



Transformations on coordinates

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the coordinate-rules lesson. Item 3 is the diagnostic and it is worth marking carefully: the answer is not “he should have multiplied” but that his triangle is STILL 3-4-5, so adding produced a congruent figure when an enlargement was asked for. A child who names the error without measuring has not seen why it is invisible — the output looks like a perfectly good triangle. Item 2 separates the length factor from the area factor, which is the other half of this standard.

Timing: 5–8 min · independent · squared paper.

1. Dilate by 3 **APPLICATION**

Answer: **(6,3), (18,3), (6,12)**

1. Multiply both coordinates by 3.
2. $(2,1) \rightarrow (6,3)$. $(6,1) \rightarrow (18,3)$. $(2,4) \rightarrow (6,12)$.

If wrong: Added 3 instead of multiplying — item 3 is that error done deliberately. Or multiplied only one coordinate, which stretches the figure instead of enlarging it.

2. The new side lengths **APPLICATION**

Answer: **9, 12 and 15**

1. Every length is multiplied by the scale factor, so $3 \times 3 = 9$, $4 \times 3 = 12$, $5 \times 3 = 15$.
2. Check one directly: $(6,3)$ to $(6,12)$ is 9 units, and the original $(2,1)$ to $(2,4)$ was 3.
3. 9, 12, 15 is a 3-4-5 triangle scaled up, so it is still right-angled.

If wrong: Multiplied the lengths by 9. The AREA goes up by 9; the lengths go up by 3.

3. Ravi's error REASONING

Answer: **the sides are still 3, 4 and 5 — he translated the triangle instead of enlarging it**

1. (5,4) to (9,4) is 4. (5,4) to (5,7) is 3. (9,4) to (5,7) is 5.
2. So his triangle is exactly the same size as the original, only moved.
3. Adding the same amount to every coordinate is the rule for a TRANSLATION, and a translation cannot change size.
4. His figure is congruent to the original when he was asked for an enlargement, and nothing about the answer looks like a failure to enlarge — which is why the check is measuring a side.

If wrong: Said the answer is wrong without working out the sides. The measurement is the whole point: it shows the error produced the wrong KIND of figure, not just wrong numbers. Also common: saying he should have multiplied, which is right, and does not say what his answer actually is.

4. Two reflections of (6,1) APPLICATION

Answer: **in the x-axis: (6,-1), x stayed. In the y-axis: (-6,1), y stayed**

1. Reflecting in the x-axis flips y and keeps x.
2. Reflecting in the y-axis flips x and keeps y.
3. The axis you reflect in names the coordinate that survives.

If wrong: Swapped the two. Naming the surviving coordinate is what makes the rule recoverable rather than memorised.

Quick answers for fast checking:

#	Level	Answer
1	application	(6,3), (18,3), (6,12)
2	application	9, 12 and 15
3	reasoning	still 3-4-5 — he translated it
4	application	(6,-1) x stays; (-6,1) y stays

Practice Worksheet — answer key

Use: In-class practice after the deck. Items 6 and 8 are the length-versus-area factor, which is the most common lasting error from this standard; item 8's answer is stronger if it names what 10 would be right for. Item 7 pairs with item 4 deliberately — the same swap, once done correctly and once not — and the orientation check from the earlier skills is what separates them. Item 9 is the summary worth having on the board.

Timing: 15–20 min · pairs · squared paper · items 7, 8 and 9 on the board.

1. Translation rule FLUENCY

Answer: $(x, y) \rightarrow (x + a, y + b)$

1. Add the horizontal shift to x and the vertical shift to y.

If wrong: Wrote it with a and b swapped. a moves horizontally, so it belongs with x.

2. Reflection rules FLUENCY

Answer: **x-axis:** $(x, y) \rightarrow (x, -y)$. **y-axis:** $(x, y) \rightarrow (-x, y)$

1. The axis you reflect in is the coordinate that survives.
2. So an x-axis reflection keeps x and flips y.

If wrong: Swapped them. The survivor rule is the recovery route when the two blur together.

3. Translate 3 left, 2 up APPLICATION

Answer: $(-1,3), (3,3), (-1,6)$

1. Subtract 3 from each x, add 2 to each y.
2. Left is negative.

If wrong: Added 3 instead of subtracting.

4. Rotate 90° anticlockwise APPLICATION

Answer: $(-1,2), (-1,6), (-4,2)$

1. $(x, y) \rightarrow (-y, x)$.
2. $(2,1) \rightarrow (-1,2)$. $(6,1) \rightarrow (-1,6)$. $(2,4) \rightarrow (-4,2)$.

If wrong: Swapped without the sign change, which is item 7's error — a reflection in the line $y = x$.

5. Rotate 180° APPLICATION

Answer: $(-2,-1), (-6,-1), (-2,-4)$

1. Both coordinates change sign.

If wrong: Changed only one sign, which is a reflection.

6. Dilate by 3, and the area APPLICATION

Answer: **(6,3), (18,3), (6,12), area 54**

1. Multiply both coordinates by 3.
2. Lengths triple: 3, 4, 5 becomes 9, 12, 15.
3. Area is multiplied by $3^2 = 9$, so 6 becomes 54.
4. Check by the formula: half of 9×12 is 54.

If wrong: Gave the area as 18, having multiplied by 3 rather than by 9. Lengths scale by k ; area scales by k squared, because both dimensions grew.

7. Priya's transformation REASONING

Answer: **a reflection in the line $y = x$, not a rotation**

1. Swapping coordinates sends (2,4) to (4,2); the 90° anticlockwise turn sends it to $(-4,2)$.
2. A bare swap reflects the figure in the line $y = x$.
3. So she has performed a real transformation and named it wrongly — harder to catch than an answer that is simply nonsense.
4. Reading the corners round settles it: her image has reversed orientation, and a rotation never reverses it.

If wrong: Said she just forgot a minus sign. True in form, and it misses that the result is a genuine transformation with a different name — and that orientation is what distinguishes them.

8. Area under a dilation of 2 REASONING

Answer: **20 — because both dimensions doubled, so the area is multiplied by 2^2**

1. A dilation stretches the figure in both directions at once.
2. Doubling the width doubles the area; doubling the height doubles it again.
3. So the area factor is $2 \times 2 = 4$, and 5 becomes 20.
4. 10 would be the answer if only one dimension had grown — and that is a stretch, not a dilation.

If wrong: Answered 10. Naming what 10 WOULD be right for — a one-directional stretch — is more useful than only correcting the number.

9. Which change size REASONING

Answer: **only the dilation — you can tell because its rule multiplies the coordinates**

1. Translation adds, reflection changes a sign, rotation swaps and changes a sign. None of those scales a distance.
2. Dilation multiplies both coordinates by k , and multiplying every coordinate by k multiplies every distance by k .
3. So the rule itself tells you: multiplication is the only operation of the four that can change a length.
4. Which is why doing a dilation by adding cannot possibly enlarge anything.

If wrong: Listed the dilation without saying how the rule shows it. The point is that the operation — multiply, rather than add or negate — is what makes size change possible.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$(x, y) \rightarrow (x + a, y + b)$
2	fluency	x-axis keeps x; y-axis keeps y
3	application	$(-1,3), (3,3), (-1,6)$
4	application	$(-1,2), (-1,6), (-4,2)$
5	application	$(-2,-1), (-6,-1), (-2,-4)$
6	application	$(6,3), (18,3), (6,12)$, area 54
7	reasoning	a reflection in $y = x$
8	reasoning	20 — the area factor is k squared
9	reasoning	only dilation, because its rule multiplies

Homework — answer key

Use: Homework after the practice sheet. Item 4 wants the DIFFERENCE argument, not a restatement of the rule: a length is a difference between coordinates, and adding the same amount to both ends leaves a difference untouched. That single sentence also explains why multiplying works, so it is the most valuable answer on the sheet. Item 6 reverses the area relationship, which catches children who know $\text{area} \times k^2$ only in one direction. Item 7 is the orientation thread again, now with a dilation in the list.

Timing: 20–25 min · independent · squared paper · items 4, 6 and 8 first when marking.

1. Reflect in the y-axis APPLICATION

Answer: **$(-3,2)$, $(-7,2)$, $(-3,5)$**

1. x changes sign, y survives.

If wrong: Flipped y instead, which is the x-axis reflection.

2. Dilate by 4 APPLICATION

Answer: **$(4,8)$, $(12,8)$, $(4,24)$**

1. Multiply both coordinates by 4.

If wrong: Added 4, which translates the figure and leaves its size alone.

3. A square dilated by 5 APPLICATION

Answer: **side 15, area 225**

1. Side: $3 \times 5 = 15$.
2. Area: $9 \times 5^2 = 9 \times 25 = 225$.
3. Check directly: $15 \times 15 = 225$.

If wrong: Gave the area as 45, multiplying by 5 rather than 25. Checking 15×15 catches it immediately.

4. Why adding cannot enlarge REASONING

Answer: **because adding the same amount to every coordinate is a translation — every point moves together, so no distance between points changes**

1. A length depends on the DIFFERENCE between two coordinates.
2. Add 5 to both ends and the difference is unchanged: $(x + 5) - (y + 5) = x - y$.
3. So every side comes out the same length and the figure is congruent, only moved.
4. The 3-4-5 triangle from class is the proof: add 3 and it is still 3-4-5.

If wrong: Said adding is “the wrong rule”. The reason is that a length is a difference, and adding the same amount to both ends leaves a difference untouched — that argument also explains why multiplying DOES change it.

5. Two 90° turns REASONING

Answer: **a rotation of 180° about the origin**

1. $(4,1) \rightarrow (-1,4) \rightarrow (-4,-1)$.
2. $(-4,-1)$ is what the 180° rule gives directly.
3. Two turns of 90° in the same direction make one of 180°, which is what you would expect.

If wrong: Said it returns to the start. That takes four 90° turns, not two.

6. Area $\times 16$ REASONING

Answer: **scale factor 4**

1. Area is multiplied by k^2 , so $k^2 = 16$.
2. $k = 4$, taking the positive root since a scale factor here is a length ratio.
3. Check: lengths $\times 4$ means area $\times 16$.

If wrong: Answered 16, or 8. The area factor is the SQUARE of the length factor, so recovering the length factor means taking a square root.

7. Orientation REASONING

Answer: **translations, rotations and dilations preserve it; reflections reverse it**

1. Sliding, turning and scaling all keep the corners running the same way round.
2. A reflection is the only one of the four that reverses the order.
3. A dilation with a positive scale factor changes size and not direction, so it preserves it too.
4. This is the test from the congruence work, and it still separates a reflection from everything else.

If wrong: Included dilation as reversing. Enlarging a figure does not turn it over — check by reading the corners of a dilated triangle.

8. Why the effect, not just the rule REASONING

Answer: **because a rule tells you where points go, and the effect tells you what happened to the figure — which is what you need to answer a question about size, shape or orientation**

1. Knowing $(x, y) \rightarrow (3x, 3y)$ is not the same as knowing lengths triple and area goes up ninefold.
2. Someone who only has the rule can compute new coordinates and still not say whether the figure is congruent.
3. The effect is also what makes an error visible: adding instead of multiplying looks fine at the coordinate level and is obviously wrong once you ask what happened to the sides.
4. And the effects are what the next skills use — congruence and similarity are statements about effects, not about rules.

If wrong: Said the effect is easier to remember. The stronger answer is that the effect is what the question is usually about, and it is what makes a wrong rule detectable.

Quick answers for fast checking:

#	Level	Answer
1	application	$(-3,2)$, $(-7,2)$, $(-3,5)$
2	application	$(4,8)$, $(12,8)$, $(4,24)$
3	application	side 15, area 225
4	reasoning	a length is a difference, and adding leaves it unchanged
5	reasoning	a 180° rotation
6	reasoning	scale factor 4
7	reasoning	only reflections reverse it
8	reasoning	the effect is what the question asks and what makes an error visible

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Frame the four rules as descriptions of an EFFECT. The standard asks what happens to the figure, so every rule gets paired with what it does to lengths.
2/8	0:45	Translation. Quick — they have it from G-01.
3/8	1:30	Reflections, and give them the survivor rule. It is the one they forget under pressure, and “the axis you reflect in is the coordinate that stays” is shorter than either rule on its own.
4/8	2:30	Rotations. Insist on swap AND sign. A bare swap is a real transformation — a reflection in $y = x$ — so this error produces a valid image with the wrong name, which is harder to spot than nonsense.

5/8	3:45	Dilation, and say MULTIPLY plainly before anyone has a chance to add.
6/8	4:45	THE slide. Do it on the board: add 3 to every coordinate of the 3-4-5 triangle and measure. Still 3-4-5. Adding did not enlarge it, it moved it. Let the class find that rather than telling them — the surprise is what makes it stick.
7/8	6:30	Lengths $\times k$, area $\times k^2$. Count squares on the 3-4-5 and the 9-12-15 to see 6 become 54.
8/8	8:00	Both transformations on the same triangle, then the exit ticket.