



Volume of cones, cylinders and spheres

GRADE 8 MATH

Answer keys for the exit ticket, practice worksheet, and homework, plus notes for the lesson slides. Each answer shows the steps and the most likely wrong answer (“If wrong”), so you can tell what a mistake means, not just that it is one. Questions go from **fluency** to **application** to **reasoning**.

Exit Ticket — answer key

Use: End of the volume lesson and the last Grade 8 Geometry exit ticket. Item 3 is the diagnostic and it asks for something more than the error: the FACTOR. Exactly 4 times too big is a doubled radius, and an arithmetic slip would not land on a whole-number multiple — so the size of a wrong answer tells a child where to look. Item 4's containing cylinder must have height $2r$, and taking it as r is the common slip; the 2 : 3 relationship is what makes four thirds memorable rather than arbitrary. Item 2 is marked on whether they used the relationship or restarted from the formula — both are correct, and the relationship is the point of the lesson.

Timing: 5–8 min · independent · calculator allowed, answers as multiples of π .

1. The can **APPLICATION**

Answer: $160\pi \text{ cm}^3$

1. Diameter 8, so $r = 4$. Write that first.
2. $V = \pi r^2 h = \pi \times 16 \times 10 = 160\pi \text{ cm}^3$.

If wrong: Used 8 as the radius, giving 640π — which is item 3. Halving the diameter is the first line, not a detail.

2. The cone **APPLICATION**

Answer: $160/3 \pi \text{ cm}^3$, about 53.3π

1. A cone is a third of the cylinder on the same base and height.
2. $160\pi \div 3 = 160/3 \pi \text{ cm}^3$.
3. No need to start again from the formula — the relationship gives it directly.

If wrong: Gave 160π , dropping the third. Three times too big, and an answer three times too big looks perfectly reasonable, which is why it survives.

3. Omar's error REASONING

Answer: **he used the diameter 8 as the radius — and 640π is exactly 4 times 160π , which is the signature of that mistake**

1. The correct radius is 4, giving 160π . He used 8.
2. Doubling the radius multiplies r^2 by 4, so the volume comes out 4 times too big.
3. $640 \div 160 = 4$ exactly, so the factor identifies the cause rather than merely showing something went wrong.
4. On a sphere the same error would give a factor of 8, because r is cubed there.

If wrong: Said he “used the wrong number”. The question asks how the SIZE of the error shows it — exactly 4 times is the diagnosis, and an arithmetic slip would not land on a whole-number factor.

4. The sphere and its cylinder REASONING

Answer: **sphere $36\pi \text{ cm}^3$; the cylinder is $54\pi \text{ cm}^3$, and 36 is two thirds of 54**

1. Sphere: $V = \frac{4}{3} \times \pi \times 3^3 = \frac{4}{3} \times 27\pi = 36\pi \text{ cm}^3$.
2. The smallest containing cylinder has the same radius, 3, and height equal to the diameter, 6.
3. Its volume is $\pi \times 9 \times 6 = 54\pi \text{ cm}^3$.
4. Two thirds of 54π is 36π , which matches — and this is why the sphere formula is worth remembering as a relationship rather than as four thirds appearing from nowhere.

If wrong: Gave the cylinder a height of 3 rather than 6. The containing cylinder's height is the sphere's DIAMETER, since the sphere has to fit top to bottom.

Quick answers for fast checking:

#	Level	Answer
1	application	$160\pi \text{ cm}^3$
2	application	$160/3 \pi \text{ cm}^3$
3	reasoning	diameter used as radius; exactly 4 times too big
4	reasoning	36π and 54π, a ratio of 2 : 3

Practice Worksheet — answer key

Use: In-class practice after the deck. Item 8 gives no dimensions on purpose — a child who hunts for a radius has not seen that the cone-to-cylinder relationship is the whole method. Items 6 and 7 are the same arithmetic twice, once as an error and once deliberately, which is the most efficient way to make the factor of 8 stick. Item 5's containing cylinder needs height $2r$, and that picture is what makes four thirds look inevitable rather than arbitrary.

Timing: 15–20 min — pairs — calculator allowed — items 6, 7 and 9 on the board.

1. The three formulas FLUENCY

Answer: **cylinder $\pi r^2 h$; cone $\frac{1}{3}\pi r^2 h$; sphere $\frac{4}{3}\pi r^3$**

1. The cone is the cylinder's formula divided by 3.
2. The sphere is the only one with r cubed and no h — a sphere has no separate height.

If wrong: Wrote πr^2 for the sphere, or gave the cone without the third. Noticing that the sphere formula has no h is a useful check: a sphere has only one measurement.

2. Cylinder FLUENCY

Answer: **90π**

1. $\pi \times 9 \times 10 = 90\pi$.

If wrong: Multiplied 3 by 10 without squaring.

3. Cone APPLICATION

Answer: **30π**

1. A third of the cylinder: $90\pi \div 3 = 30\pi$.
2. Same answer from the formula: $\frac{1}{3} \times \pi \times 9 \times 10$.

If wrong: Gave 90π , dropping the third.

4. Sphere APPLICATION

Answer: **36π**

1. $\frac{4}{3} \times \pi \times 27 = 36\pi$.
2. $3^3 = 27$, and four thirds of 27 is 36.

If wrong: Squared the radius instead of cubing, giving 12π .

5. Containing cylinder and the ratio APPLICATION

Answer: **54π , and the ratio is 2 : 3**

1. The cylinder has radius 3 and height 6, since the sphere must fit top to bottom.
2. $\pi \times 9 \times 6 = 54\pi$.
3. $36 : 54$ simplifies to 2 : 3, so the sphere fills two thirds of the cylinder.

If wrong: Used height 3, forgetting that the height must be the sphere's diameter.

6. Ella's error REASONING

Answer: **she used the diameter 6 as the radius — she is out by a factor of 8**

1. Diameter 6 means $r = 3$, and the correct volume is 36π .
2. $288 \div 36 = 8$.
3. The factor is 8 because r is CUBED in the sphere formula, and $2^3 = 8$.
4. On a cylinder the same mistake would cost a factor of 4, since r is only squared there.

If wrong: Said she doubled the radius without giving the factor, or gave 4. The exponent decides it: cubed means 8.

7. Doubling the radius REASONING

Answer: **the volume is multiplied by 8**

1. $V = \frac{4}{3} \pi r^3$. Replacing r by $2r$ gives $\frac{4}{3} \pi (2r)^3$.
2. $(2r)^3 = 8r^3$, so the volume is 8 times as big.
3. Check with numbers: radius 3 gives 36π , radius 6 gives 288π , and $288 \div 36 = 8$.
4. This is the same arithmetic as Ella's error, met deliberately instead of by accident.

If wrong: Answered 2, or 6. The cube applies to the whole of $2r$, not only to r .

8. Cone from a cylinder APPLICATION

Answer: **0.5 litres**

1. A cone is a third of the cylinder on the same base and height.
2. $1.5 \div 3 = 0.5$ litres.
3. No formula needed — and no radius or height was given, which is the hint.

If wrong: Tried to find the radius. The question gives no dimensions on purpose; the relationship is the whole method.

9. Why relationships REASONING

Answer: **because it turns three formulas into one plus two facts, and it makes them checkable and recoverable**

1. $\pi r^2 h$ is the one to know: base area times height, like any prism.
2. A third of it is the cone; two thirds of the containing cylinder is the sphere.
3. So a half-remembered formula can be rebuilt instead of guessed.
4. And it explains where four thirds comes from, which otherwise looks arbitrary — the containing cylinder is $2\pi r^3$, and two thirds of that is $\frac{4}{3}\pi r^3$.

If wrong: Said it is easier. The stronger answer is that a relationship can be rebuilt and checked, while a forgotten formula leaves nothing to fall back on.

Quick answers for fast checking:

#	Level	Answer
1	fluency	$\pi r^2 h$, $\frac{1}{3}\pi r^2 h$, $\frac{4}{3}\pi r^3$
2	fluency	90π
3	application	30π
4	application	36π
5	application	54π and $2 : 3$
6	reasoning	diameter as radius; a factor of 8
7	reasoning	$\times 8$, since $(2r)^3 = 8r^3$
8	application	0.5 litres
9	reasoning	one formula plus two facts, rebuildable

Homework — answer key

Use: Homework after the practice sheet, and the last of Grade 8 Geometry. Item 6 runs the diagnostic backwards: a factor of 4 on a sphere RULES OUT the diameter error, because that one always costs 8. A child who answers yes has the association without the mechanism. Item 4 is worth a comment — dividing gives a cylinder smaller than the cone inside it, which is impossible, so asking which solid should be bigger catches it without redoing anything. Item 7 is the only item in units, and both the $r =$ line and the litre conversion are places it is lost.

Timing: 20–25 min · independent · calculator allowed · items 4, 6 and 8 first when marking.

1. Cylinder from a diameter APPLICATION

Answer: 175π

1. Diameter 10, so $r = 5$.

2. $\pi \times 25 \times 7 = 175\pi$.

If wrong: Used 10 as the radius, giving 700π — exactly 4 times too big.

2. Cone APPLICATION

Answer: 60π

1. $\frac{1}{3} \times \pi \times 36 \times 5 = \frac{1}{3} \times 180\pi = 60\pi$.

If wrong: Gave 180π , dropping the third.

3. Sphere from a diameter APPLICATION

Answer: 288π

1. Diameter 12, so $r = 6$.

2. $\frac{4}{3} \times \pi \times 216 = 288\pi$.

If wrong: Used 12 as the radius, giving 2304π — 8 times too big.

4. Cylinder from a cone APPLICATION

Answer: **600 ml**

1. The cylinder is 3 times the cone on the same base and height.

2. $200 \times 3 = 600$ ml.

3. This is the relationship run backwards, so multiply rather than divide.

If wrong: Divided by 3, giving about 67 ml — a cylinder smaller than the cone it contains, which is impossible. Checking which solid should be bigger catches it.

5. Sphere of radius 5 and its cylinder APPLICATION

Answer: **sphere $500/3 \pi$, cylinder 250π , and the ratio is 2 : 3**

1. Sphere: $4/3 \times \pi \times 125 = 500/3 \pi$, about 166.7π .
2. Cylinder: radius 5, height 10, so $\pi \times 25 \times 10 = 250\pi$.
3. Two thirds of 250 is $500/3$, so the relationship holds.
4. Keeping the sphere as $500/3 \pi$ rather than a decimal makes the check exact.

If wrong: Used height 5 for the cylinder. The height must be the diameter, 10.

6. Could a factor of 4 be that error? REASONING

Answer: **no — on a sphere that mistake gives a factor of 8, because r is cubed. A factor of 4 must be something else**

1. Using $2r$ in place of r multiplies r^3 by $2^3 = 8$.
2. So the diameter error always costs exactly 8 on a sphere, never 4.
3. A factor of 4 would come from squaring where he should have cubed — or from some other slip.
4. Which is the useful point: the factor narrows down the cause, so it is worth working out.

If wrong: Said yes. The exponent is what fixes the factor, and this item is the diagnostic idea used in reverse — a factor of 4 rules that error OUT.

7. The tank in litres APPLICATION

Answer: **about 37 680 litres**

1. Diameter 4, so $r = 2$.
2. $V = 3.14 \times 4 \times 3 = 37.68 \text{ m}^3$.
3. $37.68 \times 1000 = 37\,680$ litres.

If wrong: Used 4 as the radius, giving four times too much, or forgot the litre conversion. The $r =$ line and the units line are both worth writing down.

8. Why 4 and why 8 REASONING

Answer: **because r appears squared in the cylinder formula and cubed in the sphere formula, so doubling r multiplies the volume by $2^2 = 4$ or $2^3 = 8$**

1. Using the diameter means using $2r$ in place of r .
2. Cylinder: $(2r)^2 = 4r^2$, so the volume is 4 times too big.
3. Sphere: $(2r)^3 = 8r^3$, so 8 times too big.
4. The height is untouched in the cylinder case, which is why only the squared term matters there.

If wrong: Said the sphere is “bigger” or “more three-dimensional”. The exponent on r is the whole answer, and it is worth writing $(2r)^2$ and $(2r)^3$ out to see where 4 and 8 come from.

Quick answers for fast checking:

#	Level	Answer
1	application	175π
2	application	60π
3	application	288π
4	application	600 ml
5	application	$500/3 \pi$ and 250π , ratio 2 : 3
6	reasoning	no — that error gives 8 on a sphere
7	application	about 37 680 litres
8	reasoning	$(2r)^2 = 4r^2$ and $(2r)^3 = 8r^3$

Lesson slides — teaching notes & timing

Mini-lesson slides, then the practice worksheet. Slide numbers match the deck.

Slide	~Time	Note
1/8	0:00	Say the plan: three formulas, but really one plus two relationships.
2/8	0:45	The $r =$ habit, and spend real time here. Every worked example on the board starts with it, including the ones where the radius is given directly — the habit is worth more than the seconds it costs.
3/8	1:30	Cylinder. Connect to prisms: base area times height, nothing new.
4/8	2:15	Cone as a third of its cylinder. If there are nesting solids and rice or water in the cupboard, pour three cones into the cylinder — it is the demonstration children remember for years.
5/8	3:15	Sphere. The tightest-cylinder picture is what makes $4/3 \pi r^3$ memorable rather than arbitrary: height $2r$ gives $2\pi r^3$, and two thirds of that is $4/3 \pi r^3$.

6/8	4:45	THE slide. Work radius 3 and then the same solid with 6 by mistake. Get 90π and 360π . Ask why it is exactly four times, and let them find the squaring.
7/8	6:15	The diagnostic half, which is the useful part: an answer exactly 8 times too big on a sphere is a doubled radius. The factor identifies the error, so a checked answer in the wrong order of magnitude tells you WHERE to look.
8/8	7:45	The can and the cone, then the exit ticket. Ask what their first line said.