answer: **an explanation that two numbers of opposite sign square to the same positive value, while only one number cubes to a given value because cubing preserves sign**

1. Squaring: 4 and -4 both give 16, because the signs cancel in pairs.
2. Cubing: only 2 gives 8. $(-2)^3 = -8$, a different answer.
3. So a square equation gains a second solution and a cube equation does not.

If wrong: Said cube roots are “just different”. Sign preservation is the reason and is the answer.

6. Cube of volume 27 APPLICATION

Answer: **3 m — a length cannot be negative, and a cube root is unique anyway**

1. $\sqrt[3]{27} = 3$, so each edge is 3 m.
2. Two reasons not to write $\pm$: the cube root of 27 is uniquely 3, and even if it were ambiguous a length could not be negative.
3. The $\pm$ belongs to SQUARE equations, and there is no square equation here.

If wrong: Wrote ± 3 , applying the plus-or-minus habit where nothing calls for it. Both reasons are worth having; either one earns the credit.

7. $x^2 = 5$ APPLICATION

Answer: **$x = \pm\sqrt{5}$, which is irrational**

1. No whole number squares to 5: $2^2 = 4$ and $3^2 = 9$.
2. So $x = \pm\sqrt{5} \approx \pm 2.236$.
3. 5 is not a perfect square, so $\sqrt{5}$ is irrational.

If wrong: Gave 2.24 as the exact answer. The exact answer is $\pm\sqrt{5}$; the decimal is a rounding of it.

8. Correct Priya REASONING

Answer: **$\sqrt{36} = 6$ — the symbol has one value; the $\pm$ comes from an equation**

1. $\sqrt{36} = 6$, by the convention that the radical denotes the principal root.
2. She is thinking of $x^2 = 36$, which does have two solutions: $x = \pm 6$.
3. So her $\pm$ is real, but it belongs to the equation rather than to the symbol.
4. Being precise matters: if $\sqrt{}$ meant two values, it could not be used inside a formula.

If wrong: Agreed with her, or corrected the value without explaining where her $\pm$ legitimately comes from. Naming the correct home of the $\pm$ is what stops the confusion returning.

Quick answers for fast checking:

#	Level	Answer
1	fluency	± 10
2	fluency	10 and 2
3	application	-5
4	fluency	11 and 12
5	reasoning	cubing preserves sign; squaring cancels it
6	application	3 m

7	application	$\pm\sqrt{5}$, irrational
8	reasoning	$\sqrt{36} = 6$; the $\pm$ belongs to the equation

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Set up the whole lesson as two questions that sound the same when spoken. Say both aloud so the class hears that speech cannot distinguish them and notation must.
2/8	1:00	Solve the equation first, and get the negative root from the class rather than announcing it. Someone always spots it once you ask “anything else?”
3/8	2:30	Now the symbol. State plainly that it denotes the principal root by convention — this is a definition, not something they should be able to derive.
4/8	4:00	THE slide. The plus-or-minus belongs to the equation, not to the radical. Leave this on the board for the rest of the unit; it is the line children come back to.
5/8	5:30	Cube roots. One answer, sign preserved. Contrast deliberately with squares, where there were two.
6/8	7:00	The dividing error, killed by cubing 9. Same shape as the halving error they met locating roots, and worth naming as the same habit.
7/8	8:30	Squares to 144 and cubes to 125. Fluency here is what makes the rest quick; keep them visible for the practice sheet.
8/8	10:00	$x^2 = 2$ connects back to rational and irrational numbers, then the exit ticket. Item 2 there is the one to watch.