

GRADE 8 · FUNCTIONS · ANSWER KEY

WHAT A FUNCTION IS

Today's goal: I can understand that a function is a rule that assigns to each input exactly one output and understand that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 6 is the sheet's centre: a child who says the two sentences mean the same thing has the misconception in plain view, which is more useful than a wrong answer anywhere else. Item 4 catches a subtler slip — repeating a whole pair does not break the rule, only a repeated INPUT with a different output does. **Timing:** 15–20 min · pairs · items 6 and 7 on the board.

1 no **2** $y = x^3$ is a function; a circle is not **3** $y = x^2$, $y = x^3$, $y = 2_x$ or similar

4 any four pairs where one input appears twice with different outputs, with that pair identified

5 a rule where every input has exactly one output

6 the first is the definition of a function; the second is a stronger, different property that many functions do not have

7 because a vertical line holds x fixed, so it asks how many outputs one input has

8 no — the same input gives different outputs

1 **MILD** (5,1) (6,2) (5,9)

no

1. Input 5 appears twice, giving 1 and 9.

Watch for: Said yes by checking the outputs.

2 **MEDIUM** $y = x^3$ and a circle

$y = x^3$ is a function; a circle is not

1. $y = x^3$: each x gives exactly one cube. Function.

2. A circle: $x = 0$ on a unit circle gives $y = 1$ AND $y = -1$. One input, two outputs, so not a function.

3. A vertical line through a circle crosses it twice, which is the same fact as a picture.

Watch for: Called $y = x^3$ a non-function because it is not straight, or called the circle a function because it is a smooth shape with an equation.

3 **MEDIUM** A function that is not straight

$y = x^2$, $y = x^3$, $y = 2_x$ or similar

1. Any curve where each x gives one y .
2. $y = x^2$ is the easiest.

Watch for: Left it blank, or gave a circle. Grade 8 spends so long on linear functions that non-linear ones stop feeling permitted.

4 **MEDIUM** Build a non-function

any four pairs where one input appears twice with different outputs, with that pair identified

1. Pick an input and use it twice.
2. Give it two different outputs.

Watch for: Repeated an entire PAIR — (2,5) twice — which is still a function: that input has one output, listed twice. The output must differ, and this is a genuinely subtle distinction worth discussing.

5 **SPICY** The definition, in your own words

a rule where every input has exactly one output

1. Say what goes in and what comes out.
2. The word that matters is EXACTLY one.

Watch for: Wrote “an equation with x and y ”, importing formulas; or “a straight line”, importing linearity. Neither is in the definition.

6 **SPICY** Input-side versus output-side

the first is the definition of a function; the second is a stronger, different property that many functions do not have

1. Each input has one output: ask a question, get one answer. That is a function.
2. Each output has one input: no two questions share an answer. That is a different demand.
3. $y = x^2$ passes the first and fails the second, and it IS a function.
4. The two sentences differ by one word and describe different things.

Watch for: Said they mean the same thing. That answer IS the misconception, so it is the most useful thing on the sheet to find.

7 **SPICY** Why VERTICAL

because a vertical line holds x fixed, so it asks how many outputs one input has

1. Every point on a vertical line shares the same x .
2. So the crossings show all the outputs for that ONE input.
3. Two crossings means one input with two outputs, which is exactly the failure.
4. A horizontal line holds y fixed and asks the OTHER question — how many inputs share an output — which is allowed.

Watch for: Said “because that is the test”. A child who can explain why vertical has the definition; a child who cannot is remembering a procedure and will draw it horizontally under pressure.

8 **EXTRA SPICY** The vending machine

no — the same input gives different outputs

1. Press B2 (the input) and you get something different each time.
2. One input, several possible outputs, so it is not a function.
3. A machine that always gave the same snack for B2 WOULD be one.

Watch for: Said yes because a machine has a rule. Having a mechanism is not the same as assigning one output per input, and this item is where that distinction becomes concrete.

Use: Homework after the practice sheet. Item 7 is the best item in the skill: the definition applied to something a child cannot dismiss as a trick, and the asymmetry — a function that stops being one when reversed — is the entire input/output distinction in one sentence. **Timing:** 20–25 min · independent · items 6 and 7 first when marking.

1 yes

2 yes

3 yes

4 any sketch where some vertical line crosses the curve twice — a circle, a sideways parabola, or a vertical line itself

5 every input has exactly one output; a formula, a straight line, or distinct outputs are all NOT required

6 two inputs may share an output — that is allowed

7 student → date of birth is a function; the reverse need not be, because two students can share a birthday

1

MILD

(1,1) (2,4) (3,9)

yes

1. Each input once.

Watch for: Said no for lack of a stated rule.

2

MILD

(1,6) (2,6) (3,6) (4,6)

yes

1. Every input appears once; the shared output 6 is allowed.

Watch for: Said no because every output is the same — the backwards reading again, and this is its starkest form.

3

MEDIUM

 $y = \sqrt{x}$ for $x \geq 0$

yes

1. The radical means the PRINCIPAL root, so each x gives exactly one value: $\sqrt{9} = 3$, not ± 3 .
2. One output per input, so it is a function.
3. This is the same convention as the roots lesson — and it is exactly WHY that convention exists.

Watch for: Said no by thinking $\sqrt{9} = \pm 3$. Worth linking back: the principal-root convention is what makes $\sqrt{x}$ usable as a function at all.

4

MEDIUM

A graph failing the vertical test

any sketch where some vertical line crosses the curve twice — a circle, a sideways parabola, or a vertical line itself

1. Draw something that doubles back.
2. Check with a ruler held upright.

Watch for: Drew a steep line, confusing steep with failing. A vertical line ITSELF fails, which is worth pointing out.

5

SPICY

Definition, plus something not required

every input has exactly one output; a formula, a straight line, or distinct outputs are all NOT required

1. The definition is about inputs and outputs only.
2. Anything else you were expecting — an equation, a line, a pattern — is optional.

Watch for: Gave the definition and no exclusion. Naming what is NOT required is the harder half and the reason the item asks for both.

6 **SPICY** Correct Ravi

two inputs may share an output — that is allowed

1. The rule restricts what one INPUT may do, not what an output may be shared by.
2. (1,6) and (2,6) is a perfectly good function.
3. What breaks it is ONE input with two outputs.
4. Ravi has stated the reverse property, which is real but different.

Watch for: Agreed with him. Same misconception as the exit ticket's item 2, phrased from the other direction, so a child can miss one and catch the other — read them together.

7 **EXTRA SPICY** Student → date of birth, and the reverse

student → date of birth is a function; the reverse need not be, because two students can share a birthday

1. Each student has exactly one date of birth, so the first direction is a function.
2. Reversed, one date could point to two students — twins, or a coincidence.
3. So a function can fail to be a function when you turn it round.
4. That asymmetry is the whole content of the input/output distinction.

Watch for: Said both, or neither. This item is the definition applied to something real, and the asymmetry is the point — it is the same fact as $y = x^2$ in a setting a child cannot dismiss as a maths trick.

Exit Ticket

4 problems

Use: End of the definition lesson, and the first Grade 8 Functions exit ticket. 5–8 min, independent. Item 3 is the diagnostic and it is worth reading first: a child who agrees with Maya has the definition reversed and will misjudge every non-injective function for the rest of the unit. **Timing:** 5–8 min · independent.

1 yes — it is a function

2 no — it is not a function

3 no — it is a function

4 yes — a formula is not required

1 **MILD** (1,4) (2,6) (3,8) (4,10)

yes — it is a function

1. Every input appears once: 1, 2, 3, 4.
2. So each has exactly one output.

Watch for: Said no because the outputs are all different, or because no rule was given. Neither is the test.

2 **MILD** (0,1) (1,2) (0,4)

no — it is not a function

1. Input 0 appears twice, giving 1 and then 4.
2. One input, two different outputs, so it fails.

Watch for: Said yes by scanning the outputs (1, 2, 4 are all different) instead of the inputs. The test is on the INPUT side.

3 **SPICY** Is Maya right about $y = x^2$?

no — it is a function

1. She has the rule backwards. The definition is that each INPUT has one output.
2. Put in 2 and you get 4, every time. Put in -2 and you get 4, every time. Neither input is ambiguous.
3. Two inputs SHARING an output is allowed. What is forbidden is one input having two outputs.
4. Her version — each output from one input — is a real property, but it is a different one, and most functions do not have it.

Watch for: Agreed with her. This is the central misconception of the topic: the two sentences are nearly identical in words and completely different in meaning. Reading both aloud back to back is the fastest correction.

4 **SPICY** A table with no formula

yes — a formula is not required

1. The definition says nothing about formulas.
2. If each age in the table has exactly one height beside it, it is a function.
3. It fails only if the same age appeared twice with different heights.
4. Six years of “find the rule” makes a formula feel compulsory. It is not.

Watch for: Said no because there is no equation. Worth naming out loud that finding a rule and being a function are different questions.

Slide	Time	What to do
1	0:00	Name both misreadings up front. Nearly every wrong answer on this topic is one of the two, and hearing them early makes them recognisable.
2	1:00	State the definition and say what is NOT in it: no formula, no line, no pattern. Children import all three.
3	2:00	The only failure mode, concretely. One input with two outputs. Write the pairs and circle the repeated input.
4	3:00	THE slide. Say both sentences aloud, back to back, and let the class hear how similar they sound. Then $y = x^2$: two inputs share an output and it is still a function, which is the case the backwards reading gets wrong.
5	4:30	A constant function. Children resist this one because it feels like nothing is happening — but every input has exactly one output, which is the whole test.
6	5:30	No formula required. Worth being explicit: “find the rule” has been the question for six years and it is not the question here.
7	6:30	The vertical line test as a PICTURE of the definition, not a separate fact. Ask why vertical before telling them — a child who can answer that has the definition.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	8:00	Three to sort, then the exit ticket. The circle is the one to discuss. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #5 on the practice sheet.
11		Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12		Launch the practice sheet. Exit ticket in the last 5–8 minutes.