



MATH WITH MR DJ

GRADE 8 • GEOMETRY

WHY PYTHAGORAS WORKS

Today's goal: I can explain a proof of the Pythagorean Theorem and its converse.



Tan



Digit



Blaze



Pi



The claim

- 1 In a right triangle with legs a , b and hypotenuse c : $a^2 + b^2 = c^2$.
- 2 c is the side OPPOSITE the right angle. Always the longest.



BIG IDEA 2

One square, four triangles

- 1 Draw a square of side $a + b$.
- 2 Fit four copies of the right triangle inside, corners out.
- 3 What is left is a square on the hypotenuse.



BIG IDEA 3

Count it with numbers first

1 $a = 3, b = 4$. Big square: $7 \times 7 = 49$.

2 Four triangles: $4 \times 6 = 24$.

3 Left over: $49 - 24 = 25 = 5^2$.



BIG IDEA 4

But that proved **ONE** triangle

- 1 Check 5-12-13. Works. Check 8-15-17. Works.
- 2 Three out of infinitely many.

REMEMBER

Examples are evidence. They are not a proof.



BIG IDEA 5

Now the same picture in letters

- 1 Big square: $(a + b)^2 = a^2 + 2ab + b^2$.
- 2 Four triangles: $4 \times \frac{1}{2}ab = 2ab$.
- 3 Left over: $a^2 + b^2$. And that leftover is c^2 .

REMEMBER

No number appears. So it covers EVERY right triangle at once.



BIG IDEA 6

The converse is a different statement

1 Theorem: right angle $\Rightarrow a^2 + b^2 = c^2$.

2 Converse: $a^2 + b^2 = c^2 \Rightarrow$ right angle.

REMEMBER

The converse is what lets you TEST a triangle. It needs its own reason.



Remember

- 1 $a^2 + b^2 = c^2$, with c the hypotenuse.
- 2 The square proof: $(a + b)^2 - 2ab = a^2 + b^2 = c^2$.
- 3 Letters, not numbers, are what make it a proof.



Pick your pepper!



MILD

State the Pythagorean theorem, saying clearly which side c is.

#1 on your practice sheet



MEDIUM

Do the square count for legs 8 and 15: big square, four triangles, leftover. Check against the hypotenuse.

#2 on your practice sheet



SPICY

Do the square count for legs 5 and 5. What is the hypotenuse squared? What does that tell you about the hypotenuse itself?

#4 on your practice sheet

How's it going?



3 • Got it!

I could teach it



2 • Almost

One more example, please



1 • Not yet

I need some help

NOW YOU!

Practice sheet: everyone does Mild & Medium, then climb the peppers.

Stuck? Look back at Mr DJ's example at the top of the sheet.

Need it again? Watch “The Pythagorean Theorem Explained ($a^2 + b^2 = c^2$)” on YouTube @MathWithMrDJ

Brain break: 3 new daily puzzles at puzzles.mathwithmrdj.com

