



Using Pythagoras

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the using-Pythagoras lesson. Item 3 is the diagnostic and it asks for something specific: not what Leo did wrong, but how he could have KNOWN. A child who answers “he added instead of subtracting” has diagnosed the error and missed the transferable part — that a leg longer than the hypotenuse is impossible, so this class of mistake is self-announcing. That check costs one glance and works without the answer. Item 4's negative coordinate is deliberate: $3 - (-2) = 5$ is where this item is usually lost.

Timing: 5–8 min · independent · calculator allowed.

1. Find the hypotenuse **FLUENCY**

Answer: **10**

1. The hypotenuse is missing, so add: $6^2 + 8^2 = 36 + 64 = 100$.
2. The square root of 100 is 10.
3. Check: 10 is longer than both legs, as a hypotenuse must be.

If wrong: Answered 100, stopping at c squared. Writing “ $c^2 = 100$ ” on its own line makes the missing root visible.

2. Find the other leg **APPLICATION**

Answer: **24**

1. A leg is missing, so subtract: $25^2 - 7^2 = 625 - 49 = 576$.
2. The square root of 576 is 24.
3. Check: 24 is shorter than the hypotenuse 25. Good.

If wrong: Added, getting 674 and about 26 — which is longer than the hypotenuse, so the check catches it immediately.

3. Leo's error REASONING

Answer: **a leg cannot be longer than the hypotenuse, and $13.9 > 13$ — so his answer is impossible. The correct leg is 12**

1. The hypotenuse is the longest side of a right triangle, always.
2. His answer 13.9 is longer than the hypotenuse 13, so it cannot be a leg — no calculation is needed to see that.
3. He added when he should have subtracted. Correctly: $169 - 25 = 144$, and the square root of 144 is 12.
4. 12 is shorter than 13, so the check passes. This check works on every leg problem and never needs the right answer.

If wrong: Said he added instead of subtracting, which is true and is not what the question asks. The question asks how he could TELL, and the answer is the comparison with the hypotenuse — a check he can run himself on any problem of this kind.

4. Distance between two points APPLICATION

Answer: **13**

1. Sketch the right triangle. Horizontal leg: $3 - (-2) = 5$. Vertical leg: $13 - 1 = 12$.
2. $5^2 + 12^2 = 25 + 144 = 169$.
3. The square root of 169 is 13.

If wrong: Used the coordinates rather than their differences, or mishandled the negative — $3 - (-2)$ is 5, not 1. Sketching the triangle makes the legs obvious and turns the formula back into Pythagoras.

Quick answers for fast checking:

#	Level	Answer
1	fluency	10
2	application	24
3	reasoning	a leg cannot exceed the hypotenuse; the leg is 12
4	application	13

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 7 is the three-dimensional case and it rewards keeping the base diagonal as a SQUARE — carrying 25 into the second step avoids rounding an irrational length in the middle of a calculation. Item 4 is worth discussing: whole-number answers are the exception in this topic, not the rule, and $c^2 = 13$ is a complete answer. Item 8's algebraic version — c^2 is a^2 plus something positive — is the neatest form of the self-check.

Timing: 15–20 min · pairs · calculator allowed · items 7 and 8 on the board.

1. Legs 9 and 12 FLUENCY

Answer: **15**

1. $81 + 144 = 225$, and the square root of 225 is 15.

If wrong: Left the answer as 225.

2. Hypotenuse 17, leg 8 APPLICATION

Answer: **15**

1. $289 - 64 = 225$, so the leg is 15.

2. $15 < 17$, so the check passes.

If wrong: Added, giving 353 and about 18.8 — longer than the hypotenuse.

3. Hypotenuse 10, leg 6 APPLICATION

Answer: **8 — and 8 is shorter than 10, so it passes**

1. $100 - 36 = 64$, so the leg is 8.

2. $8 < 10$. A leg must be shorter than the hypotenuse, and it is.

If wrong: Answered 64, or added to get about 11.7 — which the check rejects at once.

4. Legs 2 and 3 REASONING

Answer: **$c^2 = 13$, and that is exact because 13 is not a perfect square, so c is irrational**

1. $4 + 9 = 13$.

2. 13 has no whole-number square root, so c is the square root of 13.

3. Any decimal for it is rounded, so $c^2 = 13$ is the only exact statement.

4. Most right triangles are like this — the whole-number ones are the special cases.

If wrong: Gave 3.6 as the answer without saying it is rounded. Worth noticing that triples are the exception, not the rule.

5. (1,2) to (5,5) APPLICATION

Answer: **5**

1. Legs: $5 - 1 = 4$ and $5 - 2 = 3$.

2. $16 + 9 = 25$, so the distance is 5.

If wrong: Squared the coordinates instead of the differences. Sketching the triangle prevents it.

6. $(-3, -1)$ to $(1, 2)$ APPLICATION

Answer: **5**

1. Horizontal: $1 - (-3) = 4$. Vertical: $2 - (-1) = 3$.
2. $16 + 9 = 25$, so the distance is 5.

If wrong: Mishandled the negatives, getting legs of 2 and 1. Subtracting a negative adds — a sketch makes the gap visible and confirms it.

7. The longest rod in a box APPLICATION

Answer: **13**

1. The longest rod lies along the space diagonal, corner to opposite corner.
2. First the base diagonal: $3^2 + 4^2 = 25$.
3. Then the space diagonal: $25 + 12^2 = 25 + 144 = 169$, so 13.
4. Pythagoras applied twice, and the base diagonal's SQUARE carries straight into the second step — no need to find its length.

If wrong: Used only two of the three measurements, or added all three. The two-step method is the point, and keeping 25 as a square avoids rounding in the middle.

8. Why a leg cannot exceed the hypotenuse REASONING

Answer: **because the hypotenuse is opposite the right angle, which is the largest angle, and the longest side is always opposite the largest angle**

1. In any triangle the biggest side faces the biggest angle.
2. The angles sum to 180, so with one angle at 90 the other two are each less than 90.
3. So the right angle is the largest, and the side facing it is the longest.
4. Algebraically too: $c^2 = a^2 + b^2$ is bigger than a^2 , so c is bigger than a .

If wrong: Said “because it is the hypotenuse”. The reason is the angle argument, and the algebraic version is a good second answer — c^2 is a^2 plus something positive.

9. Why the order does not matter REASONING

Answer: **because the differences are squared, and squaring removes the sign**

1. From (1,2) to (5,5) the horizontal difference is 4; the other way round it is -4 .
2. $(-4)^2 = 16 = 4^2$.
3. So both orders give the same squared distance and the same distance.
4. Which is sensible — the distance from A to B is the distance from B to A.

If wrong: Said you must always subtract the smaller from the larger. That is a safe habit and it is not required, and knowing why saves worrying about the order.

Quick answers for fast checking:

#	Level	Answer
1	fluency	15
2	application	15
3	application	8, and $8 < 10$
4	reasoning	$c^2 = 13$, exact because 13 is not a perfect square
5	application	5
6	application	5
7	application	13
8	reasoning	the longest side faces the largest angle
9	reasoning	squaring removes the sign

Homework — answer key

Use: Homework after the practice sheet. Item 6 asks for the self-check explicitly and forbids redoing the arithmetic, which is the point: a leg of 15 against a hypotenuse of 12 is impossible on sight. Item 5 is the same structure in words, and identifying the ladder as the hypotenuse is where it is won or lost. Item 7 rewards keeping the floor diagonal as a square — here 100 is a perfect square so rounding costs nothing, which makes it a good moment to name the habit before it does.

Timing: 20–25 min · independent · calculator allowed · items 5, 6 and 8 first when marking.

1. Legs 5 and 12 FLUENCY

Answer: **13**

1. $25 + 144 = 169$, so 13.

If wrong: Left as 169.

2. Hypotenuse 26, leg 10 APPLICATION

Answer: **24, and $24 < 26$ so it passes**

1. $676 - 100 = 576$, so the leg is 24.
2. 24 is shorter than 26. Good.

If wrong: Added, giving 776 and about 27.9 — longer than the hypotenuse, which the check rejects.

3. Legs 1 and 1 APPLICATION

Answer: **2**

1. $1 + 1 = 2$, so $c^2 = 2$.
2. The hypotenuse is the square root of 2, which is irrational — so 2 is the exact answer to what was asked.

If wrong: Answered 1.41, which is a rounded length rather than the square that was asked for.

4. (2,-3) to (7,9) APPLICATION

Answer: **13**

1. Horizontal: $7 - 2 = 5$. Vertical: $9 - (-3) = 12$.
2. $25 + 144 = 169$, so 13.

If wrong: Took the vertical leg as 6, mishandling $9 - (-3)$. Subtracting a negative adds; a sketch shows the gap is 12.

5. The ladder APPLICATION

Answer: **8 m**

1. The ladder is the hypotenuse, since it is opposite the right angle at the base of the wall.
2. $100 - 36 = 64$, so the height is 8 m.
3. $8 < 10$, so the check passes — and it must, since the ladder is the longest of the three.

If wrong: Added, getting about 11.7 m — a wall reached higher than the ladder is long. Identifying the ladder as the hypotenuse is the first step.

6. Priya's answer REASONING

Answer: **a leg can never be longer than the hypotenuse, and $15 > 12$ — so the answer is impossible whatever the working says**

1. The hypotenuse is opposite the right angle, which is the largest angle, so it is the longest side.
2. 15 exceeds 12, so 15 cannot be a leg of that triangle.
3. The likely cause is adding where a subtraction was needed, and the check finds the error without repeating the arithmetic.
4. This is worth running on every leg answer, since it costs one comparison.

If wrong: Redid the calculation. That finds the right answer and misses the point of the question — the error is visible from the answer alone.

7. A room's diagonal APPLICATION

Answer: **about 10.4 m — exactly, the square root of 109**

1. Floor diagonal squared: $6^2 + 8^2 = 100$.
2. Space diagonal squared: $100 + 3^2 = 109$.
3. So the distance is the square root of 109, about 10.4 m.
4. Keep 100 as a square rather than taking its root first — that avoids rounding inside the calculation.

If wrong: Added all three measurements, or rounded the floor diagonal to 10 and then worked with 10 — which happens to be exact here, and is a habit that costs accuracy when it is not.

8. Why the distance formula is not new REASONING

Answer: **because it IS Pythagoras — the horizontal and vertical gaps are the two legs of a right triangle, and the distance is its hypotenuse**

1. Plot the two points and draw the horizontal and vertical lines between them: that makes a right triangle.
2. The legs are the differences in x and in y.
3. So the distance is the hypotenuse, and $a^2 + b^2 = c^2$ gives it.
4. Seeing it that way also prevents the common error of using the coordinates instead of the differences — a sketch makes the legs obvious.

If wrong: Said it is “similar to” Pythagoras. It is not similar to it, it is the same theorem applied to a triangle you draw yourself — and that is why sketching is the reliable method.

Quick answers for fast checking:

#	Level	Answer
1	fluency	13
2	application	24, and $24 < 26$
3	application	2
4	application	13
5	application	8 m
6	reasoning	$15 > 12$, so impossible
7	application	the square root of 109, about 10.4 m
8	reasoning	it IS Pythagoras on a triangle you draw

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Frame the lesson as a decision rather than a formula: which side is missing decides whether you add or subtract.
2/8	0:45	Insist on labelling the hypotenuse before any arithmetic. It costs five seconds and prevents most of the errors in this topic.
3/8	1:30	The easy direction. Fast.
4/8	2:15	Subtraction. Say why it makes sense: the hypotenuse is already the largest, so finding a leg means removing part of it, not adding more.
5/8	3:30	THE slide. Do the wrong version on the board deliberately and get 13.9. Then ask the class what is wrong with it — someone will see that a leg cannot be longer than the hypotenuse. That is the check they should run every time, and it works without knowing the right answer.
6/8	5:00	The square root. The habit of writing “ $\text{leg}^2 = 144$ ” on its own line makes a missing root visible on the page.
7/8	6:15	Distance. Sketch the triangle every time at first — the formula is Pythagoras, and children who meet it as a new formula use the coordinates rather than the differences. Ask what the legs are before writing anything.
8/8	8:00	Both problems, then the exit ticket. Remind them of the check.