

GRADE 8 · EXPRESSIONS & EQUATIONS · ANSWER KEY

SYSTEMS OF TWO EQUATIONS

Today's goal: I can understand that solutions to a system of two linear equations correspond to points of intersection of their graphs, solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing and solve real-world and mathematical problems leading to two linear equations in two variables.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

9 problems

Use: In-class practice after the deck. Item 3 is deliberately a point that passes BOTH checks — so a child who checks only one is right by luck and has learned nothing. Items 8 and 9 are the parallel pair from the one-variable lesson in graphical clothing. **Timing:** 15–20 min · pairs · graph paper for item 6 · items 7 and 9 on the board.

1 (5, 2)

2 (3, 6)

3 yes — it satisfies both

4 13 and 7

5 (4, 2)

6 (2, 3)

7 because every point on a line satisfies that line's equation, so passing one tells you only that the point is on one line

8 none — parallel

9 infinitely many — the same line

1 **MEDIUM** $x + y = 7, x - y = 3$
(5, 2)

1. Add: $2x = 10$, so $x = 5$.
2. Then $y = 7 - 5 = 2$.
3. Check: $5 + 2 = 7$ ✓ and $5 - 2 = 3$ ✓.

Watch for: Stopped at $x = 5$.

2 **MEDIUM** $y = 2x, x + y = 9$
(3, 6)

1. Substitute $y = 2x$ into the second: $x + 2x = 9$, so $3x = 9$ and $x = 3$.
2. Then $y = 2(3) = 6$.
3. Check: $3 + 6 = 9$ ✓ and $6 = 2(3)$ ✓.

Watch for: Gave (3, 3), substituting into the wrong equation for y .

3 **MEDIUM** Does (2,5) solve both?

yes — it satisfies both

1. $y = 2x + 1$ at $x = 2$ gives 5. ✓
2. $x + y = 2 + 5 = 7$. ✓
3. Both hold, so (2,5) IS the solution.
4. Both checks were needed to know that.

Watch for: Checked one and concluded. Here both happen to pass, which is exactly why the habit matters — a child who checks one and is right has learned nothing.

4 **MEDIUM** Add to 20, differ by 6

13 and 7

1. Let the numbers be x and y : $x + y = 20$ and $x - y = 6$.
2. Add: $2x = 26$, so $x = 13$, and $y = 7$.
3. Check: $13 + 7 = 20$ ✓ and $13 - 7 = 6$ ✓.

Watch for: Halved 20 to get 10 and 10, then adjusted by 6 to get 16 and 4 — which differ by 12, not 6. Setting up the two equations avoids the guesswork.

5 MEDIUM $3x + 2y = 16$ and $x = 4$

(4, 2)

1. $x = 4$ is already one coordinate.
2. Substitute: $12 + 2y = 16$, so $2y = 4$ and $y = 2$.
3. Check: $3(4) + 2(2) = 16$ ✓.

Watch for: Stopped at $x = 4$, treating the given equation as the answer. It is half of it.

6 MEDIUM Sketch and solve

(2, 3)

1. Plot both lines: one rising, one falling.
2. They cross at (2, 3).
3. Algebraically: $x + 1 = -x + 5$, so $2x = 4$ and $x = 2$, then $y = 3$.
4. The sketch predicts the answer and the algebra confirms it.

Watch for: Read the crossing point off an inaccurate sketch and did not check. A graph gives an estimate; the algebra gives the value, and the standard asks for both.

7 SPICY Why one check is not enough

because every point on a line satisfies that line's equation, so passing one tells you only that the point is on one line

1. Take $x + y = 10$. The points (6,4), (7,3) and (2,8) all satisfy it.
2. Only one of them is also on the other line.
3. So passing the first equation says the point is somewhere on the first line, which was never in doubt.
4. The solution has to be on BOTH lines, so it must pass BOTH.

Watch for: Said “to make sure you did not make a mistake”. That is a reason to check at all; this item asks why one check is insufficient, and the answer is that a line has infinitely many points on it.

8 SPICY $y = 4x + 3$ and $y = 4x - 1$

none — parallel

1. Same slope 4, different intercepts.
2. Equating gives $3 = -1$, which is false.
3. They never meet.

Watch for: Gave a point, or answered infinitely many.

9 EXTRA SPICY $y = x + 2$ and $3y = 3x + 6$

infinitely many — the same line

1. Divide the second by 3: $y = x + 2$.
2. Identical to the first.
3. Every point on the line satisfies both.

Watch for: Answered no solution because the equations look different. Every term must divide through for the lines to coincide, and here they do.

Homework

8 problems

Use: Homework after the practice sheet, and the last sheet in Grade 8 Expressions & Equations. Item 8 is the core diagnostic of the whole skill: $x = 6$ describes a vertical line rather than a point, and once a child sees the answer as a LOCATION the missing coordinate stops being a rule to remember. **Timing:** 20–25 min · independent · items 3, 7 and 8 first when marking.

1 (7, 5)

2 (4, 12)

3 yes — it satisfies both

4 adult £7, child £4

5 none — parallel

6 infinitely many — same line

7 the point where the two lines cross

8 the y-coordinate — the solution is a point

1 MEDIUM $x + y = 12, x - y = 2$

(7, 5)

1. Add: $2x = 14, x = 7$, then $y = 5$.
2. Check both: $12 \checkmark$ and $2 \checkmark$.

Watch for: Stopped at $x = 7$.2 MEDIUM $y = 3x, x + y = 16$

(4, 12)

1. $x + 3x = 16$, so $x = 4$ and $y = 12$.
2. Check: $4 + 12 = 16 \checkmark$.

Watch for: Gave (4, 4).

3 MEDIUM Does (3,4) solve both?

yes — it satisfies both

1. $x + y = 3 + 4 = 7 \checkmark$.
2. $2x - y = 6 - 4 = 2 \checkmark$.
3. Both hold, so (3,4) IS a solution.

Watch for: This is a second deliberate case where both checks pass. Marking it requires reading whether BOTH were done, not whether the verdict is right.

4 MEDIUM Two tickets, £11, £3 apart

adult £7, child £4

1. $a + c = 11$ and $a - c = 3$.
2. Add: $2a = 14$, so $a = 7$ and $c = 4$.
3. Check: $7 + 4 = 11 \checkmark$ and $7 - 4 = 3 \checkmark$.

Watch for: Guessed £8 and £3, which differ by 5. Or halved to £5.50 each and added 3 to one. Setting up two equations removes the guessing.5 SPICY $y = 5x + 2$ and $y = 5x + 9$

none — parallel

1. Same slope, different intercepts. Equating gives $2 = 9$.

Watch for: Gave a point.6 SPICY $y = 2x - 3$ and $4y = 8x - 12$

infinitely many — same line

1. Divide the second by 4: $y = 2x - 3$.
2. Identical.

Watch for: Answered no solution.

7 SPICY What the solution means on a graph

the point where the two lines cross

1. Each equation is a line: all the points satisfying it.
2. A point on both lines satisfies both equations.
3. So the solution is the intersection, and its coordinates are the x and y values.

Watch for: Said “the answer”. The geometric meaning is what makes a pair obviously necessary.

8 EXTRA SPICY What Ali is missing

the y-coordinate — the solution is a point

1. A system in x and y asks where two lines meet.
2. A meeting place has two coordinates, so the answer is (6, something).
3. $x = 6$ on its own describes a whole vertical line, which is not a point.
4. He needs to substitute $x = 6$ back into either equation to find y , then check the pair in both.

Watch for: Said “he forgot y ” without the reason. Naming the answer as a LOCATION is what makes it obvious rather than a rule about writing.

Exit Ticket

4 problems

Use: End of the systems lesson and the last of Grade 8 Expressions & Equations. 5–8 min, independent. Item 1's diagnostic is not the arithmetic — it is whether the answer is a PAIR and whether both equations were checked. A single-coordinate answer is the error to look for. **Timing:** 5–8 min · independent. Graph paper available.

1 (4, 3) 2 because the solution is a point where two lines meet, and a point needs both coordinates

3 parallel — no solution 4 infinitely many — it is the same line twice

1  MEDIUM $2x + y = 11$ and $x - y = 1$

(4, 3)

1. Add the two equations to eliminate y : $3x = 12$, so $x = 4$.
2. Substitute into $x - y = 1$: $4 - y = 1$, so $y = 3$.
3. Check both: $2(4) + 3 = 11$ ✓ and $4 - 3 = 1$ ✓.
4. Solution: (4, 3).

Watch for: Stopped at $x = 4$, or gave y without x . Or checked only the first equation — which any point on that line would pass.

2  SPICY Why $x = 5$ is incomplete

because the solution is a point where two lines meet, and a point needs both coordinates

1. A system of two equations in x and y asks WHERE the lines cross.
2. A crossing point on a grid has two coordinates.
3. $x = 5$ names a vertical line, not a point — it says nothing about how far up.
4. So the answer must be (5, something), and the something has to be found by substituting back.

Watch for: Said “he needs to find y as well” without saying why. True and worth partial credit; the reason — that the answer is a location — is what stops it recurring.

3  SPICY $y = 2x + 1$ and $y = 2x + 5$

parallel — no solution

1. Both have slope 2, so they never converge.
2. Setting them equal: $2x + 1 = 2x + 5$, which leaves $1 = 5$. False.
3. So there is no point on both lines.
4. The vertical gap is 4 at every x : at $x = 0$ it is 1 against 5, at $x = 50$ it is 101 against 105.

Watch for: Answered $x = 0$, or gave a point. The variables cancelling is the same signal as in the one-variable lesson, and this is where that recognition pays off.

4  EXTRA SPICY $y = 3x - 2$ and $2y = 6x - 4$

infinitely many — it is the same line twice

1. Halve the second equation: $y = 3x - 2$.
2. That is the first equation exactly.
3. So the two lines are identical and every point on the line solves the system.
4. Check at $x = 4$: both give 10. At $x = -1$: both give -5.
5. Infinitely many solutions.

Watch for: Answered “no solution” by seeing the equations look different, or gave one point. Dividing the second equation through is the move, and the two coefficients and the constant must ALL halve for the lines to coincide.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Lead with the shape of the ANSWER. A child who does not know they are looking for two numbers will stop after one and believe they have finished.
2	1:00	Introduce the system and say plainly that both must hold at once.
3	2:00	Graph both. Seeing the crossing point is what makes ‘a pair’ obvious — a point on a grid manifestly needs two coordinates.
4	3:30	Elimination. Two unknowns, two steps: find one, then substitute back for the other. Naming that structure prevents stopping halfway.

5	5:00	THE slide on checking. Offer (6,4): it satisfies $x + y = 10$ perfectly and fails the other. Any point on a line satisfies that line's equation, so one check is worthless.
6	6:30	Parallel. Point out that the algebra behaves exactly as it did in the one-variable lesson: the variable vanishes, the constants disagree.
7	8:00	Identical lines. Same cancelling, constants agree, infinitely many. The parallel structure with the “Solving linear equations” lesson is worth saying out loud.
9	9:30	One to solve, with both checks, then the exit ticket. Whiteboards: everyone tries Mild, then climbs. These are #1, #7 on the practice sheet.
10		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
11		Launch the practice sheet. Exit ticket in the last 5–8 minutes.