

GRADE 8 · EXPRESSIONS & EQUATIONS · ANSWER KEY

WHY SLOPE IS CONSTANT

Today's goal: I can use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane and derive the equation $y = mx$ for a line through the origin and $y = mx + b$ for a line intercepting the vertical axis at b .



How to use this key. Problem numbers match the student sheets. Each answer shows the steps and the slip Mr DJ sees most, so a wrong answer tells you what to reteach. Pepper levels: **Mild** warm-up, **Medium** stretch, **Spicy** challenge, **Extra Spicy** big brain.

Practice

8 problems + Mr DJ's worked example

Use: In-class practice after the deck. Item 5 is the standard itself and it is worth marking hardest: computing two slopes and finding them equal is a demonstration on one case, while the argument has to cover every case. Item 6 is built so the bigger number sits in the position that does not decide steepness. **Timing:** 15–20 min · pairs · graph paper · items 5 and 8 on the board.

1 5 **2** 5 — yes, it must match **3** $y = 3x$ **4** $y = 3x - 4$

5 an explanation that both triangles share the line's angle, so they are similar and their leg ratios are equal

6 $y = 5x + 2$ is steeper **7** yes to both — $-1 + 3x$ is $3x + 1$ rearranged

8 an explanation that a proportional relationship needs $y = mx$, passing through the origin, and the $+3$ means $x = 0$ gives $y = 3$

1 **MILD** (0, -2) to (2, 8)

5

1. Rise 10, run 2.
2. $10/2 = 5$.

Watch for: Inverted to $1/5$, or mishandled the negative y .

2 **MEDIUM** (1, 3) to (4, 18)

5 — yes, it must match

1. Rise 15, run 3, so $15/3 = 5$.
2. It matches, and it had to: both pairs are on the same line.

Watch for: Got a different value and accepted it. Two slopes on one line is a signal to check the arithmetic, not a discovery.

3 **MEDIUM** Through the origin and (4, 12)

$y = 3x$

1. Slope = $12/4 = 3$.
2. Through the origin means $b = 0$, so $y = 3x$.

Watch for: Wrote $y = 3x + 4$, importing a coordinate as the intercept.

4 **MEDIUM** Slope 3, intercept -4

$y = 3x - 4$

1. $m = 3$, $b = -4$.

Watch for: Wrote $y = -4x + 3$, swapping the roles.

5 **SPICY** The similar-triangles argument

an explanation that both triangles share the line's angle, so they are similar and their leg ratios are equal

1. Take any two points and drop a right triangle: horizontal run, vertical rise.
2. Do it again with two different points on the same line.
3. Both triangles have a right angle, and both share the angle the line makes with the horizontal.
4. Two equal angles make the triangles similar.
5. Similar triangles have proportional sides, so $\text{rise} \div \text{run}$ is identical.
6. That ratio is the slope, so the slope cannot depend on which points were chosen.

Watch for: Described computing two slopes and finding them equal. That is a demonstration on one example; the item asks for the argument that covers all of them.

6 **SPICY** $y = 2x + 5$ or $y = 5x + 2$?

$y = 5x + 2$ is steeper

1. Steepness is m , the coefficient of x .
2. $5 > 2$, so $y = 5x + 2$ is steeper.
3. The $+5$ in the other equation moves it UP, not steeper — at $x = 0$ it starts higher and is overtaken.

Watch for: Chose $y = 2x + 5$ because 5 appears and looks large. The position of the number is what matters, not its size, and this pair is built to make that unavoidable.

7 **SPICY** Same line?

yes to both — $1 + 3x$ is $3x + 1$ rearranged

1. The first pair are identical as written.
2. The second pair are the same equation with the terms in a different order: addition can be done either way round.
3. Both describe slope 3 and intercept 1.

Watch for: Said the second pair differ because they look different. Recognising an equation in an unfamiliar arrangement is a real skill and this is where it starts.

8 **EXTRA SPICY** Why $y = 2x + 3$ is not proportional

an explanation that a proportional relationship needs $y = mx$, passing through the origin, and the $+3$ means $x = 0$ gives $y = 3$

1. Proportional means y is a constant multiple of x : $y = mx$.
2. Here $x = 0$ gives $y = 3$, not 0, so the line misses the origin.
3. It is LINEAR — a straight line with a constant slope — but not proportional.
4. Concretely: a taxi with a £3 flag fall. The rate is constant; the total is not a multiple of the distance.

Watch for: Said it is proportional because it is straight. Straight and proportional are different, which is what the “Slope as a unit rate” lesson’s last item was setting up and what this item completes.

Homework

7 problems + Mr DJ’s worked example

Use: Homework after the practice sheet. Item 7 completes a thread begun in the “Slope as a unit rate” lesson: every proportional relationship is linear and not every linear one is proportional. That nesting is the same shape as a square being a rectangle, and a child who found that hard will find this hard for the same reason. **Timing:** 20–25 min · independent · items 5 and 7 first when marking.

1 3 2 $y = 4x$ 3 $y = -2x + 9$ 4 parallel; the gap is 4

5 an explanation that the rise and run both scale up together, so the ratio is unchanged — the triangles are similar

6 $y = 6x - 10$ is steeper 7 it is linear but not proportional — $x = 0$ gives $y = 1$

1 MILD (0,4) to (3,13)

3

1. Rise 9, run 3.

Watch for: Inverted.

2 MEDIUM Origin and (5,20)

$y = 4x$

1. Slope $\frac{20}{5} = 4$, and $b = 0$.

Watch for: Wrote $y = 4x + 5$.

3 MEDIUM Slope -2, intercept 9

$y = -2x + 9$

1. $m = -2$, $b = 9$.

Watch for: Wrote $y = 9x - 2$.

4 MEDIUM $y = 5x + 2$ and $y = 5x - 2$

parallel; the gap is 4

1. Same slope 5, so parallel.

2. Intercepts 2 and -2, so the gap is $2 - (-2) = 4$.

3. Check at $x = 3$: 17 and 13. Difference 4. And at $x = 10$: 52 and 48. Still 4.

Watch for: Said the gap is 0, or 2. Checking at two x values confirms the gap is constant, which is what parallel means.

5 SPICY Why distant points do not give a bigger slope

an explanation that the rise and run both scale up together, so the ratio is unchanged — the triangles are similar

1. Moving further along the line increases the run AND the rise.
2. They increase in the same proportion, because the triangles under the line are similar.
3. A ratio is unchanged when both parts are multiplied by the same amount: $\frac{2}{1}$ and $\frac{20}{10}$ are both 2.
4. So a bigger triangle is not a steeper one.

Watch for: Said “because the slope is always the same”, which is the fact rather than the reason. The both-parts-scale-together argument is what makes it explicable.

6 SPICY $y = 6x - 10$ or $y = 3x + 10$?

$y = 6x - 10$ is steeper

1. Compare the coefficients of x : 6 against 3.
2. The constants -10 and +10 move the lines up and down; they do not change steepness.

Watch for: Chose the second because +10 looks bigger than -10. The constant has nothing to do with steepness.

7 EXTRA SPICY Correct Jo

it is linear but not proportional — $x = 0$ gives $y = 1$

1. Straight line, yes: it is linear.
2. Proportional needs y to be a constant multiple of x , so $x = 0$ must give $y = 0$.
3. Here $x = 0$ gives $y = 1$, so it misses the origin and is not proportional.
4. Every proportional relationship is linear; not every linear one is proportional.

Watch for: Agreed with her. The last sentence is the one worth having — the two categories are nested, not identical, and that is the same one-name-per-thing difficulty as squares and rectangles.

Exit Ticket

4 problems

Use: End of the why-slope-is-constant lesson. 5–8 min, independent. Item 3 is the standard: computing both slopes is item 1, and explaining why they had to agree is the part that makes a line have a slope at all. “Because it is the same line” restates the question. **Timing:** 5–8 min · independent. Graph paper helps item 3.

1 2 both times

2 slope 3, y-intercept 5

3 because the right triangles formed under the same line are similar, so their leg ratios are equal

4 not the same line — same slope, different intercept

1 MILD Slope from two different pairs

2 both times

1. (1,3) to (3,7): rise 4, run 2, so $\frac{4}{2} = 2$.
2. (0,1) to (10,21): rise 20, run 10, so $\frac{20}{10} = 2$.

Watch for: Got different answers, which means an arithmetic slip rather than a conceptual one — on one line they cannot differ.

2 MILD $y = 3x + 5$

slope 3, y-intercept 5

1. $m = 3$ is the coefficient of x .
2. $b = 5$ is the constant, and the line crosses the y -axis at (0,5).

Watch for: Swapped them, giving slope 5 and intercept 3. Substituting $x = 0$ settles which is which: $y = 5$.

3 SPICY Why they had to match

because the right triangles formed under the same line are similar, so their leg ratios are equal

1. Drop a right triangle under each pair of points.
2. Both triangles sit under the SAME line, so they have the same angle at the bottom.
3. Triangles with equal angles are similar — same shape, different size.
4. Similar triangles have equal ratios of corresponding sides, and slope IS that ratio.
5. So the rise and run grow together and the ratio cannot change.

Watch for: Said “because it is the same line”, which restates the question. Or said “because I calculated both”, which is evidence rather than a reason. The similar-triangles argument is the content of this standard.

4 SPICY $y = 4x$ and $y = 4x - 7$

not the same line — same slope, different intercept

1. Both have $m = 4$, so both climb at the same rate. They are PARALLEL.
2. The intercepts differ: 0 and -7 .
3. So the second is the first slid down 7. At every x the gap is exactly 7 — at $x = 5$, 20 against 13.
4. Same steepness, different position. Changing b slides a line; changing m would tip it.

Watch for: Said yes because the $4x$ matches, or said the lines are completely unrelated. The useful answer names both what is shared and what is not.

Lesson slides

teaching notes & timing

Slide	Time	What to do
1	0:00	Pose it as a genuine question, because it is one. Nothing so far has told them a line has only one slope.
2	1:00	Let the suspicion stand for a moment. Both the rise and the run are bigger for the distant pair, so expecting a bigger slope is reasonable.
3	2:00	Compute both and see them agree. Now they want to know why.
4	3:00	THE slide. Similar triangles: the line fixes the angle, the angle fixes the shape, and similar shapes have equal side ratios. This is the standard, and it is the difference between a rule and a reason.
5	4:30	Derive $y = mx$ from the origin case. One line of algebra.
6	6:00	Then $y = mx + b$. Start at b , climb at m .
7	7:30	b slides, m tips. Graph $y = 2x$ and $y = 2x + 1$ together and point at the constant vertical gap.
8		Show the problem and the slip. Pairs: what went wrong, and what is the right answer? Take two answers before moving on.
9		Solve it together, one step at a time. It is also the worked example at the top of the practice sheet, so students can look back at it.
10	9:00	

Two pairs on one line, then the exit ticket. Whiteboards: everyone tries Mild, then climbs. These are #1, #2, #5 on the practice sheet.

11	Fingers up: 3 = got it, 2 = almost, 1 = not yet. Pull the 1s for a quick reteach while everyone else starts the practice sheet.
12	Launch the practice sheet. Exit ticket in the last 5–8 minutes.
